Two-Phase Flow Instabilities in Boiling and Condensing Systems*

Leonardo C. RUSPINI**, Carlos DORAO** and Maria FERNANDINO**

** Department of Energy and Process Engineering
Faculty of Engineering Science and Technology, Norwegian University of Science and Technology
E-mail: leonardo.c.ruspini@ntnu.no

Abstract
In this work Density Wave Oscillations (DWO) and Ledinegg instabilities are analyzed for boiling and condensing systems in a single tube. The analysis is based on a numerical model solved with a least squares spectral element method which is characterized by negligible numerical diffusion and high accuracy. Stability limits are constructed and compared with available correlations. The analysis is extended to sub-cooled, saturated and over-heated inlet conditions. Finally a discussion regarding the occurrence of these phenomena in condensing systems is presented.

Key words: Instabilities, Density Waves, Ledinegg, Boiling, Condensing, Two-Phase Flows

1. Introduction
The departure from stable operation conditions can cause severe damages in many industrial systems, such as heat exchangers, nuclear reactors, re-boilers, steam generators, thermal-siphons, etc. Oscillations and instabilities induced by boiling two-phase flows are of relevance for the design and operation of these systems. For this reason, the stability in thermal-hydraulic variables such as flow, pressure and temperature should be studied in detail to better understand and characterize the conditions for the occurrence of these phenomena. In particular Ledinegg and Density Wave Oscillations are the two most studied instabilities due to their occurrence and relevance.

Ledinegg instability(21) is considered the most common type of static instability. The occurrence of this instability is related to the slope of the pressure drop vs. flow characteristic curve of the system. There is considerable research related to stability analysis in two-phase flow systems, for example: heat exchanger stability analysis(10); stability analysis of boiling nuclear reactors(19); micro-channels study of Ledinegg instability(36); and some other stability margins prediction for general two-phase flow problems(13), (23), (25), (36). Recently the determination of the dynamic evolution of the instability was discussed by(28).

The phenomenon called Density Wave Oscillations (DWO), or thermally induced two-phase flow instability, is the most common type of dynamic instability occurring in real systems. There exist several experimental works describing the occurrence of this phenomenon (9),(14),(31),(33)--(35). Regarding the modeling and theoretical background of density waves oscillations, the investigations carried out in(14),(15) constitute the theoretical basis in the understanding of density wave phenomenon. In these works a thermal equilibrium model is used to describe the system in a one-dimensional model. The use of the Ishii-Zuber stability map, introduced in this work, continues being the most used kind of stability map for two-phase flow stability analysis. In(31) the use of a non-equilibrium theory is proposed. For low sub-cooling this model seems to fit better to the experimental data. Nevertheless, in the high sub-cooling cases the equilibrium model fits better the experimental data. In(11) the validity of the homogeneous model is discussed. In this last work several pressure, sub-cooling and heat capacity models are compared with experimental data. It is proven that in general terms...
The best approximation is made with no sub-cooling model and heat capacity of the wall when that mechanism could be important (massive tubes). More recently\(^{(26)}\), a homogeneous equilibrium (no sub-cooling) model is used to study the phenomenon. Several aspects of the classical theoretical description of DWO are critically discussed. The introduction of non-uniform heating is discussed in\(^{(24)}\), \(^{(26)}\). The use of commercial codes and simplified lumped methods based on the homogeneous equilibrium models are described in\(^{(1)}\) – \(^{(3)}\), \(^{(20)}\). According to all of these works, the responsible mechanisms of the oscillatory behavior are the change in the exit density (related with the spatial slope of the density profile) and the change of the exit flow. For different sub-cooling conditions those effects have a stronger or weaker influence over the system. However it is still not clear how the density wave mechanisms affect the systems for condensing and saturated boiling flows. In\(^{(17)}\) an experimental example and some numerical simulations show the occurrence of oscillations in the condensing secondary side of a heat exchanger, stating that condensation processes are similar to the boiling processes in sense of occurrence of density wave oscillations. However no theoretical analysis is presented to support the previous statement. Moreover several works describe the occurrence of oscillations in condensing systems\(^{(5)}\) – \(^{(7)}\), \(^{(22)}\). According to these experimental works the nature of the oscillations seems to be very different from density waves phenomena, regarding the time periods.

The purpose of this work is to study how the involved mechanisms in density wave and Ledinegg phenomena affect condensation and boiling systems with sub-cooled, saturated and over-heated inlet conditions. A homogeneous model is presented to study the nature of these different phenomena for different operation regions. Non-dimensional stability maps, \(N_{Zu} vs N_{sub}\), are constructed and several simulations show the behavior of a simple thermal-hydraulic system. In addition the application of a high-order method to solve the conservation equations is described and implemented.

2. Model

The thermal-hydraulic system used to study these kinds of instabilities consists of two constant pressure tanks, two valves and a horizontal heated section, as shown in Figure 1. In this model, the pressure difference between both tanks acts as the driving force (external characteristic) and, according to the \(K_v\) valve opening, the external characteristic results in a quadratic decreasing curve. The implemented model is based on the following assumptions,

- One-dimensional model.
- Thermodynamic equilibrium conditions.
- Two-phase homogeneous model.

The mathematical model used to describe the thermal-hydraulic evolution of the heated section is based on the mass, momentum and energy conservation. They can be expressed as

\[
\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial z} = 0
\]  

(1)
The pressure drop in the valves is calculated using a homogeneous pressure drop concentrated value, $K_i$, for each valve (in, out). Friction losses are neglected in the energy equation. Finally the friction factor; $f$ in Eq. (2) is given by the known Colebrook correlation for the single phase regions, liquid or gas, and by the Müller-Steinhagen and Heck correlation for the two-phase region\(^{(32)}\). In contrast with previous models\(^{(15),(31)}\), in this work the profile of density is updated according to the local enthalpy. For this updating a thermodynamic equilibrium is assumed.

3. Numerical approximation

The introduction of numerical diffusion by low-order methods can modify the nature of the problem and this issue is particular relevant when studying thermal-hydraulic problems\(^{(4),(29)}\). The least squares formulation provides a numerical discretization with negligible numerical diffusion. The details of the implementation of this method can be found in\(^{(28),(29)}\).

In a general case the least squares formulation is based on the minimization of a norm-equivalent functional. For simplicity, the system of equations can be represented as

\[
\begin{align*}
\mathcal{L}u &= g \quad \text{in} \quad \Omega \\
B u &= u_f \quad \text{on} \quad \Gamma \subset \partial \Omega
\end{align*}
\]  

(4)

(5)

with $\mathcal{L}$ a linear partial differential operator and $B$ the trace operator. We assume that the system is well-posed and the operator $(\mathcal{L}, B)$ is a continuous mapping between the function space $X(\Omega)$ onto the space $Y(\Omega) \times Y(\Gamma)$. The norm equivalent functional becomes

\[
\mathcal{J}(u) \equiv \frac{1}{2} \| \mathcal{L}u - g \|_{Y(\Omega)}^2 + \frac{1}{2} \| B u - u_f \|_{Y(\Gamma)}^2
\]

(6)

Based on variational analysis, the minimization statement is equivalent to:

\[
\lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} \mathcal{J}(u + \varepsilon v) = 0 \quad \forall v \in X(\Omega)
\]

(7)

Hence, the necessary condition for the minimization of $\mathcal{J}$ is equivalent to:

Find $f \in X(\Omega)$ such that

\[
\mathcal{A}(u, v) = \mathcal{F}(v) \quad \forall v \in X(\Omega)
\]

(8)

with

\[
\begin{align*}
\mathcal{A}(u, v) &= (\mathcal{L}u, \mathcal{L}v)_{Y(\Omega)} + (Bu, Bv)_{Y(\Gamma)} \\
\mathcal{F}(v) &= (g, \mathcal{L}v)_{Y(\Omega)} + (u_f, Bv)_{Y(\Gamma)}
\end{align*}
\]

(9)

(10)
where $A : X \times X \to \mathbb{R}$ is a symmetric, continuous bi-linear form, and $F : X \to \mathbb{R}$ a continuous linear form.

The introduction of the boundary residual allows the use of spaces $X(\Omega)$ that are not constrained to satisfy the boundary conditions. The boundary terms can be omitted and the boundary conditions must be enforced strongly in the definition of the space $X(\Omega)$. Finally, the searching space is restricted to a finite dimensional space and the problem becomes:

Find $u_h \in X_h(\Omega) \subset X(\Omega)$, such that

$$A(u_h, v_h) = F(v_h) \quad \forall v_h \in X_h(\Omega)$$  \hspace{1cm} (11)

### 3.1. Numerical description of the problem

From Eqs. (1-3) it is possible to see that the system is highly non-linear. For that reason it is necessary to find a linear form for this set of equations in order to use LSSM (Least Square Spectral Method). Non-linear effects are considered by implementing an iterative Picard loop. Hence, using the operator description of Eq. (4), it is possible to rewrite the linearized system, described in Eqs. (1-3), as

$$L = \begin{bmatrix}
\frac{\partial \rho}{\partial t}
& 0 & 0 \\
0 & \frac{\partial \rho}{\partial t} + G^* \frac{\partial \rho}{\partial z}
& 0 \\
0 & 0 & \rho^* 
\end{bmatrix}$$  \hspace{1cm} (12)

$$g = \begin{bmatrix}
-\frac{\partial \rho^*}{\partial t} \\
-\frac{\partial}{\partial z} \left( G^* \rho^* \right) \\
\frac{f}{D_H} + \sum_i K_i \delta(x - x_i)
\end{bmatrix}$$  \hspace{1cm} (13)

$$u = \begin{bmatrix}
G \\
P \\
h
\end{bmatrix}$$  \hspace{1cm} (14)

where $G^*$ and $\rho^*$ correspond to the old values of flow and density in the non-linear loop. The operator $B$ is the matrix where the corresponding initial and boundary nodes are set to one and the $u_Γ$ corresponds to the initial values for flow, pressure and enthalpy. Boundary conditions for enthalpy and pressure are set in to the boundary vector $u_Γ$ at $x = 0$ and the third boundary condition, outlet pressure, is set in $u_Γ$ at $x = L$.

### 3.2. Spectral element approximation

The computational domain $\Omega$ is divided into $N_e$ non-overlapping sub-domains $\Omega_e$ of diameter $h_e$, called spectral elements, such that

$$\Omega = \bigcup_{e=1}^{N_e} \Omega_e, \hspace{1cm} \Omega_e \cap \Omega_l = \emptyset, \hspace{0.5cm} e \neq l$$  \hspace{1cm} (15)

The global approximation in $\Omega$, $u_h$, is constructed by gluing the local approximations $u_{h_e}^*$, i.e.

$$u_h = \bigcup_{e=1}^{N_e} u_{h_e}^*$$  \hspace{1cm} (16)

The local approximation solution $u_{h_e}^*$ can be expressed like

$$u_{h_e}^*(x, t) = \sum_{j=0}^{N_h} \sum_{i=0}^{N_h} u_{ij}^* \varphi_i(x) \varphi_j(t), \hspace{0.5cm} \text{with} \hspace{0.5cm} u_{ij}^* = u(x_i, t_j)$$  \hspace{1cm} (17)

where $\varphi_i(x)$ and $\varphi_j(t)$ are the one dimensional basis functions. The basis functions used in this work are the Lagrangian interpolants polynomials through the Gauss–Lobatto–Legendre (GLL) collocation points, see\cite{8}.
4. Stability analysis

The system stability is analyzed by the construction of a non-dimensional stability map. In this work the sub-cooling and phase-change (Zuber) numbers are used\(^{(14)}\). They correspond with

\[
N_{\text{sub}} = \frac{h_f - h_{in}}{h_{fg}} \frac{\rho_f}{\rho_g} \\
N_{\text{Zu}} = N_{\text{pch}} = \frac{Q}{GA_{s}h_{fg}} \frac{\rho_f}{\rho_g}
\]

\[(18)\]

To evaluate the stability of the numerical solution the evolution of the inlet flow is fitted with the function,

\[
f(t) = Ae^{-\alpha t} \sin(\beta t + \gamma) + B
\]

\[(19)\]

then a stability criterion could be defined as

\[
\alpha > 0 \quad \text{Stable} \quad \alpha < 0 \quad \text{Unstable}
\]

\[(20)\]

Figure 2 shows two different simulations and its corresponding fitted curve, Eq. (19). These simulations correspond respectively with a stable and an unstable cases. For these two cases the \((N_{\text{pch}}, N_{\text{sub}})\) pairs are (5.2, 1.1) and (10, 0.6) respectively.

5. Numerical results

The numerical order of approximation of time and space was \(N_x = N_t = 4\). The number of elements in which the space is discretized is \(N_e = 50\) and the time step is \(\Delta t = 10^{-2}\) sec for all the cases. The non-linear relative error tolerance for the Picard loop is \(10^{-6}\). All the simulations in this section are done in a system with the following characteristics:

- Fluid: R134a
- \(L = 1\) m, \(D_H = 5\) mm
- \(P_{out} = 8 \times 10^5\) Pa, \(P_{in} = P_{\text{stationary}}(G_{in} = 500 \text{ [kg/m}^2\text{s]})\)
- \(K_{in} = 2, K_{out} = 1\)
5.1. Boiling region

Figure 3 shows a three-dimensional map of the stability function $\alpha$ as a function of $(N_{pch}, N_{sub})$. This plot is generated by a total number of 355 simulations where according to the evolution of the system the $\alpha$ coefficient is computed. The boiling region is defined for positive $N_{pch}$ numbers.

It is observed that for low negative sub-cooling numbers ($N_{sub} < -5$) the system response becomes stable and with a fast time response ($\alpha > 1$). This situation corresponds with a high-saturated ($x_{in} > 0.5$) and over-heated inlet conditions. The stability of this region is due to the low density change from the saturated to the over-heated states. On the other hand, for $N_{sub} > -5$ the stability of the system decreases and in some regions alpha becomes negative. The stability of the low saturated and sub-cooled region is analyzed in the following sections.

Sub-cooled inlet region  

The stability map for the sub-cooled region is presented in Figure 4. In this plot instabilities due to Ledinegg and density wave instabilities are observed. For the cases of Ledinegg excursion $\alpha$ is imposed to $\alpha = -0.1$ and no fitting is made. An example of Ledinegg excursion is plotted in Figure 5, where a the system evolution to a new operation point is observed. The classical limit introduced by Guido(12) for Ledinegg instability is plotted in red line. It does not fit the boundary found in this work for any sub-cooled region. This is probably due to the simplifications used in Guido’s lumped model, such as not friction in the pipe, constant liquid and vapor densities, linearization of the system, etc. The most common DWO stability limits are also presented in the figure. The simplified correlation presented by Ishii(14) seems to predict conservatively the limit of stability for DWO. Nevertheless the differences between Ishii’s correlation and the limit obtained in this work could be due
to the density profile used in present model. In contrast with Ishii’s model, where the liquid density and vapor density are assumed constant, in this work the density profile of the fluid, in the single- and two-phase regions, is updated according to the local enthalpy and pressure. Saha’s limit, introduced in (31), for low subcooling numbers is also plotted. It is necessary to add that this limit was constructed in order to describe the system behavior in the case of sub-cooling boiling. Even when the model used in this work does not take into account the occurrence of sub-cooling boiling, this limit was presented as conservative even for the case when sub-cooling boiling is not considered. Moreover this limit is not an independent criteria since it is necessary to give the critical $N_{pch}$ for which $N_{sub} = 0$. In conclusion the usage of this limit does not seem a valuable tool for predicting the occurrence of DWO in low subcooling systems. The simplified Guido’s limit for DWO is also presented in the figure. It does not predict accurately the unstable limit for DWO. The main reason of this notable discrepancy is that in this last model the friction losses in the pipe are neglected. Consequently, the results of these criteria (Ledinegg and DWO) are only valid in those cases where the local pressure losses (inlet and outlet valves) are much more higher than the pressure loss in the pipe.

**Saturated inlet region** In Figure 6 the stability map of the saturated inlet region is presented. This region is characterized by a low stability. Oscillatory behavior is observed. In general terms, the tendency of the stability limit follows the same trend as in the low part of the sub-cooled region ($N_{sub} > 0$). An inflection point in the stability limit is observed when the outlet reaches the single phase gas conditions, red line on Figure 6. This behavior change corresponds to the difference in the local pressure drop at the exit restriction between the single gas and the two-phase flow cases. An extrapolation of Saha’s limit for DWO is plotted in the figure. For $N_{sub} < 0$, it does not exist any subcooling boiling in the system (saturated inlet) and the simplified limit predicted by Saha does not fit the limit obtained in this work. In conclusion, it is possible to say that this limit does not give an accurate representation of the main phenomena occurring in the low sub-cooling region.
**5.2. Condensing region**

An example of the response of condensing systems is presented in Figure 7. In this case, in contrast with the behavior of a boiling system, the system evolves in an over-dumped way. An exponential curve is fitted to analyze the stability of the system. The stability map for the condensation region is presented in Figure 8. In all the cases the system behaves in a stable fashion, \( \alpha > 0 \). Moreover the minimum value for the parameter \( \alpha \) is 3.5, Figure 8. A total of 250 cases have been simulated in order to obtain the stability map for condensing systems. In this stability map the cases of \( N_{\text{sub}} \) between 2 and -50 are analyzed. The region with positive sub-cooling number is not analyzed since it corresponds with the just liquid region. No Ledinegg instability is observed either. A discussion of the nature of DWO in condensing systems is presented in the next section.

**6. Discussion**

**Density wave oscillations:** As described in\(^{(27)}\) for the sub-cooled inlet conditions, density wave oscillations are mainly the consequence of the regenerative feedback between friction and delay effects. Note that even when inertia effects are important they do not define the nature of the oscillation\(^{(18)}\). In general terms it is well known that friction losses at the inlet valve stabilize the system while the outlet friction losses destabilize the system. In addition, it is also known that the variations in the flow and density at the exit restriction control the oscillation occurrence, since a constant pressure is imposed to the system. While changes on density produce a positive feedback (destabilizing), the changes on the exit flow tend to stabilize the system, as described in\(^{(27)}\). Furthermore it can be easily seen that density profile slopes of opposite sign will affect the system in opposite way, since the same perturbation will produce an opposite reaction in the two-phase friction losses (pipe and valve). For example, in the case of positive slope (cooling) any increase in the flow produces a decrease in the density at the outlet, while in the case of negative slope (heating) any increase of the flow produces an increase in the exit density. In Figure 9 the density profiles for the boiling (heating) and condensing (cooling) cases are presented. For these cases the gradient of the density profile monotonously fulfills...
Fig. 8 Stability map of \( \alpha \) as a function of \((N_{pch}, N_{sub})\). In all the cases \( \alpha \) is positive and then the system is stable. The blue points correspond with the simulations.

Fig. 9 Density profiles for boiling and condensing cases. Boiling case \((N_{pch} = 19, N_{sub} = 5)\); Condensing \((N_{pch} = -19, N_{sub} = -15)\).

\[
\frac{\partial \rho}{\partial z} < 0 \quad \text{(Heating case)} \tag{21}
\]

\[
\frac{\partial \rho}{\partial z} > 0 \quad \text{(Cooling case)} \tag{22}
\]

and these statements are valid for any distribution of heat and any inlet condition (sub-cooled, saturated or over-heated). Consequently due to the characteristic positive slope of the density profile for condensing flows, the outlet two-phase friction terms will have a stabilizing effect. Thus the system will become stable in the density wave sense, as shown by the simulations of condensing systems in the Figure 8. This last conclusion is also in accordance with the experimental data reported in \((5),(6)\), where it is shown that the oscillations in condensing systems are not according with density waves and they seem to be related with the amount of compressible volume in the system such as pressure drop oscillations for boiling systems \((9),(16)\). The main difference with the oscillations occurring in those systems are the characteristic periods of the oscillations.

**Ledinegg instability:** Regarding the occurrence of Ledinegg instability it is necessary to understand shape of the pressure drop vs. flow characteristic curve of the system. Steady simulations for both, boiling and condensing systems are presented in Figure 10. The system is equivalent to the one described in the previous sections and the inlet temperatures are \(-23^\circ C\) and \(107^\circ C\) for the boiling and condensing cases, respectively. For the boiling case when the flow tends to zero then the total pressure drop tends to the “all vapor” curve. In contrast, when the flow tends to infinite then the pipe is full of subcooled liquid and then the total pressure drop tends to the “all liquid” case. As the vapor have a higher pressure drop than the liquid, then in the two-phase flow transition the characteristic curve of the system tends to follow trend and in consequence for some cases the N-shape is observed. On the other hand for a condensing system when the flow tends to zero then the total pressure drop will tend to the
“all liquid case” and it will tend to the “all vapor” case when the flow tends to infinite. As it is possible to see in Figure 10, the pressure drop for the “all vapor” case is always greater, 2 orders of magnitude, than the ”all liquid” case. In consequence the characteristic curve for the condensing systems will show a steeper slope than the boiling case. Even when locally the two-phase flow pressure drop could be bigger than the just vapor case, the total pressure drop of the system can not be bigger than the “all vapor” case. As shown in the figure, the characteristic curve for condensing systems can not present the N-shape of boiling systems and in conclusion condensing systems can not have the problem of Ledinegg instability.

7. Conclusions

Density wave and Ledinegg phenomena are analyzed for a boiling and a condensing single tube. A high-order spectral method is used to solve the constitutive equations of the system avoiding the diffusion of low order methods. Ledinegg and density wave stability limits are analyzed for boiling and condensing systems. A comparison with previous density wave stability criteria is made. In general terms only Ishii’s simplified criterion for density wave oscillations is predicting accurately the stability limits. A saturated inlet case is analyzed, none of the stability correlations describe the stability limit in this case. The Ledinegg limits are also characterized and a significant difference between the limit obtained here and the classical criteria is observed. A condensing system is also analyzed. No unstable oscillatory behavior is observed. A discussion of the occurrence of density wave oscillations and Ledinegg phenomena in condensing systems is presented. It is found that due to the characteristic positive slope of the density profile it is not possible to find density wave phenomenon in such systems. It is also proven that Ledinegg instabilities are not occurring in condensing systems.

References


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