Computational Study of Turbulent Heat Transfer for Heating of Water in a Vertical Circular Tube*  
–Influence of Tube Inner Diameter on Thickness of Conductive Sub-Layer–

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Abstract
The steady-state turbulent heat transfer coefficients in a vertical circular Platinum (Pt) test tube for the flow velocities ($u = 4.22$ to 21.45 m/s), the inlet liquid temperatures ($T_{in} = 308.20$ to 311.77 K), the inlet pressures ($P_{in} = 834.04$ to 910.23 kPa) and the increasing heat inputs ($Q_0 \exp(t/\tau)$, exponential periods, $\tau$, of 6.04 to 32.13 s) were systematically measured by an experimental water loop comprised of a multistage canned-type circulation pump with high pump head. Measurements were made on Pt test tubes of 3, 6 and 9 mm inner diameters, 32.7, 69.6 and 49.6 mm heated lengths and 0.5, 0.4 and 0.3 mm thicknesses, respectively. Theoretical equations for turbulent heat transfer in circular tubes of 3, 6 and 9 mm in diameter and 492, 636 and 616 mm long were numerically solved for heating of water with heated sections of 3, 6 and 9 mm in diameter and 33, 70 and 50 mm long by using PHOENICS code under the same condition as the experimental one considering the temperature dependence of thermo-physical properties concerned. The surface heat flux, $q$, and the average surface temperature, $T_{s,av}$, on the circular tubes solved theoretically under the flow velocities, $u$, of 4.22 to 21.45 m/s were compared with the corresponding experimental values on heat flux, $q$, versus the temperature difference between average inner surface temperature and liquid bulk mean temperature, $\Delta T_L = (T_{in} + T_{out})/2$, graph. The numerical solutions of $q$ and $\Delta T_L$ are almost in good agreement with the corresponding experimental values of $q$ and $\Delta T_L$ with the deviations less than $\pm 10\%$ for the range of $\Delta T_L$ tested here. The numerical solutions of local surface temperature, $(T_s)_z$, and local average liquid temperature, $(T_f,av)_z$, are within $\pm 10\%$ of the corresponding experimental data on $(T_s)_z$ and $(T_f,av)_z$. The thickness of the conductive sub-layer, $\delta_{CSL} = (\Delta r)_{out}/2$, and the non-dimensional thickness of conductive sub-layer, $y^{+}_{CSL} = (f F/2)^{0.5} \rho u \delta_{CSL}/\mu$, for the turbulent heat transfer in various vertical tubes are clarified based on the numerical solutions. It was confirmed in this study that authors’ steady-state turbulent heat transfer correlation based on the experimental data [Hata and Noda, 2008] can not only describe the experimental data of steady-state turbulent heat transfer but also the numerical solutions within $\pm 10\%$ difference for the wide ranges of tube inner diameters (3, 6 and 9 mm), temperature differences between average inner surface temperature and liquid bulk mean temperature ($\Delta T_L = 8$ to 145 K) and flow velocities ($u = 4.22$ to 21.45 m/s).

Key words: Computational Study, Turbulent Heat Transfer, Heating of Water, Tube Inner Diameter, Thickness of Conductive Sub-Layer
1. Introduction

Thicknes of conductive sub-layer for turbulent heat transfer for heating of water in a vertical circular tube is important as a detailed knowledge of turbulent heat transfer in pipes and a database for the design of a diverter in a nuclear fusion facility. However, there have been little fundamental work for thickness of conductive sub-layer and little is known about the influence of tube inner diameter on thickness of conductive sub-layer.

Many researchers have experimentally studied the steady-state turbulent heat transfer in pipes and given the correlations for calculating steady-state turbulent heat transfer coefficients (1-6).

- Hata and Noda (1):
  \[ N_u = 0.02 \operatorname{Re}_d^{0.85} \operatorname{Pr}^{0.4} \left( \frac{l_{\text{eff}}}{d} \right)^{-0.08} \left( \frac{\mu_l}{\mu_w} \right)^{0.14} \]

- Dittus and Boelter (2):
  \[ N_u = 0.023 \operatorname{Re}_d^{0.8} \operatorname{Pr}^{0.4} \]

- Nusselt (3):
  \[ N_u = 0.036 \operatorname{Re}_d^{0.8} \operatorname{Pr}^{1/3} \left( \frac{\mu_l}{\mu_w} \right)^{0.14} \]

- Sieder and Tate (4):
  \[ N_u = 0.027 \operatorname{Re}_d^{0.8} \operatorname{Pr}^{1/3} \left( \frac{\mu_l}{\mu_w} \right) \]

- Petukhov (5):
  \[ N_u = \frac{(f/2)\operatorname{Re}_d \operatorname{Pr}}{1.07 + 12.7(f/2)^{1/3}(\operatorname{Pr}^{2/3} - 1)} \]

- Gnielinski (6):
  \[ N_u = \frac{(f/2)(\operatorname{Re}_d - 1000)\operatorname{Pr}}{1 + 12.7(f/2)^{1/3}(\operatorname{Pr}^{2/3} - 1)} \]

All properties in these equations are evaluated at the liquid bulk mean temperature, \( T_{\text{L}} \) \( = (T_{\text{in}} + T_{\text{out}})/2 \), except \( \mu_w \), which is evaluated at the wall temperature.

Quite recently, the authors have systematically measured the steady-state turbulent heat transfer coefficients in a vertical circular Platinum (Pt) test tube for the flow velocities \( (u=4.11 \text{ to } 41.07 \text{ m/s}) \), the inlet liquid temperatures \( (T_{\text{in}}=296.47 \text{ to } 310.04 \text{ K}) \), the inlet pressures \( (P_{\text{in}}=810.40 \text{ to } 1044.21 \text{ kPa}) \) and the increasing heat inputs \( (Q_0 \exp(\tau/t)) \), \( \tau \) of 6.04 to 23.66 s). Measurements were made on a 59.2 mm effective length and its three sections (upper, mid and lower positions), which were spot-welded four potential taps on the outer surface of the Pt test tube of a 6 mm inner diameter, a 69.6 mm heated length and a 0.4 mm thickness. Theoretical equations for turbulent heat transfer in a circular tube of a 6 mm in diameter and a 636 mm long were numerically solved for heating of water with heated section of a 6 mm in diameter and a 70 mm long by using PHOENICS code under the same condition as the experimental one considering the temperature dependence of thermo-physical properties concerned. The thickness of the conductive sub-layer, \( \delta_{\text{CSL}} \) \( = \langle \Delta r \rangle_{\text{out}}/2 \), and the non-dimensional thickness of conductive sub-layer, \( y_{\text{CSL}} \) \( = (f/2)^{1/3} \rho \mu \langle \Delta r \rangle_{\text{out}}/\mu_w \), for the turbulent heat transfer in a vertical circular tube under velocities controlled are clarified based on the numerical solutions. It was confirmed in this study that authors’ steady-state turbulent heat transfer correlation based on the experimental data (1) can not only describe the experimental data of steady-state turbulent heat transfer but also the numerical solutions within \( \pm 10 \% \) difference for the wide ranges of temperature differences between average inner surface temperature and liquid bulk mean temperature \( (\Delta T_{\text{L}}=5 \text{ to } 200 \text{ K}) \) and flow velocities \( (u=4.01 \text{ to } 41.07 \text{ m/s}) )^{(7-9)} \).

The present work has the following objectives: (1) to measure the steady-state turbulent heat transfer coefficients on Platinum test tubes of 3, 6 and 9 mm inner diameters divided into three sections (upper, mid and lower positions) for the wide ranges of test tube surface temperature \( (T_s) \) and flow velocity \( (u) \), (2) to obtain the numerical solutions of surface heat flux, \( q_s \), and surface temperature, \( T_s \), on the heated length, and liquid temperature, \( T_L \), and flow velocity, \( u \), in the 3-, 6- and 9-mm inner diameters from theoretical equations for turbulent heat transfer under the same condition as the experimental one, (3) to compare above results with experimental data of surface heat flux, \( q_s \), local surface temperature, \( (T_s) \), and local average liquid temperature, \( (T_L) \), (4) to clarify the thickness of conductive sub-layer, \( \delta_{\text{CSL}} \)\( \langle \Delta r \rangle_{\text{out}}/2 \), and the non-dimensional thickness of...
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4. Experimental apparatus and method

2.1 Experimental water loop

The schematic diagram of experimental water loop comprised of the pressurizer is shown in Fig. 1. The loop is made of SUS304 stainless steel and is capable of working up to 2 MPa. The loop has five test sections whose inner diameters are 2, 3, 6, 9 and 12 mm. Test sections were vertically oriented with water flowing upward. The test sections of the inner conductive sub-layer, \( y_{CSL} = \frac{(fF/2)^{0.5} \rho u \delta_{CSL}}{\mu} \), for the turbulent heat transfer in Platinum test tubes of 3, 6 and 9 mm inner diameters, and (5) to discuss the influence of tube inner diameter on the thickness of conductive sub-layer, \( \delta_{CSL} \), and the non-dimensional thickness of conductive sub-layer, \( y_{VBL} \), for turbulent heat transfer in a vertical circular tube.

Nomenclature

\( C_{1e}, C_{2e}, C_{3e}, C_{d}, C_{\mu} \) coefficients in approximated turbulent transport equations

\( d \) test tube inner diameter, m

\( f_F \) Fanning friction factor

\( G = \rho u \) mass velocity, kg/m\(^2\)s

\( L \) heated length, m

\( L_{eff} \) effective length, m

\( l_m \) length of energy containing eddies, m

\( N_{It} = \frac{h d}{\lambda_{t}} \) nusselt number

\( P_{in} \) pressure at inlet of heated section, kPa

\( P_{out} \) pressure measured by outlet pressure transducer, kPa

\( P_{out} \) pressure at outlet of heated section, kPa

\( P_{opt} \) pressure measured by outlet pressure transducer, kPa

\( \Pr \) \( = c_{\mu}/\lambda_{t} \) Prandtl number

\( \Pr_t \) \( = c_{\mu}/\lambda_{t} \), turbulent Prandtl number

\( Q \) heat input per unit volume, W/m\(^3\)

\( Q_0 \) initial exponential heat input, W/m\(^3\)

\( q \) heat flux, W/m\(^2\)

\( Ra \) average roughness, \( \mu m \)

\( Re_d = \frac{G d}{\mu} \) Reynolds number

\( r_i \) test tube inner radius, m

\( r_o \) test tube outer radius, m

\( \Delta r_{out} \) outer control volume width for \( r \)-component, m

\( TEM \) analyzed temperature of the outer control volume, K

\( T_{in} \) average liquid temperature, K

\( T_{out} \) outlet liquid temperature, K

\( T_{in} \) inlet liquid temperature, K

\( T_{out} \) outlet liquid temperature, K

\( T_{cal} \) calculated outlet liquid temperature, K

\( T_{s} \) heater inner surface temperature, K

\( T_{ave} \) average inner surface temperature, K

\( \Delta T_L = (T_{ave}-T_{TL}) \), temperature difference between average inner surface temperature and liquid bulk mean temperature, K

\( u \) flow velocity, m/s

\( v_r \) \( r \)-component of a velocity vector

\( v_\theta \) \( \theta \)-component of a velocity vector

\( v_z \) \( z \)-component of a velocity vector

\( y_{CSL} = \frac{(fF/2)^{0.5} \rho u \delta_{CSL}}{\mu} \), non-dimensional thickness of conductive sub-layer

\( \Gamma_k \) volumetric production rate of \( k \) by gravitational force

\( \delta_{CSL} = \frac{(\Delta r)_{out}}{2} \), thickness of conductive sub-layer

\( \varepsilon \) emissivity and rate of dissipation of turbulent energy, m\(^2\)/s\(^3\)

\( \lambda_{e} \) \( = \lambda_{t} + \lambda_{l} \), effective turbulent thermal conductivity, W/mK

\( \mu_e \) \( = \mu + \mu_{l} \) effective turbulent viscosity, Ns/m\(^2\)

\( \mu_l \) viscosity of liquid, Ns/m\(^2\)

\( \mu_t \) turbulent viscosity, Ns/m\(^2\)

\( \mu_{w} \) viscosity at tube wall temperature, Ns/m\(^2\)

\( \sigma_{\varepsilon} \) effective turbulent Prandtl number for transport of \( \varepsilon \)

\( \sigma_{\mu} \) effective turbulent Prandtl number
diameters of 3, 6 and 9 mm were used in this work. The circulating water was distilled and deionized with about 20 μS/m specific resistivity. The circulating water through the loop was heated or cooled to keep a desired inlet temperature by pre-heater or cooler. The flow velocity was measured by a mass flow meter using a vibration tube (Nitto Seiko, CLEANFLOW 63FS25, Flow range=100 and 750 kg/min). The mass velocity was controlled by regulating the frequency of the three-phase alternating power source to the canned type circulation pump (Nikkiso Co., Ltd., Non-Seal Pump Multi-stage Type VNH12-C4 C-357SP, pump flow rate=12 m³/h, pump head=250 m) with an inverter installed a 4-digit LED monitor (Mitsubishi Electric Corp., Inverter, Model-F720-30K). The pump input frequency shows the net pump input power and pump discharge pressure free of slip loss. The water was pressurized by saturated vapor in the pressurizer in this work. The pressure at the outlet of the test tube was controlled within ±1 kPa of a desired value by using a heater controller of the pressurizer.

2.2 Test section

The cross-sectional view of 3-, 6- and 9-mm inner diameter test sections used in this work is shown in Fig. 2. The Platinum (Pt) test tubes for the test tube inner diameter, \( d \), of 3, 6 and 9 mm, the heated lengths, \( L \), of 32.7, 69.6 and 49.6 mm with the commercial finish of inner surface were used in this work. The Platinum test tube is highly sensitive for a resistance thermometry. Wall thicknesses of the test tube, \( \delta \), were 0.5, 0.4 and 0.3 mm. Four fine 0.07-mm diameter platinum wires were spot-welded on the outer surface of the 6 mm inner diameter test tube as potential taps: the first one is at the position of 5.2 mm from the leading edge of the test tube, and the second to forth ones are at 19.6, 20.0 and 19.6 mm from the previous ones, respectively. The effective length, \( L_{\text{eff}} \), of the 6 mm inner diameter test tube between the first potential tap and forth one on which average heat transfer was measured was 59.2 mm. The silver-coated 5-mm thickness copper-electrode-plates to supply heating current were soldered to the surfaces of the both ends of the test tube. The both ends of test tube were electrically isolated from the loop by Bakelite plates of 14-mm thickness. The inner surface condition of the test tube was observed by the scanning electron microscope (SEM) photograph (JEOL JXA8600) and inner surface roughness was measured by Tokyo Seimitsu Co., Ltd.’s surface texture measuring instrument (SURFCOM 120A). Figure 3 shows the SEM photographs of the Platinum (Pt) test tubes for \( d=3 \), 6 and 9 mm with commercial finish of inner surface. The values of inner surface roughness for \( R_a \), \( R_{\text{max}} \) and \( R_z \) were measured 0.40, 2.20 and 1.50 μm for \( d=3 \) mm, 0.45, 2.93 and 1.93 μm for \( d=6 \) mm, and 0.78, 3.90 and 2.64 μm for \( d=9 \) mm, respectively.

2.3 Method of heating test tube

The Pt test tube has been heated with an exponentially increasing heat input supplied from a direct current source (Takasago Ltd., NL035-500R, DC 35 V-3000 A) through the two copper electrodes shown in Fig. 4. Heat transfer processes caused by exponentially
increasing heat inputs, \( Q_0 \exp(t/\tau) \), were measured for the Pt test tube. The exponential periods, \( \tau \), of the heat input ranged from 6.04 to 32.13 s. The common specifications of the direct current source are as follows. Constant-voltage (CV) mode regulation is a 4.75 mV minimum, CV mode ripple is 500 \( \mu \)V r.m.s. or better and CV mode transient response time is less than 200 \( \mu \)sec (Typical) against 5 % to full range change of load.

The outer surface temperature of the test tube with heating was observed by an infrared thermal imaging camera (NEC Avio Infrared Technologies Co., Ltd. Thermography TVS-200EX). The accuracy is \( \pm 2 \) % of reading. The outer surface of the test tube was uniformly painted black with black body spray (Japan Sensor Corporation, JSC-3, emissivity, \( \varepsilon \), of 0.94) in this work.

2.4 Measurement of heat flux, temperature and pressure for test tube

The average temperature of the Pt test tube, \( T \), was measured with resistance thermometry participating as a branch of a double bridge circuit for the temperature measurement. The output voltages from the bridge circuit, \( V_{T1} \), together with the voltage drop across the potential taps of the test tube (first and forth potential taps, \( V_R=IR_T \), first and second ones, \( V_{R1}=IR_{T1} \), second and third ones, \( V_{R2}=IR_{T2} \), and third and fourth ones,
The temperatures of the heater inner and outer surfaces, \( T_i \) and \( T_{io} \), and \( C \) in Eq. (11) can be described as follows:

\[
T_i = T(r_i) = \frac{q_T}{4 \pi r_i^2} \int_0^{r_i} 2 \pi r T(r) dr - \frac{q_T}{2 \pi r_i^2} \ln r + C
\]

\[
T_{io} = T(r_{io}) = \frac{q_T}{4 \pi r_{io}^2} \int_0^{r_{io}} 2 \pi r T(r) dr - \frac{q_T}{2 \pi r_{io}^2} \ln r + C
\]
where $\overline{T}$, $q_l$, $\lambda$, $r_i$, and $r_o$ are average temperature of the test tube, heat flux, thermal conductivity, test tube inner radius and test tube outer radius, respectively.

In case of the 3-, 6- and 9-mm inner diameter test sections, before entering the test tube, the test water flows through the tube with the same inner diameter of the Pt test tube to form the fully developed velocity profile. The entrance tube lengths, $L_e$, are given 282, 333 and 333 mm ($L_e/d=94, 55.5$ and $37$). The values of $L_e/d$ for $d=3, 6$ and 9 mm in which the center line velocity reaches 99 % of the maximum value for turbulence flow were obtained ranging from 9.8 to 21.9 ($1.50 \times 10^4 \leq Re \leq 1.575 \times 10^5$) by the correlation of Brodkey and Hershey (19) as follows:

$$L_e/d = 0.693 Re^{1/4}$$

The inlet and outlet liquid temperatures, $T_{in}$ and $T_{out}$, were measured by 1-mm o.d., sheathed, K-type thermocouples (Nimblox, sheath material: SUS316, hot junction: ground, response time (63.2 %): 46.5 ms) which are located at the centerline of the tube at the upper and lower stream points of 262 and 53 mm from the tube inlet and outlet points for the 3-mm inner diameter test section and at the upper and lower stream points of 283 and 63 mm from the tube inlet and outlet points for the 6- and 9-mm inner diameter test sections. The inlet and outlet pressures, $P_{in}$ and $P_{out}$, were measured by the strain gauge transducers (Kyowa Electronic Instruments Co., Ltd., PHS-20A, Natural frequency: approximately 30 kHz), which were located near the entrance of conduit at upper and lower stream points of 53 mm from the tube inlet and outlet points for $d=3$ mm inner diameter test section, and at upper and lower stream points of 63 mm from the tube inlet and outlet points for the $d=6$- and 9-mm inner diameter test sections. The thermocouples and the transducers were installed in the conduits as shown in Fig. 2.

The inlet and outlet pressures, $P_{in}$ and $P_{out}$, for the 3-, 6- and 9-mm inner diameter test sections were calculated from the pressures measured by inlet and outlet pressure transducers, $P_{in}$ and $P_{out}$, as follows:

$$P_{in} = P_{opt} - \frac{(P_{opt})_{nabh} - (P_{opt})_{nabh}}{L_{opt} + L + L_{opt}}$$

$$P_{out} = P_{in} - \frac{(P_{in} - P_{opt})} {L + L_{opt}}$$

where $L_{opt}=0.053$ m and $L_{opt}=0.053$ m for the 3-mm inner diameter test section, and $L_{opt}=0.063$ m and $L_{opt}=0.063$ m for the 6- and 9-mm inner diameter ones. Experimental errors are estimated to be $\pm 1$ K in inner tube surface temperature and $\pm 2$ % in heat flux. Mass velocity, inlet and outlet subcoolings, inlet and outlet pressures and exponential period were measured within the accuracy $\pm 2$ %, $\pm 1$ K, $\pm 4$ kPa and $\pm 2$ %, respectively.

3. Numerical solution of turbulent heat transfer

3.1 Fundamental equations for $k-\varepsilon$ turbulence model with high Reynolds number form

Theoretical equations for $k-\varepsilon$ turbulence model (11) in a circular tubes of 3, 6 and 9 mm in diameter and 492, 636 and 616 mm long were numerically solved for heating of water with heated sections of 3, 6 and 9 mm in diameter and 33, 70 and 50 mm long by using PHOENICS code under the same conditions as the experimental ones considering the temperature dependence of thermo-physical properties concerned (12). The unsteady fundamental equations for turbulent heat transfer are expressed in the three dimensional coordinate shown in Fig. 5 as follows (13).

(Transport Equation for $k$) Cylindrical coordinates ($r, \theta, z$):

$$\rho \frac{\partial k}{\partial t} + \rho \frac{\partial}{\partial r}\left(r \frac{\partial k}{\partial r}\right) + \rho \frac{\partial}{\partial \theta}\left(\frac{1}{r} \frac{\partial k}{\partial \theta}\right) + \rho \frac{\partial}{\partial z}\left(\frac{1}{r^2} \frac{\partial k}{\partial z}\right) + \frac{\partial}{\partial \theta}\left(\frac{1}{r^2} \frac{\partial k}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\frac{1}{r} \frac{\partial k}{\partial z}\right) + \rho \left(P_h + \Gamma_k - C\right) =$$

(21)

(Transport Equation for $\varepsilon$) Cylindrical coordinates ($r, \theta, z$):
Table 1 The values of the constants in the Chen-Kim $k$-$\varepsilon$ turbulence model \(^{(11)}\)

<table>
<thead>
<tr>
<th>$\sigma_k$</th>
<th>$\sigma_\varepsilon$</th>
<th>$C_{1e}$</th>
<th>$C_{2e}$</th>
<th>$C_{3e}$</th>
<th>$C_\mu$</th>
<th>$C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>1.3</td>
<td>1.0</td>
<td>1.15</td>
<td>1.9</td>
<td>0.25</td>
<td>0.5478</td>
</tr>
</tbody>
</table>

\[
\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial r} \left( \rho \frac{\varepsilon}{\rho} \right) + \frac{\partial}{\partial \theta} \left( \rho \frac{\varepsilon}{\rho} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \mu + \frac{\mu_s}{\sigma_s} \right) \frac{\partial \varepsilon}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \mu + \frac{\mu_s}{\sigma_s} \right) \frac{\partial \varepsilon}{\partial \theta} + \frac{\partial}{\partial z} \left( \mu + \frac{\mu_s}{\sigma_s} \right) \frac{\partial \varepsilon}{\partial z} + \frac{\rho_c}{k} \left( 1 - k \varepsilon - C_{1e} \varepsilon \right)
\]

\[(22)\]

(Energy Equation)

Cylindrical coordinates \((r, \theta, z)\):

\[
\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial r} \left( \rho \frac{\varepsilon}{\rho} \right) + \frac{\partial}{\partial \theta} \left( \rho \frac{\varepsilon}{\rho} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \lambda + c_{\varepsilon} \frac{\mu_s}{\sigma_s} \right) \frac{\partial \varepsilon}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \lambda + c_{\varepsilon} \frac{\mu_s}{\sigma_s} \right) \frac{\partial \varepsilon}{\partial \theta} + \frac{\partial}{\partial z} \left( \lambda + c_{\varepsilon} \frac{\mu_s}{\sigma_s} \right) \frac{\partial \varepsilon}{\partial z} + Q
\]

\[(23)\]

where

\[
v_r = C_{d} k^{3/2} \frac{l_m}{l_w}
\]

\[(24)\]

\[
\varepsilon = C_{d} \frac{k^{3/2}}{l_w}
\]

\[(25)\]

\[
P_s = \nu \left[ 2\left( \frac{\partial v_r}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_r}{r} \right)^2 + \left( \frac{\partial v_\theta}{\partial \theta} \right)^2 + \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta}{r} \right)^2 + \left( \frac{\partial v_z}{\partial z} \right)^2 + \left( \frac{\partial v_r}{\partial \theta} + \frac{v_r}{r} \right)^2 + \left( \frac{\partial v_\theta}{\partial \theta} \right)^2 \right]
\]

\[(26)\]

\[
\Gamma_b = -\nu \left( \frac{\partial v_r}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right)
\]

\[(27)\]

\[
h = c_{\varepsilon} \varepsilon
\]

\[(28)\]

\[
\lambda_v = \lambda_r + \lambda
\]

\[(29)\]

\[
\lambda_r = \frac{c_{\varepsilon} \mu_s}{\sigma_s}
\]

\[(30)\]

$v_r$, $v_\theta$ and $v_z$ are the $r$, $\theta$ and $z$ components of a velocity vector, respectively. The constants, $\sigma_k$, $\sigma_\varepsilon$, $\sigma_s$, $C_{1e}$, $C_{2e}$, $C_{3e}$, $C_\mu$, $C_d$ and $C_{1\mu}$ appearing in Eqs. (21) to (25), (27) and (30) take the values given in Table 1.
3.2 Boundary conditions

The fundamental equations are numerically analyzed together with the following boundary conditions. On the outer boundary of heated section: constant heat flux, and non-slip condition.

\[ q = -\lambda \frac{\partial T}{\partial r} = \text{constant} \]  

(31)

At the outer boundary of non-heated section:

\[ \frac{\partial T}{\partial r} = 0 \]  

(32)

At the lower boundary:

\[ T = T_{in}, \quad v_r = 0, \quad v_g = 0 \quad \text{and} \quad v_z = u \quad \text{for in-flow} \]  

(33)

where \( T_{in} \) and \( u \) are a inlet liquid temperature and a flow velocity at the entrance of the test section.

3.3 Method of solution

The control volume discretization equations were derived from these fundamental equations by using the hybrid scheme \(^{14}\). The thermo-physical properties for each control volume are given as those at each volume temperature. The procedure for the calculation of the flow field is the SIMPLE algorithm which stands for Semi-Implicit Method for Pressure-Linked Equations \(^{15}\).

The surface heat fluxes, \( q_s \), for the heated length were equally given in the range of \( 2.60 \times 10^5 \) to \( 3.20 \times 10^7 \) W/m\(^2\) as an initial condition, and numerical calculation was continued until the steady-state was obtained. The surface temperature on the test tube, \( T_s \), was calculated from the analyzed temperature of the outer control volume on the test tube surface, \( TEM \), which is located on the center of the control volume, by solving the heat conduction equation in liquid as follows \(^{7-9, 16}\).

\[ T_s = \frac{q_s (\Delta r)_{out}}{2 \lambda} + TEM \]  

(34)

where, \((\Delta r)_{out}\) is the outer control volume width for the \( r \)-component. Average heat transfer coefficient on the test tube surface was obtained by \( T_{s,av} \) averaging the calculated local surface temperatures, \((T_s)_z\), at every 0.5 mm in the heated length, \( L \). All the calculations were made by using the PHOENICS code \(^{12}\).

4. Experimental results and discussion

4.1 Experimental conditions and Parameters used for Calculation

Steady-state heat transfer processes on the Pt test tubes of 3, 6 and 9 mm inner diameters that caused by the exponentially increasing heat inputs, \( Q_0 \exp(t/\tau) \), were measured. The exponential periods, \( \tau \), of the heat input ranged from 6.04 to 32.13 s. The initial experimental conditions such as inlet flow velocity, inlet liquid temperature, inlet pressure and exponential period for the single-phase flow heat transfer experiment were determined independently each other before each experimental run.

The experimental conditions were as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heater material</td>
<td>Platinum</td>
</tr>
<tr>
<td>Surface condition</td>
<td>Commercial finish of inner surface</td>
</tr>
<tr>
<td>Surface roughness for Ra, Rmax and Rz</td>
<td>0.40, 2.20 and 1.50 μm for ( d=3 ) mm, 0.45, 2.93 and 1.93 μm for ( d=6 ) mm, 0.78, 3.90 and 2.64 μm for ( d=9 ) mm</td>
</tr>
<tr>
<td>Inner diameter (( d ))</td>
<td>3, 6 and 9 mm</td>
</tr>
<tr>
<td>Heated length (( L ))</td>
<td>32.7, 69.6 and 49.6 mm</td>
</tr>
<tr>
<td>Effective Length (( L_{eff} ))</td>
<td>32.7, 59.2 and 49.6 mm</td>
</tr>
<tr>
<td>( L_{12}, L_{23} ) and ( L_{34} )</td>
<td>19.6 mm, 20.0 mm and 19.6 mm for ( d=6 ) mm, 19.6 mm, 20.0 mm and 19.6 mm for ( d=6 ) mm</td>
</tr>
<tr>
<td>( L/d )</td>
<td>10.9, 11.6 and 5.51</td>
</tr>
<tr>
<td>( L_{eff}/d )</td>
<td>10.9, 9.87 and 5.51</td>
</tr>
<tr>
<td>Wall thickness (( \delta ))</td>
<td>0.5, 0.4 and 0.3 mm</td>
</tr>
<tr>
<td>Inlet flow velocity (( u ))</td>
<td>4.22, 7.31, 10.41, 13.72 and 21.45 m/s</td>
</tr>
<tr>
<td>Inlet pressure (( P_{in} ))</td>
<td>834.04 to 910.23 kPa</td>
</tr>
<tr>
<td>Outlet pressure (( P_{out} ))</td>
<td>816.32 to 845.31 kPa</td>
</tr>
</tbody>
</table>
Inlet subcooling (ΔT_{sub,in}) 133.81 to 138.36 K
Outlet subcooling (ΔT_{sub,out}) 127.84 to 134.39 K
Inlet liquid temperature (T_{in}) 308.20 to 311.77 K

Exponentially increasing heat input (Q) Q_0 exp(t/τ), τ=6.04 to 32.13 s

The parameters used for calculation were as follows:
Inner diameter (d) 3, 6 and 9 mm
Heated length (L) 33, 70 and 50 mm
Entrance length (L_{en}) 282, 333 and 333 mm
Exit length (L_{ex}) 177, 233 and 233 mm
Test section length (L_{ts}) 492, 636 and 616 mm
Heat flux (q) 2.63 \times 10^5 to 3.20 \times 10^7 W/m^2 (q_0 exp(t/τ), τ=22.89 to 32.13 s)
Inlet flow velocity (u) 4.22, 7.31, 10.41, 13.72 and 21.45 m/s
Inlet liquid temperature (T_{in}) 308.20 to 311.77 K

Coordinate system cylindrical coordinate (r, θ, z)
Control volume number (17 to 41, 60, 894 to 938)
Physical model k-ε turbulence model with high Reynolds number form

4.2 Steady-state turbulent heat transfer characteristics

Figures 6, 7 and 8 show the typical examples of the steady-state turbulent heat transfer curves for Platinum circular tubes of d=3, 6 and 9 mm and L=32.7, 69.6 and 49.6 mm with the exponential period, τ, of around 24 s at the flow velocities, u, of 13.32, 13.51 and 13.65 m/s, respectively. The experimental data were compared with the values derived from

\[ ΔT_L = \left( T_{s,av} - T_L \right) \]

for circular tube of d=3 mm and L=32.7 mm with Tin=308.73 K and u=13.32 m/s at P_n=852.36 kPa

\[ ΔT_L = \left( T_{s,av} - T_L \right) \]

for circular tube of d=6 mm and Leff=59.2 mm with Tin=302.68 K and u=13.51 m/s at P_n=841.69 kPa

\[ ΔT_L = \left( T_{s,av} - T_L \right) \]

for circular tube of d=9 mm and L=49.6 mm with Tin=311.77 K and u=13.65 m/s at P_n=849.26 kPa

\[ \Delta T_L \text{ (K) } \]
\[ q \text{ (W/m}^2\text{) } \]

**Fig. 6 Relationship between q and ΔT_L [=(T_{s,av}-T_L)]**
Fig. 7 Relationship between q and ΔT_L [=(T_{s,av}-T_L)]
for circular tube of d=3 mm and L=32.7 mm with for circular tube of d=6 mm and Leff=59.2 mm with
T_{in}=308.73 K and u=13.32 m/s at P_n=852.36 kPa

T_{in}=302.68 K and u=13.51 m/s at P_n=841.69 kPa

\[ \Delta T_L \text{ (K) } \]
\[ q \text{ (W/m}^2\text{) } \]

**Fig. 8 Relationship between q and ΔT_L [=(T_{s,av}-T_L)]**
for circular tube of d=9 mm and L=49.6 mm with T_{in}=311.77 K and u=13.65 m/s at P_n=849.26 kPa
The authors' correlation of the steady-state turbulent heat transfer for the empty tube, Eq. (1), at the flow velocities, \( u \), of 13.32, 13.51 and 13.65 m/s, respectively. The heat fluxes gradually become higher with an increase in the temperature difference between average inner surface temperature and liquid bulk mean temperature, \( \Delta T_L (=T_{inner}-T_{outer}) \), on the steady-state turbulent heat transfer curve derived from Eq. (1) (1).

The numerical solutions for the relation between the heat flux, \( q \), and the temperature difference between average inner surface temperature and liquid bulk mean temperature, \( \Delta T_L \), are shown for the heat flux, \( q \), ranging from 6.47 \times 10^5 to 1.67 \times 10^7 \text{ W/m}^2 \text{ from 3.41 \times 10^3 to 8.49 \times 10^6 W/m^2 and from 5.82 \times 10^3 to 7.55 \times 10^6 W/m^2 at the flow velocities of } 13.32, 13.51 and 13.65 m/s as red solid circles, respectively. The 14, 12 and 11 different values for the numerical solutions are plotted for the \( \Delta T_L \) ranging from 8.74 to 216.89 K, from 4.93 to 119.11 K and from 9.16 to 115.98 K on the log-log graph, respectively. These solutions become also higher with an increase in the \( \Delta T_L \) along the curve derived from Eq. (1). The numerical solutions solved by the theoretical equations for turbulent heat transfer, Eqs. (21) to (33), are in good agreement with the experimental data and the values derived from Eq. (1) within \( \pm 10 \% \) difference. The outer control volume widths for the \( r \)-component, \( (\Delta r)_{outer} \), are 13.04, 13.04 and 14.9 \mu m, respectively.

These experimental data are compared with the values derived from other researchers' correlations of the steady-state turbulent heat transfer in pipes, Eqs. (2) to (7). The experimental data for \( d=3, 6 \) and 9 mm at the high heat flux points shown in Figs. 6, 7 and 8 are 17.6 to 53.8 \% higher, 24.7 to 62.7 \% higher and 21.6 to 55.0 \% higher than the values derived from these correlations at a fixed temperature difference between average inner surface temperature and liquid bulk mean temperature, \( \Delta T_L (=\text{constant}) \), and 20 to 64 K lower, 26 to 75 K lower and 17.81 to 41.80 K lower than the values derived from these correlations at a fixed heat flux \( (q=\text{constant}) \), respectively. We confirm in this study that the sixth term, \( (\mu/\mu_0)^{0.14} \), for authors' correlation of the steady-state turbulent heat transfer in pipe, Eq. (1), will play an important role in expressing the experimental data well for the wide range of the \( \Delta T_L \). The heat transfer coefficients for the experimental data become gradually higher with an increase in \( \Delta T_L \).

4.3 Inner and outer surface temperatures, \( T_s \) and \( T_{outer} \), heat fluxes, \( q \), and heat transfer coefficients, \( h \), for 1st, 2nd and 3rd positions of three sections, and inlet and outlet liquid temperatures, \( T_{in} \) and \( T_{outer} \) for 6 mm inner diameter test tube

Figure 9 shows inner surface temperatures (\( T_{s1}, T_{s2} \) and \( T_{s3} \)-black solid circles), outer surface temperatures (\( T_{outer1}, T_{outer2} \) and \( T_{outer3} \)-black open circles), heat fluxes (\( q_{1}, q_{2} \) and \( q_{3} \)-black solid triangles) and heat transfer coefficients (\( h_{1}, h_{2} \) and \( h_{3} \)-black open triangles) for first, second and third positions of the sections between first and second potential taps, second and third ones and third and fourth ones, and inlet and outlet liquid temperatures (\( T_{in} \) and \( T_{outer} \)-black solid squares) at the flow velocity, \( u \), of 13.59 m/s with the heat flux, \( q \), of 8.44 MW/m². The liquid temperatures at the center of each section (heated lengths, \( L \), of 15, 34.8 and 54.6 mm-black open squares) are linearly estimated from the values of the inlet liquid

![Fig. 9 Inner and outer surface temperatures, heat fluxes and heat transfer coefficients for first, second and third positions of three sections, and inlet and outlet liquid temperatures for the exponential period of 21.85 s at the flow velocity of 13.59 m/s compared with numerical solutions of inner surface temperature and outer surface temperature observed by an infrared thermal imaging camera](image-url)
temperature, \( T_{in} \) and outlet one, \( T_{out} \). The inner surface temperatures (\( T_{so1}, T_{so2} \) and \( T_{so3} \)) and outer surface temperatures (\( T_{out1}, T_{out2} \) and \( T_{out3} \)) were obtained from Eqs. (15) and (16) with the measured average temperature and surface heat flux. The inner and outer surface temperatures for each position of the sections (\( T_{si}, T_{s2} \) and \( T_{s3} \), \( T_{so1}, T_{so2} \) and \( T_{so3} \)) become gradually higher with an increase in the heated length from the leading edge of the test tube, whereas the heat fluxes (\( q_1, q_2 \) and \( q_3 \)) are almost constant for each position of the sections. The increasing rate of the inner surface temperature shows nearly the same trend of that of the liquid temperature from the inlet to the outlet (\( T_{in} \) to \( T_{out} \)). The heat transfer coefficients for each position of the sections (\( h_1, h_2 \) and \( h_3 \)) become almost constant as shown in the figure.

The experimental solutions of the average liquid temperature of the outlet control volume on the test tube surface, \( TEM \), are shown at the heat fluxes of 8.49 MW/m\(^2\) with the flow velocities of 13.59 m/s as a red 1-dot dashed line. The surface temperatures on the test tube, \( T_s \), were calculated from the analyzed liquid temperature of the outer control volume on the test tube surface, \( TEM \), which is located on the center of the control volume, by solving the heat conduction equation in liquid as given in Eq. (34). The \( z \)-axis variations in the inner surface temperature of the test tube at every 0.5 mm in the heated length, \( L \), are shown as red solid line in the figure. They become gradually higher with an increase in the \( z \)-axis distance from the leading edge of the test tube. The numerical solutions of the inner surface temperatures solved by the theoretical equations for turbulent heat transfer, Eqs. (21) to (33), are in good agreement with the experimental data given by Eq. (15) which are obtained from the steady one-dimensional heat conduction equation.

The outer surface temperature of the test tube was observed by an infrared thermal imaging camera. The \( z \)-axis variations in the outer surface temperature of the test tube, \( T_{os} \), for the exponential period of 21.85 s at the flow velocity of 13.59 m/s with the heat flux of 8.44 MW/m\(^2\) are shown in Fig. 9. The outer surface temperature of the test tube steeply increases from 312.54 K at the electrode of the inlet up to 430 K at the heated length of 5 mm from the leading edge of the test tube, and gradually becomes higher, lower and higher again up to the highest value of around 455 K to about 5 mm short of the outlet of the test tube. Finally, it linearly falls down to 322.34 K at the electrode of the outlet again. The both ends of the Platinum test tube were soldered to the copper-electrode-plates which have a width of 80 mm, a length of 120 mm and a thickness of 5 mm. The heat capacities of the copper-electrode-plates with subcooled water flow would be far larger in comparison with the joule heat of the test tube which has a wall thickness, \( \delta \), of 0.4 mm. The outlet liquid temperature (\( T_{out}=301.09 \) K) by the uniform heating of the test tube with \( \phi=8.44 \) MW/m\(^2\). It is observed from this figure that the outer surface temperatures for first, second and third positions of three sections on the blue solid line (---) gradually increase with an increase in the heated length. The increasing rate of the outer surface temperature becomes the same with that of the liquid temperature from inlet to outlet. It will be considered from this fact that the heat transfer coefficients on the inner surface of the test tube will be almost constant for the heated length in the non-boiling region as mentioned above. The outer surface temperatures (a blue broken line, --) measured with \( \phi=0.94 \) for first, second and third positions of three sections were 10 to 20 % lower than the values calculated from Eq. (16) derived by Hata and Masuzaki (17), although the infrared thermal imaging camera has the accuracy of \( \pm 2 \% \) of reading. The temperature curve (-----) on the figure was asymptotically revised to the values calculated from Eq. (16) and plotted.

The numerical solutions of average liquid temperature, \( T_{f,av} \), from inlet to outlet are also shown as red broken line in Fig. 9 for comparison. The average liquid temperatures, \( T_{f,av} \), were calculated from the analyzed liquid temperatures of the control volumes on the \( r \) and \( \theta \) axis grid numbers of the \( r-\theta \) plane for each \( z \)-axis grid number. Those become linearly higher with an increase in the \( z \)-axis distance, \( Z \), and become almost equal to the outlet liquid temperature, \( T_{out} \), at the exit of the heated length. Therefore, it was confirmed from this fact that the heat transfer coefficients, \( h \), calculated from the heat fluxes, \( q \), and the liquid temperatures, \( T_i \) at the heated lengths, \( L \), of 15, 34.8 and 54.6 mm which were obtained by the use of linear interpretation between the inlet and outlet liquid temperatures, \( T_{in} \) and \( T_{out} \), would also be true, although these inlet and outlet liquid temperatures were measured by one 1-mm o.d., sheathed, K-type thermocouple in the small diameter tube of \( d=6 \) mm, respectively.
4.4 Thickness of conductive sub-layer, $\delta_{CSL}$, and the non-dimensional thickness of conductive sub-layer, $y_{+CSL}$, for steady-state turbulent heat transfer

The numerical solutions solved by the theoretical equations for steady-state turbulent heat transfer, Eqs. (21) to (33), are in good agreement with the experimental data and the values derived from Eq. (1) within \( \pm 10\% \) difference as shown in Figs 6 to 8. In Fig. 10, the test tube surface is located at $r=-3$ mm and the conductive sub-layer \((18,19)\) exists on the test tube surface. The liquid temperatures in the conductive sub-layer on the test tube surface will become linearly lower with a decrease in the radius by the heat conduction from the surface temperature on the test tube, $T_f=T_s-\frac{\Delta qr}{\lambda l}$. And let those, $T_f$, equal the analyzed liquid temperature of the outer control volume on the test tube surface, $TEM$, in the turbulent flow region, which is located on the center of the control volume as given in Eq. (34). Half the outer control volume width for the $r$-component, $(\Delta r)_{out}/2$, would become the thickness of the conductive sub-layer, $\delta_{CSL}$, for the turbulent heat transfer in a vertical circular tube under two-phase model classified into conductive sub-layer and inner region of the turbulent flow. Relationships between $\delta_{CSL}$ and the non-dimensional thickness of conductive sub-layer, $y_{+CSL}$, for the steady-state turbulent heat transfer numerically solved, and $u$ are shown for the temperature difference between average inner surface temperature and liquid bulk mean temperature, $\Delta T_L = (T_{in}+T_{out})/2$, of $100$ K in Fig. 11. Two non-dimensional thicknesses of conductive sub-layer, $(y_{+CSL})_{TEM}$ and $(y_{+CSL})_{TL}$, are given as it will be very difficult to solve the analyzed temperature of the outer control volume on the test tube surface, $TEM$. All properties in the equation are evaluated at the analyzed temperature of the outer control volume on the test tube surface, $TEM$, and the liquid bulk mean temperature, $T_L = (T_{in}+T_{out})/2$, respectively. The non-dimensional thickness of conductive sub-layer, $y_{+CSL}$, is defined as follows:

$$y_{+CSL} = \left( \frac{f_F}{2} \right)^{0.3} \frac{\rho_l \delta_{CSL}}{\mu_l}$$

Fig. 10 Liquid temperatures in the conductive sub-layer for circular tube of $d=6$ mm

Fig. 11 Relationships between $\delta_{CSL}$, $(y_{+CSL})_{TEM}$ and $(y_{+CSL})_{TL}$ for the turbulent heat transfer numerically solved, and $u$ with $\Delta T_L=100$ K
f_F = 0.126 Re_d^{0.25}

\text{for the empty tube}

(36)

where \( f_F \) is Fanning friction factor \(^{(20, 21)}\). The values of \( \delta_{CSL} \) for \( d=3, 6 \) and 9 mm become linearly lower with an increase in the flow velocity, \( u \), on the log-log graph, although those of \((y^+_{CSL})_{TEM}\) and \((y^+_{CSL})_{TL}\) become a little higher with an increase in the flow velocity, \( u \), but are almost constant in the whole numerical range. These numerical solutions of \( \delta_{CSL} \), \((y^+_{CSL})_{TEM}\) and \((y^+_{CSL})_{TL}\) can be expressed for the \( u \) ranging from 4.11 to 21.45 m/s by the following correlations:

\[ \delta_{CSL} = 139.98 \times 10^{-6} d^{0.15} u^{-0.85} \]

for \( \Delta T_L=100 \) K

(37)

\[ (y^+_{CSL})_{TEM} = 9.08u^{0.1} \]

for \( \Delta T_L=100 \) K

(38)

\[ (y^+_{CSL})_{TL} = 6.05u^{0.1} \]

for \( \Delta T_L=100 \) K

(39)

Eq. (37), \( 1/\delta_{CSL} \propto d^{0.15} u^{-0.85} \), represents authors’ steady-state turbulent heat transfer correlation, Eq. (1), derived from the experimental data on the test tube, \( h \propto d^{0.15} u^{-0.85} \). These numerical solutions of \((y^+_{CSL})_{TEM}\) and \((y^+_{CSL})_{TL}\) for various fluids are also shown versus the Reynolds number, \( Re_d \), with the temperature differences between average inner surface temperature and liquid bulk mean temperature, \( \Delta T_L \), of 40, 80, 100 and 120 K in Figs. 12 and 13, respectively. These numerical solutions of \((y^+_{CSL})_{TEM}\) and \((y^+_{CSL})_{TL}\) can be expressed for the \( Re_d \) ranging from 1.89\(^{10^4}\) to 3.74\(^{10^5}\) by the following correlations:

\[ (y^+_{CSL})_{TEM} = 2.95 e^{205/Re_d} Re_d^{0.88} \]

for \( \Delta T_L=40, 80, 100 \) and 120 K

(40)

\[ (y^+_{CSL})_{TL} = 2.95 Re_d^{0.88} \]

for \( \Delta T_L=40, 80, 100 \) and 120 K

(41)

5. Conclusions

The steady-state turbulent heat transfer coefficients in a vertical circular Platinum (Pt) test tube for the flow velocities \( (u=4.22 \text{ to } 21.45 \text{ m/s}) \), the inlet liquid temperatures \( (T_{in}=308.20 \text{ to } 311.77 \text{ K}) \), the inlet pressures \( (P_{in}=834.04 \text{ to } 910.23 \text{ kPa}) \) and the increasing heat inputs \( (Q_{exp}t/\tau) \), \( \tau \) of 6.04 to 52.13 s) were systematically measured. Measurements were made on Pt test tubes of 3, 6 and 9 mm inner diameters, 32.7, 69.6 and 49.6 mm heated lengths and 0.5, 0.4 and 0.3 mm thicknesses, respectively. Theoretical equations for turbulent heat transfer in circular tubes of 3, 6 and 9 mm in diameter and 492, 636 and 616 mm long were numerically solved for heating of water with heated sections of 3, 6 and 9 mm in diameter and 33, 70 and 50 mm long by using PHOENICS code under the same condition as the experimental one considering the temperature dependence of
thermo-physical properties concerned. Experimental and computational study results lead to the following conclusions.

1) The numerical solutions for the relation between the heat flux, \( q \), and the temperature difference between average inner surface temperature and liquid bulk mean temperature, \( \Delta T_{\text{L}} \), solved by the theoretical equations for turbulent heat transfer, Eqs. (21) to (33), are in good agreement with the experimental data and the values derived from Eq. (1) within \( \pm 10\% \) difference for the heat flux, \( q \), ranging from \( 2.60 \times 10^5 \) to \( 3.20 \times 10^7 \) W/m\(^2\) at the flow velocities of 4.22 to 21.45 m/s.

2) The inner and outer surface temperatures for each position of the three sections (\( T_{s1}, T_{s2}, T_{s3}, T_{o1}, T_{o2}, T_{o3} \)) become gradually higher with an increase in the heated length from the leading edge of the test tube, whereas the heat fluxes (\( q_1, q_2, q_3 \)) are almost constant for each position of the sections. The increasing rate of the inner surface temperature shows nearly the same trend of that of the liquid temperature from the inlet to the outlet (\( T_{in} \) to \( T_{out} \)). The heat transfer coefficients for each position of the sections (\( h_1, h_2, h_3 \)) become almost constant.

3) The numerical solutions of the inner surface temperatures solved by the theoretical equations for turbulent heat transfer, Eqs. (21) to (33), are in good agreement with the experimental data (\( Ts_{1}, Ts_{2}, Ts_{3} \)) given by Eq. (15) which are obtained from the steady one-dimensional heat conduction equation.

4) The thickness of the conductive sub-layer, \( \delta_{\text{CSL}} \), which is half the outer control volume width for \( r \)-component, \((\Delta r)_{\text{out}}/2\), become linearly lower with an increase in the flow velocity, \( u \), on the log-log graph, although the values of the non-dimensional thickness of conductive sub-layer, \( y_{\text{CSL TEM}}^{CSL} \) and \( y_{\text{CSL TL}}^{CSL} \), become a little higher with an increase in the flow velocity, \( u \), but are almost constant in the whole numerical range. These numerical solutions of \( \delta_{\text{CSL}}, y_{\text{CSL TEM}}^{CSL} \) and \( y_{\text{CSL TL}}^{CSL} \) can be expressed for the \( u \) ranging from 4.22 to 21.45 m/s by Eqs. (37) to (41).

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References


