Proposal and validation of mathematical model for resistance training

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Abstract A set of mathematical models for resistance training (RT model) with repeated dynamic and voluntary activations of knee extensor muscles was proposed and validated. RT model predicts muscle activity, fatigue and recovery, and can be used to predict the mechanical impulses identified in different muscle fiber types. Several resistance training program variables (e.g. load, velocity, number of sets, and duration of rest intervals) were addressed in this study. Experimental data from six subjects were taken under several different training protocols. For the protocols studied, the model precisely predicted the maximum number of repetitions during knee extension resistance exercise ($R^2 = 0.87$). This study demonstrates the applicability of the first simple mathematical model for voluntary dynamic exercise of knee extensor muscles. The developed model calculates the mechanical impulse of fast twitch fiber for the selected protocols. For instance, the model shows high intensity training tends to induce more fast-twitch fiber recruitment than that in low intensity training in the case of non-failure. This model is expected to help in understanding how impulses of different fiber types contribute and affect training.

Keywords : training variable, mathematical modeling, muscle fiber recruitment, repetition maximum

Introduction

Resistance training is considered as the most effective method to develop musculoskeletal strength and improve health and fitness status¹⁾. Understanding the dose-response relationships in exercise training is important for maximizing the effects of resistance training.

Activation of the fast twitch (FT) fibers is considered an important factor for muscular hypertrophy since muscle cells readily increase in size through the activation of mTORC1 signaling²⁾. There are two types of major muscle fibers: slow twitch (ST) and fast twitch (FT) fibers. FT fibers are less resistant to fatigue, but develop contractile force more quickly than ST fibers³⁾. In general, activation of FT fibers follows the “size-principle”, in which smaller motor units are recruited more predominantly than larger ones with a gradual increase in contraction force⁴⁾. Since FT fibers are mainly involved in larger motor units, it has been thought that the recruitment of FT fibers requires high intensity. Many studies have shown that mid-high [$>60\%$ 1 repetition maximum (RM)] intensity is ideal for inducing muscle hypertrophic effects^{1,5,6)}.

On the other hand, several studies have demonstrated that low-load resistance exercises can also induce muscle hypertrophy when repetition until exhaustion is per-

formed. It is suggested that training to failure can induce recruitment of FT fibers with lower training load⁷⁻¹¹⁾. However, the relationships between the muscle hypertrophic effects and training variables such as intensity, volume, and interval between sets remain unclear.

Even though the importance of FT fiber recruitment is well-known, recruitment and fatigue of FT fibers during resistance training have not been studied in detail. One of the reasons for this lack is because of the limitations in observing FT fiber activation during dynamic resistance training⁹⁾.

One of the effective ways to predict the recruitment and fatigue of FT fibers is to use mathematical models¹²⁾. Some useful models have been proposed to predict muscle activation and endurance time in different tasks, such as isometric contraction¹³⁾. In addition, several recent studies proposed mathematical models to predict the relationships between muscle activation and fatigue. Liu and colleagues have shown the time course of muscle fatigue in activated, fatigued, and resting states¹⁴⁾. Xia and Frey-Law further improved this model by adding parameters related to muscle fiber types and activation rules of the fiber¹⁵⁾. There are also other models that have proposed to predict the intensity-endurance time relationships in muscle contraction¹⁶⁾.

In this study, we developed a set of mathematical models for dynamic knee extension exercise, which is often

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included in conventional resistance training (RT) programs, and validated the models with healthy male participants. The developed set of models (RT model) can be used to predict the recruitment and fatigue of each type of fiber under various training conditions and variables.

Materials and Methods

Mathematical modeling for resistance training (RT)

Table 1 summarizes the parameters used in the RT model. They are divided into three categories: (i) training-related parameters, (ii) subject-related parameters, and (iii) other parameters. The training- and subject-related parameters are assumed to be constant during a training session of a subject, while the other parameters are considered time-dependent. In dynamic contractions, one of the factors having a considerable effect on the muscle force is the torque-angle-velocity (TAV) relationship. The TAV relationship was assumed to be independent of fatigue¹⁷⁾.

Equation (1) describes the knee extension movement in resistance training, where contractile force is balanced with an external load so that the movement velocity is constant.

$$E(t) \cdot A(\theta, v) = B(\theta) \cdot L \quad (1)$$

The $E(t)$ is the activation scale [$0 \leq E(t) \leq 100$], which represents a force relative to maximum voluntary contraction (MVC) with no fatigue ($= 100$). $A(\theta, v)$ denotes the TAV relationship where θ is the anatomical knee joint angle (0 degrees represents fully extended position) and v is the contraction velocity (≥ 0 represents concentric and isometric contractions and $v < 0$ represents eccentric contraction). $B(\theta)$ is a factor influencing the intensity at the joint angle θ , which varies with the exercise movement and equipment used. For example, torque-knee angle relationship of leg extension exercise varies depending on the leg extension machine used¹⁸⁾. L is the training load.

In the present RT model, in order to make $\max_{\theta \in ROM} (\frac{B(\theta)}{A(\theta, v)} \cdot L)$ represent 100% 1RM, the maximum value of $\frac{B(\theta)}{A(\theta, v)}$ throughout the range of motion is defined as “1”.

The TAV relationship of dynamic knee extension exercise reported by Anderson et al.¹⁹⁾ was used to find $A(\theta, v)$, and data reported by Folland and Morris¹⁸⁾ were used to find $B(\theta)$.

For construction of the RT model, fiber-type specificity of the generated force during concentric and eccentric phases should be taken into consideration. According to the size principle⁴⁾, a divided activation scale of ST fibers and FT fibers is described by equations (2) and (3).

$$E_1(t) = \begin{cases} \frac{B(\theta)}{A(\theta, v)} \cdot L & [\text{CON}; RC_1(t) \geq \frac{B(\theta)}{A(\theta, v)} \cdot L] \\ RC_1(t) & [\text{CON}; RC_1(t) < \frac{B(\theta)}{A(\theta, v)} \cdot L] \\ X_1(t) & (\text{ECC}) \end{cases} \quad (2)$$

CON: concentric contraction, ECC: eccentric contraction

$$E_2(t) = \begin{cases} \frac{B(\theta)}{A(\theta, v)} \cdot L - E_1(t) & (\text{CON}) \\ X_2(t) & (\text{ECC}) \end{cases} \quad (3)$$

$$X_n(t) = \frac{B(\theta)}{A(\theta, v)} \cdot L \cdot \frac{RC_n}{RC_1 + RC_2} \quad (n = 1 \text{ or } 2) \quad (4)$$

Where, n represents a fiber type (1 for ST fiber and 2 for FT fiber). RC denotes residual capacity (maximum activation capacity remaining in the fiber).

In general, motor unit recruitment during concentric contraction follows the size principle, but this does not apply to eccentric contraction²⁰⁾. Some reported that recruitment of larger FT fiber was more dominant in eccentric contractions than in concentric contractions^{21,22)}. In the present RT model, we assumed the activation probability to be constant throughout the observation period, regardless of the size or type of motor unit. Thus, the force generated from FT fiber depends on the occupancy rate of RC_2 , between the sum of RC_1 and RC_2 [eq. (4)].

In this study, we considered a decrease in fatigue-induced residual capacity to be proportional to the force output from each motor unit. Thus, changes in residual capacity of each fiber can be described by eq. (5).

$$\frac{d}{dt} RC_n = -\phi_n E_n(t) \quad (n = 1 \text{ or } 2) \quad (5)$$

Table 1. Parameters used in the RT model

Training related	Test subject related	Others
L: Training load [%1RM]	$RC_n(0)$: Initial residual capacity of a fiber, where	t: Time from the beginning of a training protocol
RI: Rest interval between sets [s]	$RC_1(0) + RC_n(0) = 100$	
v: Contraction velocity [rad/s]	ϕ_n : Fatigue rate of a fiber	θ : Knee joint angle
ROM: Range of motion [rad]	ρ_n : Recovery rate of a fiber	n: Type of fiber ST fiber = 1 and FT fiber = 2

Fatigue rate, ϕ_n , represents a mean decreasing rate of residual capacity in dynamic contraction. The three-compartment model proposed by Xia and Frey-Law defined the states of muscle fiber as activated, fatigued, and resting. They assumed that transitions between each state occur at a given rate¹⁵⁾. In the RT model, changing the rate from “fatigued” to “activated” via “resting” during dynamic contractions was not included, since it was negligibly small and the lengths of the muscle fibers kept changing²³⁾.

The state when a subject is fully exhausted can be represented by eq. (6). The equation can be used to determine whether to execute the next repetition by predicting if the subject’s fatigue level is reaching one’s limit.

$$\{RC_1(t_m) + RC_2(t_m)\} = MVC_{failure} \quad (6)$$

Where, m is the count of the set and t_m represents the time of “failure” (i.e., the state when the subject fails to continue contracting the muscle due to fatigue) at the m^{th} set. $MVC_{failure}$ is the maximum voluntary activation level at t_m , the time point of “failure”. The difference between $\frac{B(\theta)}{A(\theta, v)} \cdot L$ (i.e., the activation scale required to be generated

from muscle) and $MVC_{failure}$ is related to the occurrence of central fatigue.

When performing multiple training sets, the initial residual capacity of the m^{th} set depends on the final residual capacities of the $(m-1)^{\text{th}}$ set. The residual capacity of the muscle fiber that has been fully restored during the rest period can be represented by eq. (7)²⁴⁾.

$$RC_n(t_m + RI) = RC_n(0) - \{RC_n(0) - RC_n(t_m)\} \cdot \exp(-\rho_n \cdot RI) \quad (n = 1 \text{ or } 2) \quad (7)$$

Where, RI is the length of rest interval and ρ_n is the recovery rate of each fiber.

The repetition maximum and the integrated value of activation level for each motor unit type in various training protocols can be calculated by eqs. (1) - (7) with subject-related (individual) parameters and $MVC_{failure}$ (Fig. 1).

Experimental procedure

Participants

Six healthy Japanese men [age 24.2 ± 1.6 (yrs), height 169.0 ± 7.9 (cm), weight 68.3 ± 9.6 (kg)] with no current lower extremity orthopedic problems voluntarily

Fig. 1 A flow chart summarizing the relationships between different equations used to calculate the repetition maximum (RM) of each set.

participated in this study. Young participants were chosen for safety reasons, as this study included a 1RM test and high-intensity sessions that can be risky for elderly people^{25,26}. All participants gave their signed informed consent. This study was approved by the Ethics Committee for Human Experiments at The University of Tokyo (approval number: 14-149).

Task design

To confirm the validity of the RT model and study the effects and relationships between individual parameters of each test subject, a study set consisting of 17 sessions was designed. There were 3 preparation sessions, 12 training sessions, and 2 test sessions. Each session was separated by more than 72 hours.

Preparation sessions followed the same protocol as training sessions (described later) in order for the participants to become familiar with the training movements. The training variables used were $L = 80\%$ 1RM with $v = 1.57$, $L = 30\%$ 1RM with $v = 1.57$, and $L = 30\%$ 1RM with $v = 0.39$. All three protocols used $RI = 120$ and the

number of sets was 5.

A training session consisted of five sets of unilateral knee extension exercise. Table 2 shows the training variables of 12 training protocols. The training variables such as intensity, rest interval, and contraction speed, were randomly varied. Each set was performed until the participant reached “failure” state. Maximum voluntary contraction (MVC) of knee extension was measured before starting the exercise to obtain the baseline (100%) MVC, and MVCs were also measured at the start and the half-way point of each rest interval to observe fatigued MVCs (described as percentage of baseline MVC) (Fig. 2).

During the first test session (MVC test 1), ten MVC measurements were taken. Each measurement was taken for 2 seconds, every 15 seconds. The measurement was used to observe if fatigue measured during a rest interval influences the performance of subsequent sets. During the second test session (MVC test 2), 30 MVC measurements were taken. Each measurement was taken for five seconds, every three seconds.

Table 2. Training variables used in the 12 training protocols

	L	v	RI
1	80[%1RM]	1.57[rad]	30[s]
2	80[%1RM]	1.57[rad]	120[s]
3	80[%1RM]	0.39[rad]	30[s]
4	80[%1RM]	0.39[rad]	120[s]
5	60[%1RM]	1.57[rad]	30[s]
6	60[%1RM]	1.57[rad]	120[s]
7	60[%1RM]	0.39[rad]	30[s]
8	60[%1RM]	0.39[rad]	120[s]
9	30[%1RM]	1.57[rad]	30[s]
10	30[%1RM]	1.57[rad]	120[s]
11	30[%1RM]	0.39[rad]	30[s]
12	30[%1RM]	0.39[rad]	120[s]

L, training load; v , velocity; RI, rest interval between sets.

Fig. 2 A time-flow diagram showing MVC measurement points (indicated by arrows).

Force measurements

Force output from the muscle was measured during the exercise. Tests were performed using a knee extension machine (Isotonic line; Technogym, Cesena, Italy). The range of motion (ROM) was set between 1.75 and 0.17 rad. The length of the machine wire was adjusted to reach the point just above the weight, and the wire was connected to a load cell (AS-20GB; Kyowa Electric Instruments Co., Ltd., Tokyo, Japan). Data from the load cell were collected digitally and stored using a data acquisition system (Power Lab/16SP; ADInstruments, Dunedin, NZ).

Parameter identification and model validation

The data collected from 12 protocols per subject were used to identify the individual parameters [$RC_1(0)$, ϕ_1 , ϕ_2 , ρ_1 and ρ_2] and test the validity of the RT model. Data from each protocol consisted of repetition maximums of each set and fatigued MVCs.

We randomly selected the results of six protocols per subject to identify each subject's individual parameters. By using the simulated annealing (SA) method, we estimated the individual parameters to minimize the differences between observed and predicted (i.e. calculated by the RT model) MVCs. Initial values of five parameters were set to 30, 0.01, 0.07, 0.06 and 0.03, respectively. The total number of evaluations was 10,000. Individual parameters were defined for each subject.

In order to confirm whether calculated individual parameters represented the subjects' condition, the correlation between estimated $RC_1(0)$ and 15th MVC of MVC test 2, which was supposed to reflect subjects' fiber type distribution²⁷⁾, was tested by the general linear model.

Results of the six protocols (identified parameters not included) were used to validate the model. The identified individual parameters were used to validate if the RT model predicts actual repetitions from observed fatigued MVCs. The correlation between actual and predicted numbers of repetitions was examined by generalized linear mixed model (GLMM). The predicted number of repetitions was used as a fixed effect and the factor of subjects was used as a random effect. The mean value of determination coefficient of general linear models for each subject was also calculated. Moreover, in order to check whether predicted repetitions have achieved the goal of explaining actual values, residual analysis was performed with individual data to assess whether some of the underlying assumptions of regression have been violated. Ideally, the residuals follow white noise, that is, the residuals have a zero mean, constant variance and are

uncorrelated. The Goldfeld-Quandt test was applied to check the presence of heteroscedasticity, and the Durbin Watson test was applied to detect autocorrelation in the residuals. Furthermore, in order to examine the impact of training protocol on the accuracy of the RT model, correlation between training variables and absolute values of residuals was analyzed with multivariate analysis.

Prediction of repetition maximum and impulse generated from fast motor unit

The fast motor unit's time course of activation level during each training protocol was calculated using the mean of individual parameters (shown in Table 3).

The repetition maximum (RM), the maximum number of repetitions, and the integrated value of the fast motor units' activation level (FT impulse) for the multiple training protocols were predicted using the RT model. Then, we predicted the Load-Velocity-RM relationship and Load-Velocity-FT impulse relationship using the calculated RM and FT impulses, respectively. We assumed that $MVC_{failure}$ is dependent on training variables such as training load, number of sets, and contraction velocity. Therefore, multivariate analysis was performed to identify how training variables influenced the $MVC_{failure}$. Akaike information criterion (AIC) was used to select variables in this regression model [eq. (8)]

$$MVC_{failure} = 40.053 + 0.4598 \cdot L - 1.448 \cdot m \quad (8)$$

We also simulated the cases when a set stops by dropping one's residual capacity to 80% of the baseline.

Statistical analysis

Collected data were calculated and presented as mean with standard deviation (SD) in parentheses. Fatigued MVCs measured at the start and the middle of all rest intervals in each session were calculated as percentage ratio of the baseline MVC, taken at the beginning of each session. For all statistical tests, $P < 0.05$ was considered significant. All calculations in this study were performed using R (<https://www.r-project.org/>).

Results

Results in MVC test 1 showed that MVC measurement in training sessions did not affect the performance of the subsequent exercise.

Table 3 shows the calculated individual parameters. The 95 percent confidence intervals for $RC_1(0)$, ϕ_1 , ϕ_2 , ρ_1

Table 3. Calculated individual parameters

$RC_1(0)$	ϕ_1	ϕ_2	ρ_1	ρ_2
39.4 (6.0)	0.011 (0.002)	0.076 (0.015)	0.041 (0.013)	0.006 (0.002)

and ρ_2 were (33.1, 45.7), (0.009, 0.012), (0.060, 0.092), (0.028, 0.054) and (0.004, 0.008), respectively. The correlation between the calculated RC_1 (0) and 15th MVC of MVC test 2 was significant ($R^2 = 0.77$).

The generalized linear mixed model shows the regression coefficient for the predicted number of maximum repetitions to actual measures was 1.01. The general linear models for each subject show that predicted number of maximum repetitions agreed with the actual values to the extent of $R^2 = 0.87$. Also, the residual analysis showed that the residuals of the RT model have a zero mean and have neither heteroscedasticity nor autocorrelation. It is noted that the residuals follow white noise. On the other hand, multivariate analysis showed that the absolute values of residuals had an association with training parameters, especially, training load (standardized partial regression coefficient = -0.51), contraction velocity (standardized partial regression coefficient = 0.30) and the number of sets (standardized partial regression coefficient = -0.04).

Fig. 3 shows the fast motor unit's time course of acti-

vation level under higher (80% 1RM) and lower (30% 1RM) load (Fig. 3). The FT activity level decrease was observed at the later set in low intensity protocol.

Effects of training variables on RM (Fig. 4) and FT impulse (Fig. 5) are shown. Fig. 6 also shows the load-velocity-FT impulse relationship of the case when a set stopped by dropping one's residual capacity down to 80% of the baseline.

Discussion

In this study, we proposed and validated a simple theoretical model for resistance training in the case of knee extension exercise performed by healthy males. Prediction of the proposed RT model closely matched the actual repetition measurements (i.e. time to reach "failure") with $R^2 = 0.87$. This indicates that the model can explain the decrease in MVC, which was known to be less affected by central fatigue²⁸.

For individual parameters, variability of RC_1 (0) between subjects could be explained by their variations in

Fig. 3 The fast motor unit's time course of activation level (solid line) and sum of each motor unit type (dotted line) under higher (80% 1RM) and lower (30% 1RM) load with normal ($v = 1.57$) velocity predicted by the RT model. A: higher load protocol at the 1st set, B: higher load with long rest interval (RI = 120) protocol at the 5th set, C: lower load protocol at the early stage of 1st set, D: lower load with short rest interval (RI = 30) protocol at the early stage of 5th set.

Fig. 4 Load-Velocity-RM surface: (A) result of the 1st set, (B, C) results of the 5th set. The surfaces of (B) and (C) represent RI = 30 and RI = 120, respectively.

Fig. 5 Load-Velocity-FT impulse surface: (A) result of the 1st set, (B, C) results of the 5th set, and (D, E) sum of 5 sets. The surfaces of (B, C) and (C, E) are RI = 30 and RI = 120, respectively.

Fig. 6 Load-Velocity-FT impulse surface of the cases when a set stopped by dropping the residual capacity down to 80% of the baseline. (A) represents result of the 1st set, (B, C) represent result of the sum of 5 sets. The surfaces of (B) and (C) represent RI = 30 and RI = 120, respectively.

the distribution of different fiber types ($R^2 = 0.77$). It is assumed that there are three main factors contributing to the in-between subject differences in fatigue parameters. They are predominant muscle fiber type, mobilized muscle balance, and central fatigue.

There are at least three FT fiber variations (IIa, IIx and IIb). The influence of different FT fibers on fatigue resistance and force development depend on genetic traits. In this study, FT fiber types were not identified. The difference may also affect the variation of ϕ_2 as well as ρ_2 .

The balance between the recruited muscles also affects fatigue rate. The knee extensor muscle (quadriceps femoris) consists of four individual muscles: rectus femoris, vastus lateralis, vastus medialis, and vastus intermedius, and all of them are involved in knee extension exercise. The effects from each muscle differ and the total effect is the balance of the four muscles. Pincivero et al. reported that the presence of more recruiting rectus femoris muscle might lead to a larger fatigue rate²⁹⁾.

Central fatigue is also an important factor as it influences the activation of FT fiber³⁰⁾. Therefore, a large increasing rate of central fatigue during resistance training can lead to an increase in the fatigue parameter of FT fiber (ϕ_2).

Generally, it is considered that fatigue is necessary to acquire FT fiber recruitment during low-load resistance training⁸⁻¹¹⁾. By using this simple RT model, FT fiber recruitment for several training variables can be calculated. The RT model can be useful for providing a training pro-

toocol which induces FT fiber recruitment with low-load and without full-exhaustion. This scheme is also effective in maximizing the training effects.

The RT model calculates impulse of FT fiber during resistance training of multiple protocols. Results show that the total FT fiber impulse was larger in the protocol with long rest interval between sets than in the protocol with short rest interval between sets.

The RT model shows that during low intensity training, more FT fiber was recruited than during high intensity training by having longer rest intervals as seen in Fig. 5. However, in the actual resistance training, reaching the exhaust state with low intensity was difficult. As seen in Fig. 6, high intensity training tends to induce more FT fiber recruitment than in low intensity training in the case of non-failure. Thus, a recommendation to a trainee should take the conditions of the individual into consideration.

Expression of eccentric contractions has room for future improvement. One of the most unique parts of the proposed RT model is the inclusion of eccentric contraction phase in eq. (4). The main reason the dynamic voluntary muscle activation model is difficult is the lack of information on the recruiting rule of muscles during eccentric contraction. We assumed muscle recruitment to be proportional to the occupancy of residual capacity for each fiber type. In the future, we may need to modify the RT model, when the rules of mobilization during eccentric

contraction become clear; although reproducibility of the current model ($R^2 = 0.87$) suggests that the necessity of the modification is low.

To our knowledge, the RT model proposed and validated in this study is the first simple mathematical model which describes dynamic resistance training that includes both concentric and eccentric actions. The model demonstrates muscle fatigue and recovery related to various training variables. When FT fiber recruitment calculated by the RT model can be evaluated experimentally, the model is expected to be useful for exploring a training protocol to mobilize the FT fiber better. Future studies are planned to clarify the relationships between the FT impulse and effects of resistance training, and to expand target participants and exercises.

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Conflict of Interests

The authors have no conflict of interests associated with this manuscript.

Author Contributions

SH and NI conceived and designed the study. SH performed the study and analyzed the data. SH also interpreted the data and wrote the paper. All authors approved the final version of the manuscript.

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