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Streamline Simulation of Hot Water Flooding Processes
in Heavy Oil Reservoirs

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Conventional simulations of hot water flooding processes are based on fully-implicit finite difference (FD) thermal models, which have been successfully applied to thermal simulation. However, these models require long computational times and may suffer from grid orientation effects. Streamline simulation offers a viable alternative to FD simulation if the reservoir heterogeneity and fluid mobility dominate the displacement mechanism. This paper describes the development and verification of a new streamline-based model, and applications to the simulation of hot water flooding processes in heavy oil reservoirs. For simplification of the modeling, heat was assumed to transfer only by convection, in the direction parallel with the flowing phases. This convective heat flow was solved implicitly along streamlines. An important extension of this method did not assume volumetric heat flux along streamlines as a constant but as a variable depending on the temperature and pressure. This variable was introduced as a sink or source in the streamline heat transport model. The developed model was applied to three case studies and the results were compared with those from the 5- and 9-point schemes of a commercial fully-implicit thermal simulator. The streamline results were closer to those of the 9-point scheme than the 5-point scheme. The results also demonstrated that the streamline-based model minimizes the grid orientation effects and requires less time than the FD thermal model.

Keywords

Heavy oil reservoir, Hot water flooding, Thermal simulation, Streamline simulation, Heat transport

1. Introduction

Hot water flooding is an effective recovery mechanism for heavy oil reservoirs, in particular heavy oil of 10 to 20°API gravity with *in-situ* viscosity of several hundred centipoises^{1)~5)}. Oil displacement processes are considered unfavorable if the oil is displaced immiscibly by both cold and hot water. Recovery efficiency is determined partly by the local displacement efficiency and partly by the fluid flow within the reservoir. The local displacement efficiency is primarily controlled by the improved oil mobility resulting from the reduction in the oil viscosity and the reduction in residual oil at high temperatures. Fluid flow and the associated convective heat flux are also controlled strongly by reservoir heterogeneity.

Management of hot water flooding requires an understanding of how the injected heat fluid displaces the oil to the producing wells. By quantifying these heat fluid movements, the displacement process can be actively managed to enhance the oil recovery. The possible al-

location of new wells is readily determined once the unswept areas are identified. Injection rate and temperature can be optimized to maximize oil production, and inefficiencies in the flooding such as cycling can be eliminated.

The fully-implicit FD thermal method can provide general answers for quantifying fluid behavior. This method can handle the rapidly changing properties and complex physical mechanisms of the processes. However, the time constraint prohibits the development and use of detailed models to capture complexities in the interaction of reservoir heterogeneity and fluid behavior. The solutions are sensitive to the grid system because the fluids move between discrete grid-blocks. The method could also introduce errors into the predictions of fluid movements if the numerical scheme is not carefully chosen. Much work has been done to reduce the dependency of simulation results on the orientation and the degree of refinement of the numerical grid system. A 9-point scheme can alleviate the grid orientation effects⁶⁾. For coarse grids, a high order differencing technique of up to third-order accurate total variation diminishing with 9-point schemes should be considered⁷⁾. However, simple methods providing

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a practical balance between detailed thermodynamics and computational efficiency are not available.

In contrast, streamline simulation can model detailed convective displacements within reasonable computation times, because the fluids move along streamlines, not through grid-blocks. The solutions provide a stronger indication of the flow features. In addition, relatively large time-steps can be adopted as they are not affected by the grid-based stability limitations. Therefore, streamline simulation can provide fast evaluation of reservoir performance. Under steady-state isothermal displacement, the streamline method is the ideal tool for quantifying the injector to producer relationship⁸⁾, determining the fluid flow pattern in a field⁹⁾, and quickly assessing multiple full-field development scenarios such as injection and well allocation strategies¹⁰⁾.

The present study further extended the application of the streamline method to unsteady-state thermal displacement through simulation of the hot water flooding process. The modeling conditions were two-dimensional (2D) areal, two-phase flow, with negligible capillary pressure and heat conduction. The last two assumptions are justified at high injection rates in which sharp propagation fronts are expected to develop^{4),5)}. These conditions are known to be challenging to streamline simulation. The streamline-based simulation accounted for unsteady-state thermal effects in two steps. First, the total velocity which defines the streamlines was calculated based on the changes in temperature and pressure. Second, the divergence of volumetric heat flux caused by the temperature and pressure variations along the streamline directions, previously assumed to be constant^{11),12)}, was modeled by introducing a sink or source term in the streamline heat transport equation.

The present approach was tested using artificial examples involving one-quarter, single, and double five-spot patterns in homogeneous and heterogeneous heavy oil reservoirs that had undergone cold water flooding. The results were compared with the equivalent predictions from a commercial thermal simulator (CMG STARS) using the 5- and 9-point fully implicit FD schemes¹³⁾. The streamline-based thermal model showed closer agreement with the 9-point scheme but exhibited fewer grid orientation effects. The streamline-based thermal model required less computational time than the FD thermal model.

2. Streamline Simulation

Streamline simulation is greatly facilitated by the time-of-flight concept. Using the time of flight, τ , as the coordinate space instead of the (x, y, z) coordinates, any three-dimensional (3D) convection-driven equation can be transformed into multiple one-dimensional (1D)

equations that are solved along the streamlines. A brief discussion of this coordinate transformation is given in the following section. The extension of the streamline method to the unsteady-state thermal problem is also described.

2.1. Time-of-flight and Steady-state Flow

The time-of-flight can be defined as^{14),15)}

$$\tau(x, y, z) = \int_0^s \frac{\phi}{|\vec{v}_t(s)|} ds$$

which is the time required to reach a distance s along a streamline based on the total velocity $|\vec{v}_t|$ along the streamline. The permeability, porosity, and total mobility of the 3D domain are defined along the streamline by the τ coordinate.

To determine the coordinate transform from (x, y, z) to τ , rewrite Eq. (1) as

$$|\vec{v}_t| \frac{\partial}{\partial s} = \phi \frac{\partial}{\partial \tau} \quad (2)$$

$|\vec{v}_t|$ can be stated as¹⁶⁾

$$|\vec{v}_t| = \frac{ds}{dt} = \frac{\partial s}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial s}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial s}{\partial z} \frac{\partial z}{\partial t} = \vec{v}_t \cdot \nabla s$$

Substituting Eq. (3) into Eq. (2) yields an expression for the operator equality along the streamline as

$$\vec{v}_t \cdot \nabla = \phi \frac{\partial}{\partial \tau} \quad (4)$$

This equality is used to transform any convection-driven equation from a 3D problem to a set of 1D problems along the streamlines.

To illustrate this transformation, consider the sequential formulation of the water-flood displacement in the absence of sink and source

$$\phi \frac{\partial (b_w S_w)}{\partial t} + \vec{v}_t \cdot \nabla (b_w f_w) + (b_w f_w) \nabla \cdot \vec{v}_t = 0 \quad (5)$$

For steady-state isothermal flow, the term $\nabla \cdot \vec{v}_t$ vanishes. Assuming that the reciprocal of the water formation volume factor, b_w , is constant, Eq. (5) leads to

$$\phi \frac{\partial S_w}{\partial t} + \vec{v}_t \cdot \nabla f_w = 0 \quad (6)$$

This expression is transformed into the τ coordinate using Eq. (4) as

$$\frac{\partial S_w}{\partial t} + \frac{\partial f_w}{\partial \tau} = 0 \quad (7)$$

This is the most familiar expression encountered in the streamline-based water-flood simulation. Therefore, to implement the streamline method, all that is required is the calculation of the integral in Eq. (1). The details of this calculation are given by Pollock¹⁴⁾. Note that the total volumetric flux is divergence-free in Eq. (7). If a specific volumetric flux is assigned to a

Fig. 1 Schematic Diagram of Streamline Calculations

streamline at an injector, then the same flux will be transported to a producer.

2.2. Generalization to Unsteady-state Flow

For unsteady-state thermal flow, the total volumetric flux is not divergence-free and the term $\nabla \cdot \vec{v}_t$ does not vanish. The required expression was derived from Pollock's algorithm by Cheng *et al.*¹⁷⁾. This algorithm starts by defining the velocity in a grid-block as a linear interpolation of the velocity components at the grid-block faces,

$$v_x = v_{x,o} + a_x(x - x_o), \quad a_x = \frac{v_{x,\Delta x} - v_{x,o}}{\Delta x} \quad (8a)$$

$$v_y = v_{y,o} + a_y(y - y_o), \quad a_y = \frac{v_{y,\Delta y} - v_{y,o}}{\Delta y} \quad (8b)$$

$$v_z = v_{z,o} + a_z(z - z_o), \quad a_z = \frac{v_{z,\Delta z} - v_{z,o}}{\Delta z} \quad (8c)$$

Figure 1 is a schematic representation of the above variables in a 3D (x, y, z) grid-block. According to Eqs. (8a), (8b), and (8c),

$$\nabla \cdot \vec{v}_t = \sum_{j=1}^3 a_j = a_x + a_y + a_z = a$$

Equations (9) and (4) are then used to transform any unsteady-state 3D thermal convection equation into 1D streamline equations that account for the divergence of the flux along the streamlines.

Streamlines are launched from the grid-block faces containing the injector. The initial flux for each streamline is simply assigned as¹⁸⁾

$$q_o = \frac{q_{\text{face}}}{n_{\text{face}}} \quad (10)$$

where q_{face} is the flux out of a given face, and n_{face} is the number of streamlines launched from the face. For unsteady-state thermal flow, this flux varies along streamlines according to the variables τ and a . Cheng *et al.* expressed this relationship in the form

$$q = q_o e^{\left(\frac{a\tau}{\phi}\right)} \quad (11)$$

This formulation is used for mapping the calculated variables along the streamlines back to the grid-blocks. Note that this formulation is sufficiently general for any vector field parallel to the velocity defining the streamlines, so can be applied to convective heat transfer that has components in the x, y , and z directions parallel to the x, y , and z components of the fluid flow¹⁹⁾.

3. Hot Water-flood Simulation with Streamline

The governing equations for hot water flooding based on the streamline method are discussed in the following sections.

3.1. Hot Water-flood Model

A mathematical model that describes the hot water flooding processes in heavy oil reservoirs was adapted from the two-phase black-oil model. The water and oil components are assumed to always be present as the water and oil phases. The governing equations for the 2D general thermal model, neglecting heat conduction, are given by

$$\begin{aligned} -\nabla \cdot (\rho_w \vec{v}_w) + q_w &= \frac{\partial(\phi \rho_w S_w)}{\partial t} \\ -\nabla \cdot (\rho_o \vec{v}_o) + q_o &= \frac{\partial(\phi \rho_o S_o)}{\partial t} \\ -\nabla \cdot (\rho_w H_w \vec{v}_w + \rho_o H_o \vec{v}_o) + q_w H_w^w + q_o H_o^w \\ &= \frac{\partial}{\partial t} (\phi (S_w \rho_w U_w + S_o \rho_o U_o) + (1 - \phi) \rho_r C_r T) \end{aligned} \quad (14)$$

where the superscript w stands for well conditions. The velocity, $\vec{v}_\alpha$, for 2D areal flow is given by Darcy's law

$$\vec{v}_\alpha = -k(\lambda_\alpha \nabla p), \quad \alpha = w, o \quad (15)$$

where λ_α is the relative phase mobility,

$$\lambda_\alpha = \frac{k_{r\alpha}}{\mu_\alpha} \quad (16)$$

The mass flow rate, q_α , from a layer k at a well is given by Peaceman's formulation²⁰⁾ as

$$q_\alpha = \frac{2\pi \Delta z_k}{\ln \frac{r_{e,k}}{r_{w,k}} + s_k} \frac{k k_{r\alpha} \rho_\alpha}{\mu_\alpha} (p_k^w - p_{\alpha k}) / (\Delta x \Delta y \Delta z_k) \quad (17)$$

3.2. Streamline Heat Transport Model

The streamline method is a different type of sequential approach in which streamlines are used instead of grids to solve any conservation equation in sequential steps. The formulation presented here has some similarities with our previous work¹²⁾. Equations

(12), (13) and (17) were first solved simultaneously for pressure and saturation with Newton-Raphson's procedure on a grid system. A multi-grid solver²¹⁾ was used to solve the resulting set of linear equations. The obtained pressures were then converted to the total velocity field using Eq. (15) from which the streamlines were traced. Finally, the heat conservation equation was solved along individual streamlines using a 1D formulation expressed in terms of the time-of-flight variables.

In this study, the streamlines were launched from the faces of the well blocks, as originally proposed by Batycky¹⁸⁾. Recognizing that $\partial\phi/\partial t = 0$ from the assumption of constant pressure for solving for temperature, the sequential formulation of Eq. (14) for 2D areal flow outside the well blocks takes the form as follows¹²⁾

$$\frac{\partial E}{\partial t} + \frac{1}{\phi} \nabla \cdot (\vec{v}_t F) = 0 \quad (18)$$

Here, E is the total energy per unit volume written as

$$E = S_w \rho_w U_w + S_o \rho_o U_o + \frac{1-\phi}{\phi} \rho_r C_r T$$

and F is the convective energy flow defined as

$$F = f_w \rho_w H_w + f_o \rho_o H_o \quad (20)$$

where f_α is the fractional flow of the phase α given as

$$f_\alpha = \frac{k_{r\alpha}/\mu_\alpha}{k_{rw}/\mu_w + k_{ro}/\mu_o}$$

Expanding the divergence operator, Eq. (18) leads to

$$\frac{\partial E}{\partial t} + \frac{F}{\phi} \nabla \cdot \vec{v}_t + \frac{\vec{v}_t}{\phi} \cdot \nabla F = 0 \quad (22)$$

Using Eqs. (4) and (9), the streamline heat transport model now takes the following form

$$\frac{\partial E}{\partial t} + \frac{\partial F}{\partial \tau} = -\frac{aF}{\phi} \quad (23)$$

The right-side term in Eq. (23) acts as a sink or source which accounts for the effects of the divergence of volumetric heat flux along the streamlines.

For numerical solution, an implicit approximation was employed for Eq. (23). The discretization form in terms of its residual is

$$R = \frac{1}{\Delta t_T} [E_i^{k+1} - E_i^k] + \frac{1}{\nabla \tau} \left[F_{i+\frac{1}{2}}^{k+1} - F_{i-\frac{1}{2}}^{k+1} \right] + \left[\frac{a_i^{n+1} F_i^{k+1}}{\phi_i^{n+1}} \right] = 0 \quad (24)$$

Values of the inter-block quantities were taken at the upstream block. The subscript i represents nodes along the streamline. The superscripts k and n denote iteration levels between streamline solutions and between pressure solutions, respectively. The iterative solution was considered to reach convergence when the temperature increment between iterations, δT , became smaller than the convergence tolerance. The time

Fig. 2 Starting Conditions for Hot Water Flooding in Homogeneous Reservoir, (a) Pressure (kPa), (b) Water Saturation

step, Δt_T , was taken to be the same as the pressure time step, Δt . The time step needed to be small enough to account for the rapidly changing saturation, viscosity and density involved in the coefficients of the pressure equation. The gband solver²²⁾ was used to solve the resulting set of linear equations derived from Newton-Raphson's procedure. To calculate the fluid properties on the time-of-flight grids, the pressure, temperature, and saturation must be stored together with the time-of-flight and volumetric flux along the streamlines.

The temperature calculations advanced to the desired time along each streamline were then mapped back to the grid-blocks. The average temperature in a grid-block was calculated by weighting the temperature with local volumetric flux and time-of-flight of each streamline within the grid-block as follows:

$$\bar{T} = \frac{\sum_{i=1}^{n_s} T_i q_i \Delta \tau_i}{\sum_{i=1}^{n_s} q_i \Delta \tau_i} \quad (25)$$

where n_s is the number of streamlines in the grid-block.

4. Model Validation

This phase of the study was conducted to examine the effects of the divergence of volumetric heat flux along streamlines on the solutions, and to demonstrate the performance of the simulator in terms of minimizing the grid orientation effects. This simulator is termed TESIS for the remainder of the paper.

4.1. Data for Simulation

The reservoir contains heavy oil of 14°API gravity and relatively low viscosity (100 cP at 40°C) with the pressure and water saturation distributions as depicted in Fig. 2. The oil saturation around the injector is close to the oil residual saturation, S_{or} , of 0.45. The oil saturation around the producer is still about the initial oil saturation, S_{oi} , of 0.7825, but has low mobility. These values are typical for a reservoir that has under-

Table 1 Parameters for Fluid Properties Calculations

Parameters	Water	Oil
Critical properties:		
M [kg/gmol]	0.018	0.600
T_{cr} [°C]	374.1	493.8
p_{cr} [kPa]	2.206E+4	1.118E+3
Density: $\rho = \rho^{sc} \cdot \exp[c(p - p_{sc}) - a(T - T_{sc}) - 0.5b(T^2 - T_{sc}^2)]$		
ρ^{sc} [kg/m ³]	998	972
c [kPa ⁻¹]	4.57E-7	7.30E-7
a [°C ⁻¹]	-1.91E-3	3.80E-4
b [°C ⁻²]	0.00E+0	0.00E+0
Viscosity: $\mu = a_{vis} \cdot \exp(b_{vis}/T)$		
a_{vis} [kPa·day]	9.4953E-14	1.4097E-15
b_{vis} [K]	1328.626	4264.587
Enthalpy, $C_g = c_1 + c_2T + c_3T^2 + c_4T^3$, $H_{vap} = h_{vap}(T_{cr} - T)^{e_{vap}}$		
Internal Energy: $H_g = \int_{T_{ref}}^T C_g dT$, $H_\alpha = H_g - H_{vap}$, $U_\alpha = H_\alpha - p/\rho_\alpha$		
c_1	32.24	-22.38
c_2	1.924E-3	1.939E+0
c_3	1.055E-5	-1.117E-3
c_4	-3.596E-9	2.528E-7
h_{vap}	25.1	287.1
e_{vap}	0.380	0.625

Table 2 Rock Properties and Reference Conditions

Reservoir heat capacity ($\rho_r C_r$)	[kJ/(m ³ ·°C)]	2347
Porosity at reference pressure		0.3
Rock compressibility	[1/kPa]	7.24E-5
Reference pressure	[kPa]	6890
Reference temperature	[°C]	15.6
Surface pressure	[kPa]	100
Surface temperature	[°C]	15.6

gone cold waterflooding, which was terminated before the cold water reached the producers. The permeability is uniformly distributed with 1000 mD. A quarter five-spot pattern was employed in an area of 75×75 m² with 5 m thickness, divided into 50×50 grid-blocks. The wellbore radius is 0.09 m. The producer was constrained with bottom-hole pressure at 5170 kPa. The injector was constrained to a rate of 100 m³/day cold water equivalent (CWE) at 150°C. The details of the constants used for the fluid property calculations appear in **Table 1**. Rock properties and reference conditions are given in **Table 2**. The relative permeability curves are shown in **Fig. 3**.

4.2. Volumetric Heat Flux Effects

Figure 4 shows the cell values of flux calculated by Eq. (9). The values are not divergence-free for the situation studied here. The flux divergence becomes stronger when an unstable front is generated as seen in **Fig. 4 (b)**. The values of the flux divergence are negative at the center of the reservoir, but are positive at most other parts. This trend corresponds to the presence of the oil bank as shown in **Fig. 4 (d)**. The ef-

Fig. 3 Relative Permeability Curves

Fig. 4 Velocity Gradient, a , and Water Saturation Distributions, (a) Velocity Gradient at 10 Days, (b) Velocity Gradient at 100 Days, (c) Velocity Gradient at 300 Days, (d) Water Saturation at 100 Days

fects of this divergence were modeled by adding a sink or source term in the streamline heat transport model as given at the right-hand side in Eq. (23).

The source and sink terms represent the heat flux transverse to the flow direction due to unsteady velocity. This transverse heat flux generates a change in effective volume along the streamlines. To examine the impact of this term on the solutions, simulations were run for two cases; in which the heat source or sink arising in Eq. (23) was turned off or left on. **Figure 5** shows

Fig. 5 Production Performance of Quarter 5-Spot Pattern, (a) Oil Recovery, (b) Oil Production Rate, (c) Water Cut

the production performance for both cases. The results from STARS with 5- and 9-point schemes are also included for validation purposes.

The results obtained including the flux divergence along the streamlines yielded close matches with the STARS results. A very good match was obtained for the recovery curve as shown in Fig. 5 (a). The peak of the oil production rate without the source/sink term was much higher than the others (Fig. 5 (b)) and leads to overoptimistic oil recovery estimates. Absence of the transverse heat flux caused the heat front to move faster in the diagonal direction as shown in Fig. 6. The breakthrough of the heat front was much faster as seen in Fig. 6 (b). The oil rate and water cut profiles with the flux divergence were slightly different from those obtained from STARS, as discussed in the following sections.

4.3. Grid Orientation Effects

Figures 5 and 6 show the choice of numerical scheme in STARS produced varying simulation results. As seen in Fig. 5 (b), the onset of incremental oil production is earlier for the 9-point scheme than the 5-point scheme. This is consistent with the temperature front shapes in Figs. 6 (c) and 6 (d). The breakthrough of heated water to the producer is earlier for the 9-point scheme than the 5-point scheme, because the 9-point scheme accounts for flow not only in the x - and y -directions but also in the xy -diagonal direction. The grid orientation effects are minimized by the 9-point scheme. In the 5-point scheme, the flow moves between discrete grid-blocks, so the solutions reflect stronger grid orientation effects. The superiority of the 9-point scheme over the 5-point scheme in terms

Fig. 6 Temperature Distributions for Quarter 5-Spot Pattern after 300 Days, (a) TESIS with Heat Flux Divergence, (b) TESIS without Heat Flux Divergence, (c) STARS with 5-Point Scheme, (d) STARS with 9-Point Scheme

Fig. 7 Production Performance of 50×50 and 100×100 Grid Systems, (a) Oil Recovery, (b) Oil Production Rate

of minimizing the grid orientation effects was confirmed for the immiscible thermal displacement process^{4),6)}. Therefore, our findings emphasize the accuracy of the 9-point scheme. The results of TESIS are closer to those of the 9-point scheme than the 5-point scheme, particularly in the front shape.

To examine the dependency of the simulation results on the numerical grid, the 50×50 grid-blocks were refined to 100×100 grid-blocks. Figure 7 compares the results of the two grid systems. The production profiles, especially the oil production peaks, are quite different. The oil rate of the fine grid lies at first below and later crosses over the solution of the coarse grid. The difference can also be observed in their corresponding recovery curves. The difference is more apparent in the solution of STARS with the 5-point scheme, which is inferred to result from the grid orien-

Fig. 8 Permeability Field and Starting Conditions for Hot Water Flooding in Single 5-Spot Pattern, (a) Permeability Field (mD), (b) Pressure (kPa), (c) Water Saturation

tation effects. In contrast, the solutions of STARS with the 9-point scheme and TESIS are insensitive to the number of grid-blocks, and this sensitivity is less in TESIS. The corresponding recovery curves that represent the integrated response of displacement remained unchanged.

TESIS proved to be more efficient in term of run time. For the 50×50 grid-blocks, TESIS required 1.21 min, whereas STARS required 2.38 and 1.87 min for the 5- and 9-point schemes, respectively. For the 100×100 grid-blocks, TESIS required 11.47 min, an increase by a factor of 9. STARS with the 5- and 9-point schemes needed 25.70 min and 21.77 min, increases by factors of 11 and 12.

5. Simulation with Heterogeneous Reservoirs

The performance of TESIS was then evaluated for more realistic displacement of hot water flooding in heterogeneous reservoirs with multiple wells.

5.1. Single Five-spot Pattern

The reservoir contains heavy oil of 14°API gravity and moderate viscosity (300 cP at 50°C), with the pressure and water saturation distributions as shown in Fig. 8. The reservoir has undergone cold water flooding for 10 days. The oil around the injector has been displaced by the cold water injection, which has caused low oil saturation. The cold water injection was terminated before the water reached the producers, leaving high oil saturation around the producers. This oil has low mobility, hence low well productivity. The area

Fig. 9 Field Production Performance of Single 5-Spot Pattern, (a) Oil Production Rate, (b) Water Cut

of $250 \times 250 \text{ m}^2$ with 5 m thickness was divided into 100×100 grid-blocks. Figure 8 also shows the well locations and the permeability field. The permeability values range from 142 to 5332 mD with a Dykstra-Parsons coefficient of 0.51. The producers were constrained with bottom-hole pressure at 1720 kPa. The injector was constrained with a rate of $150 \text{ m}^3/\text{day}$ CWE at 180°C . Other values were the same as those used in the previous model.

Figure 9 compares the field oil rate and water cut predictions by TESIS and STARS at the surface conditions. The production profile has three major stages. The first stage is dominated by the initial oil viscosity. The hot water front displaces both the cold water and the oil with low mobility toward the producers. Oil production tends to decrease gradually. The second stage is dominated by the oil mobility and reservoir permeability of the heated zone. The production rate is essentially proportional to the displacement rate. The last stage is dominated by depletion and the remaining fraction of mobile oil. The match between TESIS and STARS is good up to the second stage of production. A substantial difference appears in the third stage. The oil production after the peak calculated by TESIS is much lower than by STARS. The corresponding water cut predicted by TESIS at this stage is consistently higher with lower oil rate. The difference is most likely a result of less pressure depletion in TESIS as a consequence of the sequential approach. Using the temperature at the start of the time step, the overall transmissibility tended to be lower, which resulted in lower pressure drops between the injector and producers. The lower pressure drops leads to the lower production.

Figure 10 compares the oil rate and water cut performance of TESIS and STARS on a well-by-well basis. The matches are generally good except for well P3. Over-prediction of oil production by STARS with 9-point scheme is observed in the well, giving a higher field oil rate as seen in Fig. 9. This over-prediction can be explained by examining the temperature distributions at 1200 days shown in Fig. 11. The breakthrough of heated water occurred at well P3 in simu-

Fig. 10 Well Production Performance of Single 5-Spot Pattern

Fig. 11 Temperature Distributions of Single 5-Spot Pattern after 1200 Days, (a) TESIS, (b) STARS with 5-Point Scheme, (c) STARS with 9-Point Scheme

lation by STARS with the 9-point scheme, but not in simulation by TESIS or STARS with the 5-point scheme. The higher oil rate in the well P3 obtained by STARS with the 9-point scheme coincides with the breakthrough of heated water in this well.

As **Fig. 11** shows, the solutions were impacted by the choice of numerical scheme. The growth of the temperature front calculated by STARS with the 5-point scheme followed the direction of the coordinate axes, whereas STARS with the 9-point scheme generated

flow in the diagonal direction of the grid. The results of TESIS were intermediate as expected because the solution of temperature is not based on the grid, but on the streamlines that were dynamically changed to account for the changes in the phase properties.

The problem simulated here presents an important issue. The nonlinearity of the system was strong due to the significant changes in pressure and temperature caused by the high pressure drops between the injector and producers. In such a system, more iterations are necessary to converge the solutions of STARS with the 9-point scheme than with the 5-point scheme. For this example, the run time of STARS with the 9-point scheme was 52.72 min to complete simulation of 1200 days, whereas STARS with the 5-point scheme required 48.75 min. In contrast, TESIS required only 30.88 min. This example demonstrates that as the nonlinearity becomes stronger, the FD method with the 9-point scheme requires a longer run time than with the 5-point scheme.

5.2. Double Five-spot Patterns

This evaluation used the same oil properties as for the previous simulation. Cold water flooding was carried out before starting the hot water flooding. The cold water flooding was terminated before breakthrough leaving high oil saturation around the producers. The oil has low mobility associated mainly with high oil viscosity. The reservoir dimension is 100 m and 300 m in the x - and y -directions, respectively, with a thickness of 5 m. The area was divided into a regular grid of 40×120 (4800 grid-blocks), but only 4547 grid-blocks were considered as active. The model contains two producers and six injectors. The well locations and the permeability field are shown in **Fig. 12**. The permeability values range from 285 to 2987 mD with a Dykstra-Parsons coefficient of 0.32. The following boundary conditions were used. The producers were constrained with bottom-hole pressure at 10,300 kPa. The injectors were constrained with a hot water rate of 100 m³/day CWE at 150°C. Other data were the same as listed in **Tables 1** and **2**.

The simulated field performance by TESIS is shown in **Fig. 13** together with the results of STARS. The same general behavior as the previous example is observed throughout the results. Matches between TESIS and STARS were considered good up to the second stage of production. In the third stage, the field oil rate of TESIS was consistently lower than those of STARS with the 5- and 9-point schemes. Comparisons of well production performance are shown in **Fig. 14**. The growth of temperature fronts after 300 days of simulation is depicted in **Fig. 15**. Allocation patterns are clearly demonstrated in this figure, which are useful in quantifying the injector to producer relationship. The well P2 is supported by four injectors, I3 through I6, and yields much higher pro-

Fig. 12 Well Locations and Permeability Field (mD) of Double 5-Spot Patterns

Fig. 13 Field Production Performance of Double 5-Spot Patterns, (a) Oil Production Rate, (b) Water Cut

duction, whereas the well P1 is supported only by two injectors, I1 and I2.

As **Fig. 15** shows, the results of STARS with the 5-point scheme were subjected to large numerical dispersion that caused the flood fronts to smear, whereas the 9-point scheme created sharp fronts and left more unswept regions after 300 days injection. Again, the TESIS results were intermediate between the STARS schemes. Compared to the previous simulation, this example had less heterogeneity and less nonlinearity due to the lower pressure drops between the injectors and producers. As a result, STARS with the 9-point scheme was superior to the 5-point scheme in this case. To generate the solution up to 300 days, STARS with the 5- and 9-point schemes required 11.10 min and

Fig. 14 Well Production Performance of Double 5-Spot Patterns

Fig. 15 Temperature Distributions of Double 5-Spot Patterns after 300 Days, (a) TESIS, (b) STARS with 5-Point Scheme, (c) STARS with 9-Point Scheme

8.18 min of run time, respectively. In contrast, TESIS required 5.29 min.

6. Conclusions

The streamline method to model hot water flooding in water-flooded heavy oil reservoirs was evaluated. The results of streamline-based simulation were compared with equivalent simulation with a grid-based thermal simulator with the 5-point and 9-point schemes. The following conclusions were obtained:

- (1) The streamline method can model hot water flooding in heavy oil reservoirs with a reasonable balance between detailed flow modeling and computational efficiency.
- (2) The solutions were improved by including the term representing the transverse heat flux due to unsteady velocity. Transverse heat flux tended to retard advancement of the temperature front.
- (3) The streamline approach and the 9-point FD scheme can minimize the grid orientation effects, which were not strong functions of degree of grid refinement. However, the 9-point scheme was less efficient for problems with high degree of nonlinearity.
- (4) The results of streamline-based simulations were intermediate between the results of the 5-point and 9-point FD schemes.
- (5) The developed simulator required less computational time than the standard thermal simulator.

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Nomenclatures

a	: velocity gradient	[1/day]
b_w	: reciprocal of water formation volume factor	[sm ³ /m ³]
C_g	: specific heat of gas phase	[kJ/(kg·°C)]
C_r	: specific heat of rock	[kJ/(kg·°C)]
f_α	: fractional flow of phase α	[fraction]
H_α	: enthalpy of phase α	[kJ/kg]
k	: absolute permeability	[m ²]
$k_{r\alpha}$	: relative permeability of phase α	[—]
n_{face}	: number of streamline traced from a face block	[—]
p_{ref}	: reference pressure	[Pa]
q_{face}	: flux at grid-block face	[m ³ /day]
q_o	: initial flux for each streamline	[m ³ /day]
q_α	: source or sink flow rate per unit reservoir volume	[kg/(day·m ³)]
r_e	: well drainage radius	[m]
r_w	: wellbore radius	[m]
s	: local streamline coordinate	[m]
s_k	: skin factor	[—]
S_α	: saturation of phase α	[fraction]
T	: temperature	[°C]
U_α	: internal energy of phase α	[kJ/kg]
t	: time	[day]
v_α	: velocity of phase α	[m/day]
<Greeks>		
Δt	: time step size	[day]

$\Delta x, \Delta y, \Delta z$	: grid-block dimension in x -, y -, z -directions	[m]
λ_t	: total mobility	[(m ³ ·day)/kg]
λ_α	: mobility of phase α	[(m ³ ·day)/kg]
μ_α	: viscosity of phase α	[Pa·day]
ρ_r	: rock density	[kg/m ³]
ρ_α	: density of phase α	[kg/m ³]
τ	: time-of-flight	[day]
ϕ	: porosity	[fraction]
<Subscripts>		
cr	: critical	
g	: gas	
i, j, k	: grid-block indices	
o	: oil	
sc	: surface conditions	
t	: total	
vap	: vapor	
w	: water	
<Superscripts>		
k, n	: number of time step	
w	: well	
<SI metric conversion factors>		
141.5/(131.5 + °API) = g/cm ³		
cP × 1.0 [*] E − 03 = Pa·s		
mD × 9.86923 E − 16 = m ²		
*Conversion factor is exact.		

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要 旨

重質油層における熱水攻法のためのストリームライン法シミュレーション

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これまでの熱攻法シミュレーションモデルは、一般に全陰解有限差分法モデルによるもので、実際のシミュレーションを繰り返すには計算時間が長く、シミュレーション結果に対するグリッド方向の影響が不可避であり、数値モデルの設定によって大きな差異が生じる。本研究の対象とした熱水攻法では、油層の非均質性および流体の易動度が油置換メカニズムにおいて支配的な要素である場合、ストリームライン法シミュレーションは有限差分法モデルに代わる有効な手段となり得る。本論文では、ストリームライン法による熱水攻法モデルの構築と検証、ならびに非均質重質油層への適用例について報告する。モデル

構築では、熱移流は流体流動の伴う熱伝達のみによるとし、熱移流式をストリームラインに沿って陰的に解いた。特に、本モデルではストリームラインに沿う熱流束は圧力・温度の影響を受けて変動するものとし、熱移流式では生産・圧入項として扱った。商用の熱攻法モデルの5点および9点差分スキームによるシミュレーション結果と比較して、モデルの検証と実際的な非均質モデルへの適用性を確認した。本モデルでは、グリッド方向の影響が少ない9点法スキームに近い結果が得られ、計算時間も短縮された。