A Computer Simulation of Unsteady-State Gas Flow in Pipe Line

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A simulator for unsteady state gas flow in pipe lines was made in the computer. The gas flow was indicated by the Kinetic Formula and the Continuity Equation. The Simultaneous Difference Equations of these equations were used in the simulator, and the simulations were conducted to find the best simulation approach. The final simulation selected matched the field performance satisfactorily.

1. Introduction

When the pressure or the flow rate of a gas, which is flowing in a steady state in a gas pipe line, is altered by some disturbance at the end of the line, the flow is changed to an unsteady state, and the disturbance is transmitted through the line with time. A theoretical analysis of the above problem is carried out in this paper. An unsteady-state gas flow in a pipeline can be indicated by the Kinetic Formula and the Continuity Equation. These equations were developed in to the 'Simultaneous Difference Equations', and a simulator was made in the computer.

Several simulations were conducted to find the best approach to simulation and to reliability of the simulator. The final simulation results compared well with the field data as shown in **Fig. 10**. Later, the author has succeeded in the automatic computer control of the Nakajo Gas Field by applying this simulation model.¹⁾

2. Gas Flow Equation

There are many equations^{2)~4)} which express the phenomena of unsteady state gas flow in a pipeline. However, the most suitable equations may be the Kinetic Formula and the Continuity Equation.

2.1 Kinetic Formula

If it is assumed that (1) the pipeline is horizontal, (2) the cross section area is constant, (3) the body force can be neglected, and (4) heat change is isothermal, the Kinetic Formula is derived as follows:⁵⁾

$$-\frac{\partial(\rho V)}{\partial t} + V \frac{\partial(\rho V)}{\partial x} + \frac{\partial P}{\partial x} g$$

$$= -\frac{1}{2} f_m \rho V^2 \pi D / S \quad (1)$$

2.2 Continuity Equation

If the flow in the pipeline is one dimensional (x axis) and the cross section area (S) is constant, the Continuity Equation in infinitesimal volume can be shown as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = 0 \quad (2)$$

3. Difference Equations

Eqs. (1) and (2) in common field units are shown as follows:

$$\frac{\partial Q_n}{\partial t} + \frac{P_0}{P} \frac{T}{T_0} \frac{\bar{Z}}{Z_0} \frac{Q_n}{S} \frac{\partial Q_n}{\partial x} + \left(\frac{RT_0 Z_0 S g}{MGP_0} \right) \frac{\partial P}{\partial x} = - \frac{f_m P_0 T Z \pi D}{2PT_0 Z_0 S^2} Q_n^2 \quad (3)$$

$$\frac{\partial Q_n}{\partial x} + \frac{T_0 Z_0 S}{P_0 T Z} \frac{\partial P}{\partial t} = 0 \quad (4)$$

Then the 'Simultaneous Difference Equations' can be obtained as follows:

$$Q_n(x, t + \Delta t) = Q_n(x, t) + \Delta t \left[-2.679 \frac{f_m \bar{Z}}{Z_0} \frac{\bar{T} Q_n^2(x, t)}{D^3 P(x, t)} - 1.339 \times 10^{-5} \frac{\bar{Z}}{Z_0 D^2} \frac{\bar{T} Q_n(x, t)}{P(x, t)} \frac{Q_n(x + \Delta x, t) - Q_n(x - \Delta x, t)}{2 \Delta x} - 21.44 \frac{Z_0 D^2}{G} \frac{P(x + \Delta x, t) - P(x - \Delta x, t)}{2 \Delta x} \right] \quad (5)$$

$$P(x, t + \Delta t) = P(x, t) - 1.339 \times 10^{-5} \frac{\bar{Z} \bar{T}}{Z_0 D^2} \frac{\Delta t}{2 \Delta x} [Q_n(x + \Delta x, t) - Q_n(x - \Delta x, t)] \quad (6)$$

The coefficients of the equations are calculated with the following constant values:

$$P_0 = 1.034 \text{ Kg/cm}^2 \cdot \text{abs.}$$

$$T_0 = 273.2 \text{ K}$$

$$R = 847.8 \text{ Kg-m/kg-mol} \cdot \text{K}$$

$$M = 28.97 \text{ kg/kg-mol}$$

$$g = 9.807 \text{ m/sec}^2$$

The unit of flow rate, (Q_n), is m^3/hr and that

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Fig. 1 The Calculation Steps

of the pipeline length, (x) , is km. The others are the same as those shown in the list of nomenclature. The calculation steps of the Difference Equations (Eqs. (5) and (6)) are illustrated in **Fig. 1**. Therefore, these simultaneous difference equations may need the following values for initial and boundary conditions.

Initial Conditions : $Q_n(x, 0)$; $P(x, 0)$

Boundary Conditions : $Q_n(0, t)$; $P(0, t)$

$Q_n(L, t)$; $P(L, t)$

4. Steady-State Gas Flow Equation

In the steady state, Eq. (3) becomes as follows :

$$\frac{dP}{dx} = \frac{28.97G}{Rg} \left(\frac{P_0}{T_0} \frac{1}{Z_0} \frac{1}{S} \right)^2 \frac{f_m \bar{Z} D \bar{T} \pi}{2SP} Q_n^2 \quad (7)$$

The Difference Equation of the above equation is therefore developed as follows :

$$P(x \pm \Delta x) = P(x) \mp 0.1248 \frac{G f_m \bar{Z} \bar{T}}{D^5 Z_0^2 P(x)} Q_n^2(x) \Delta x \quad (8)$$

$$Q_n(x) = \sqrt{|[P(x) - P(x + \Delta x)] \frac{D^5 Z_0^2 P(x)}{0.1248 G f_m \bar{Z} \bar{T}}|} \quad (9)$$

The coefficients and units of the equation are calculated with the same values as in Eqs. (5) and (6).

5. The Simulator

The simulator is constructed by Eqs. (5) and (6), and it is started by generating the pressure or flow rate change at either end of the pipeline in which the gas is flowing in a steady state. This disturbance is assumed, for simplifying the problem, to be maintained throughout the simulation time. The other conditions of simulator are as follows.

5.1 Initial and Boundary Values

At the start of simulation, the flow is assumed to be in a steady state. The initial values ($Q_n(x, 0)$ & $P(x, 0)$) therefore, are defined in the following equation, which is obtained from Eq.

Fig. 2 The Definition of Terms in This Simulation

(7).

$$P_1^2 - P_2^2 = 0.2497 \frac{G f_m \bar{Z} \bar{T} L}{Z_0^2 D^5} Q_n^2 \quad (10)$$

The boundary values require four values, two of which can be determined by the value of generated disturbance or the initial condition, because of the assumption that the disturbance is maintained throughout the simulation time. However, the other two boundary values cannot be determined, because these values are the very answers which must be obtained by this simulation. If the section is spaced out at a very small distance from either end section of the pipeline, the gas flow between that section and the end section can be assumed immediately to be in a steady state. (Hereafter, this section is called 'the inside section'.) Hence these two boundary values (the values of the end sections) may be calculated by means of the steady state flow equations (Eqs. (8) and (9)) using the value of the inside section which can be calculated by this simulator. The whole pipeline length, therefore, is divided into several equal spans (Δx), and the inside section is

selected with half length or quarter length of the span ($\Delta x/2$, $\Delta x/4$) from the end section as shown in **Fig. 2**. (Hereafter, these are called methods A and B.) When the whole pipeline is very long, *the inside section* cannot be selected at a short distance from the end sections in methods A and B. Therefore, the flow between the end section and *the inside section* in this case cannot be assumed immediately to be in a steady state. Then another method is defined as follows: an image section, whose distance is Δx to outside of pipeline from the end section, is made (see **Fig. 2**) and the flow condition between this image section and the end section is assumed to be in a steady state at any time. (Hereafter this image section is called '*the outside section*') The order of calculation is, therefore, defined as follows in this method. (called method C).

(1) Select a section inside the pipeline where the state of flow between this section and the end section could become steady immediately. (see **Fig. 2**: Hereafter this section is called '*the selected section*') The distance between *the selected section* and the end section therefore depends on the value of disturbance and of calculation step Δt at simulation.

(2) Calculate the initial condition of each of the x_i sections and *the selected section* and *the outside section*.

(3) Calculate the flow rate or pressure at the end section at disturbance-generated time ($\Delta t = 0$). If the disturbance is pressure (flow rate), the flow rate (pressure) at the end section at disturbance generated is unknown, then calculate this value by using the initial pressure at *the selected section* and the disturbance pressure (flow rate) at the end section, by steady state flow equation (Eq. (10)).

(4) Calculate the pressure at *the outside section* by Eq. (10). [The flow rate at *the outside section* is the same as at the end section, because of the flow between *the outside section* and the end section is assumed to be in a steady state.

(5) Calculate the flow rate and pressure at each section at step Δt by Eqs. (5) and (6).

(6) Continue to calculate in the same way for step (4) and (5).

5.2 Selection of Calculation Span 'r'

Generally, the value of 'r' ($\Delta t/\Delta x$), which is smaller than the reciprocal of velocity in the pipeline, is acceptable to the differential equations such as Eqs. (5) and (6). However, if this value is selected to be smaller, the computing time will

be longer. Therefore, several values are selected in the case analysis for finding the largest value of 'r'

5.3 Procedure of Simulation

By considering the gas flow, it may be noted that if no pressure change appears around certain points (x_k) in the pipeline during a short specific time (Δt), the flow rate at that point will not change. However, in the case of simulation by using Eq. (5), even when there is no pressure change in the points ($x_k - \Delta x$, x_k , $x_k + \Delta x$) during the short time, Δt , the flow rate $Q_n(x_k, t + \Delta t)$ may not be exactly the same value as the flow rate $Q_n(x_k, t)$, which was derived previously, because of the round-off error in Eqs. (5) and (6). On the other hand, in this simulator, the round-off error may appear greater in the flow rate than in the pressure at each step, as specified in **Fig. 1** and Eqs. (5) and (6). The following procedure, therefore, was developed. When the simulator is started, at first, the pressure of each section along the pipeline ($P(x_i, t + \Delta t)$) is calculated, and if the pressure change at any three points, ($x_k - \Delta x$, x_k , $x_k + \Delta x$), is smaller than ε compare with the pressure of the previous calculation, (the pressure which is derived at time t) Eq. (5) should not be used and it, ($Q_n(x_k, t + \Delta t)$), should be fixed at the same rate as previous rate, $Q_n(x_k, t)$. Only in the case of pressure change appearing larger than ε at one of these three points, Eq. (5) can be used for calculating the flow rate at the second step ($t + \Delta t$) such as $Q_n(x_k, t + \Delta t)$. The several values of ε are selected in the next section for finding the best value.

6. Case Analysis and Results

Two kinds of analyses were carried out for seeking the best simulation approach and for exploring the reliability of the simulator. In both analyses, the pipeline data used were the actual pipeline data which are being used in NAKAJO GAS FIELD. Namely;

- (1) The diameter of the flow line is 30.47 cm (12 inches) and its length is 6.645 km.
- (2) Friction coefficient (f_m) is 0.003155, which was calculated from Eq. (10) using the actual data of the above pipeline. (If Weymouth Equation⁴⁾ is used, f_m may be calculated to be 0.003477.

6.1 Case Analysis A

The purpose of the first analysis is to find out the best simulation approach. Several case analyses therefore were carried out for each objective.

Any simulation is assumed to be generated by pressure disturbance at the outlet point, downstream, and this disturbance pressure was maintained throughout the simulation time. The following conditions, therefore, were used in this case analysis.

- (1) Initial condition; It should be selected from the actual data of the same pipeline when the flow is in a steady state. Therefore :
 Q_n : 35, 325 m³/hr
 P_1 : 8.93 Kg/cm²
 P_2 : 5.51 Kg/cm²
 $\bar{T}$: 29.35°C
 Z_0 : 0.98
 G : 0.65
- (2) The boundary condition : The disturbance is generated one time at outlet pressure (5.51→5.20) and this value is maintained. Therefore;
* The pressure at the inlet point is kept at initial pressure 8.93 Kg/cm².
* The pressure at the outlet point is kept at disturbance pressure 5.20 Kg/cm².

* The flow rate at both end sections are calculated by using methods A,B or C.

6.1.1 The Calculation Span ‘r’

Eight case analyses were carried out for finding the most suitable value of ‘r’ ($\Delta t/\Delta x$). In all of the eight case analyses, the simulation conditions were as follows.

- (1) Calculation method for boundary values : method A. (the inside section is $\Delta x/2$)
- (2) The value of ϵ : 1×10^{-5} .

The results of simulations are shown in **Table 1**. Column 4 indicates $2(\Delta t/\Delta x)$ instead of $\Delta t/\Delta x$, because method A was used.

6.1.2 The Calculation of Boundary Condition

Three case analyses were carried out for considering a suitable method to determine the boundary values. The value of ϵ used was 1×10^{-5} in all case analyses. The results of simulation are shown in **Table 2**.

6.1.3 The Value of ϵ

Four case analyses were carried out for studying the most suitable value of ϵ . The simulation conditions were as follows :

Table 1 Comparison of Simulation Results—Changing ‘r’—

1. Case	A-1	A-2	A-3	A-4	A-5	A-6	A-7	A-8
2. Δx (km)	$L/200$		$L/100$			$L/50$		$L/20$
3. Δt (sec)	0.01	0.02	0.01	0.02	0.03	0.02	0.06	0.02
4. $2\Delta t/\Delta x$ (sec/m, 10^{-4})	6.02	12.0	3.01	6.02	9.03	3.01	9.03	1.20
5. Q (m ³ /hr)	183, 880		132, 260			96, 605		66, 559
6. $1/v$ (sec/m, 10^{-4})	14.28		19.9			27.2		39.4
7. t (sec)	20—30	—	20—30	20—30	—	20—30	20—30	20—30
8. T_s (sec)	(100)	—	100	(100)	—	70	(100)	70
9. q (m ³ /hr)	(161)	—	189	(135)	—	405	(250)	672
10. $6 \div 4$	2.37	1.19	6.61	3.31	2.20	9.04	3.01	32.83
11. Judgment	○	×	○	◎	×	○	○	○

where; Q flow rate at outlet point when disturbance generate
 v velocity
 t time, when inlet flow rate is changed
 T_s stabilized time.
Gothic figures; the time when both flow rate, inlet and outlet point were getting constant
() ; the time when both flow rate are still changing but changing rate is very small
 q flow rate difference between inlet and outlet point at T_s
— illogical value
Judgment ◎ : better ○ : good × : bad
(consideration by t , T_s and q)

Table 2 Comparison of Simulation Results—Changing Calculation Method of Boundary Condition—

1. Case	A-4	A-9	A-10
2. Method (see section 5.1)	A	B	C
3. the Inside Section (km)	$\Delta x/2$	$\Delta x/4$	—
4. the Selected Section (m)	—	—	50
5. Δx (km)	$L/100$	$L/100$	$L/100$
6. Δt (sec)	0.02	0.04	0.02
7. Q (m ³ /hr)	132, 260	183, 880	111, 400
8. t (sec)	20—30	20—30	15—20
9. T_s (sec)	(100)	(100)	(100)
10. q (m ³ /hr)	(135)	(640)	(277)
11. Judgment	○	△	○

where Q , t , T_s , q are the same as Table 1

- (1) The calculation method of boundary values : method C
- (2) The selected section : 50 meters from the end section.
- (3) $\Delta x : L/30$ (4) $\Delta t : 0.05$ sec.

The results of four case analyses are as shown in **Table 3**.

6.1.4 The Selected Section

Four case analyses were carried out for finding a suitable distance of the selected section in method C. The value of ϵ used is 1×10^{-5} in these analyses.

The results of simulation are shown in **Table 4**. It is noted that A-18 is the best simulation way in the eighteen case analyses. The reasons are described in the discussion given in section 7. The simulation method in A-18 is, therefore, selected in the case analysis B in section 6.2.

The result of flow phenomenon in A-18 is also shown in **Fig. 3** to **Fig. 8**. **Fig. 3** to **6** indicate the change of pressure and flow rate around the outlet point and the inlet point since the disturbance was generated. **Figs. 7** and **8** indicate the same phenomenon as **Fig. 3** to **6** but along the whole length of the line.

6.2 Case Analysis B

The purpose of second analysis is for exploring the reliability of the simulator. Therefore, the actual phenomena of flow rate change at inlet and outlet points are used as generated disturbance and simulated the pressure change at both end sections. Then the results obtained were compared with the real pressure changes. In the analysis, therefore, the following conditions and data were used which were selected from the same pipeline as described in analysis A.

Table 3 Comparison of Simulation Results—Changing Value of ϵ —

1. Case	A-11	A-12	A-13	A-14
2. ϵ (-)	1×10^{-6}	1×10^{-5}	5×10^{-5}	2×10^{-4}
3. Q (m^3/hr)		111,400		
4. t (sec)	4	18	25	28
5. T_s (sec)	(100)	(100)	70	30
6. q (m^3/hr)	(170)	(315)	1,390	3,384
7. Judgment	○	○	△	×

where Q, t, T_s, q are the same as Table 1

Table 4 Comparison of Simulation Results—Changing the Selected Section—

1. Case	A-15	A-16	A-17	A-18
2. the Selected Section (m)	2	10	50	50
3. Δx (-)	$L/30$	$L/30$	$L/30$	$L/20$
4. Δt (sec)	0.05	0.05	0.05	0.10
5. Q (m^3/hr)	529,440	238,870	111,400	111,400
6. t (sec)	17	17	18	17
7. T_s (sec)	130	120	(150)	140
8. q (m^3/hr)	170	172	(200)	134
9. Judgment	○	○	△	○

where Q, t, T_s, q are the same as Table 1

Fig. 3 The Pressure Transition around the Outlet Point (Disturbance Point)
—Case Analysis A-18—

Fig. 4 The Flow Rate Transition around the Outlet Point (Disturbance Point)
—Case Analysis A-18—

Fig. 5 The Pressure Transition around the Inlet Point
—Case Analysis A-18—

Fig. 6 The Flow Rate Transition around the Inlet Point
—Case Analysis A-18—

- (1) Initial conditions :

$Q : 35, 100 \text{ m}^3/\text{hr}$

$P_1 : 8.95 \text{ Kg}/\text{cm}^2$

$P_2 : 5.50 \text{ Kg}/\text{cm}^2$

$\bar{T} : 29.35^\circ\text{C}$
- $Z_0 : 0.98$

$G : 0.65$

(2) Boundary conditions :

* Flow rate at both end sections; **Fig. 9.**

* Pressure at both end sections;

Fig. 7 The Pressure Transition along the Whole Length of Line
—Case Analysis A-18—

Fig. 8 The Flow Rate Transition along the Whole Length of Line
—Case Analysis A-18—

Fig. 9 The Flow Rate Transition at Both End Section in Actual and Simulation Case :
Fixed Boundary Condition —Case Analysis B—

Calculated by method C, (the selected section was two meters from the end section)

(3) $\Delta t/\Delta x=15.0 \times 10^{-4}$ sec/m ($\Delta x=L/20$ $\Delta t=0.5$ sec)

(4) $1/v=75 \times 10^{-4}$ sec/m ($Q=35,000$ m³/hr)

(5) $(4) \div (3)=5$

(6) $\varepsilon=1 \times 10^{-5}$

The results of simulation are shown in **Fig. 10**. The dotted line indicates the actual data and the other line is the simulation result.

Fig. 10 The Pressure Transition at Both End Section of Actual Data and Simulation Results
—Case Analysis B—

7. Discussion

7.1 The Span 'r'

If there is not any suction part in the flow line, the pressure does not become minus at any point. However, in the calculation of Eq. (6); if the calculation span ' r ' ($\Delta t/\Delta x$) is large or the flow rate change between $x+\Delta x$ and $x-\Delta x$ is very large or the flow rate itself is large, the pressure $P(x, t+\Delta t)$ becomes minus. The cause of this large flow rate change is also a way of selecting ' r ' in Eq. (6). Namely, if Δt is selected very long, the flow rate $Q_n(x, t+\Delta t)$ is changed greatly. On the other hand, if Δx is selected long, the second term and the third in brackets in Eq. (5) will be getting smaller. The flow rate change therefore will not become large. This means that a smaller span ' r ' is better for selecting, and it is dependent on the flow rate which is velocity in the pipeline. From the results in **Table 1** (column 4), it may be determined that the span should be selected smaller than the value of $1/2.37 v$. However, the simulation time may be long to obtain the required answer when the span ' r ' selected is a small value. Therefore, it would be better to select a value for span ' r ' in the range from $1/2.5 v$ to $1/5 v$.

7.2 The Calculation Method of Boundary Condition

It is understandable from **Table 2**, when the inside section, the outside section, and the selected section are of suitable distances, the results of methods A, B and C are not so different. Therefore, method C is better for selection, because this method does not need the inside section, which requires considering divided length (Δx), and the outside section requires no need to consider that length.

7.3 Errors (Value of ϵ)

The simulator which uses 'Simultaneous Difference Equations' may lead to several errors. However, if there is no calculation, the error will, of course, not appear, and a lesser number of calculation steps will reduce the number of errors. Therefore, the procedure, which was described in section 5.3, was used. In the simulation case A-4, a trial calculation without the procedure given in section 5.3 was made.

Accumulation of errors appeared in 10 seconds, and after 80 seconds the flow rate value at the outlet point was showing an unrealistic value such as greater than $100,000 \text{ m}^3$. In spite of that the flow rate may be calculated approximately $36,500 \text{ m}^3$ in this case under steady state conditions. Therefore, we can determine that the simulation procedure in section 5.3 is very suitable and important for eliminating the errors.

The problem is what values of ϵ should be selected. When ϵ was selected as a large value, from **Table 3**, the following points were realized.

- (1) the time when inlet flow rate is changed, (t), is getting longer,
- (2) the time when both flow rates become constant, (T_s), gets shorter,
- (3) the flow rate difference between inlet and outlet sections at T_s , (q), gets larger.

Namely, if the value of ϵ selected is a large value, the result of simulation become coarse. The transmission by small disturbance, therefore, cannot strictly be simulated. However, if the value of ϵ selected is very small, the error is accumulated and the computing time will be very long as has been described before. Therefore, it may be judged that the suitable ϵ is 1×10^{-5} by

consideration of the results of **Table 3**.

7.4 The Selected Section

The results are not so different in **Table 4**. Therefore, it can be mentioned that the position of the *selected section* is not too important in the method C. However, if the computing time is considered, A-18 should be selected because of the values of Δx and Δt .

7.5 Reliability of Simulator (Analysis B)

The best method for checking the reliability of simulation is to compare the results, which are obtained by generating the same disturbance with these of the actual field pipeline. The data, therefore, were selected from the field, and the simulation was conducted. However, the conditions of the simulation case and these of the field case could not be the same. The differences were as follows: (1) The selected field pipeline has been sales line for three chemical factories. The flow rate to the biggest company was about 98%, and the flow rate change to the other smaller companies was very small. Therefore, the assumption was made that the other two small companies were located at the same point as was company A, and their flow rates and pressures were constant. (2) The pipeline was under operation, so the control practice was constantly maintained by the operator, and there are separators at inlet point and outlet point which are connected directly to factories. This means that the pressure at both points was influenced not only by simple disturbance but also by the separator conditions and usage. (3) The starting condition assumed to be a steady state. From careful examination of **Figs. 9** and **10**, it was observed that the pressure at the outlet point had not reached a perfectly stable condition on the selected pipeline. This was still a small decrease even when the outlet flow rate had started to decrease, because of the small influence of the last disturbance of flow; however, it was neglected in the simulation case. (4) The pressure gauge and the flow rate recorder at both ends of the pipeline were different, so they might have individual, independent errors. However in the simulation case the difference was neglected.

Considering the above conditions, it may be recognized from **Fig. 10** that the results of simulation tests do resemble the actual gas flow; thus, the simulations which are described in this paper are reliable.

7.6 Result of Case Analysis A-18

The flow rate disturbance around the pipeline was kept up for approximately five seconds, then it reached a steady-state condition with large oscillations after about a half to two minutes as shown in **Fig. 8**. This phenomenon was very great at the disturbed point. Against this flow rate phenomenon, the phenomenon of pressure disturbance had small oscillations around the disturbed point as shown in **Figs. 7**. Therefore, it may be determined that the pressure disturbance propagation is simpler, clearer, and easier to read than the flow rate. These features are noteworthy phenomena and also very convenient for automatic control of a pipeline by computer.

This phenomenon was applied to the computer control of the NAKAJO GAS FIELD. Namely, in the control loop, the flow rate, which is indicated by a flow counter, was not used, and the calculated flow rate obtained by using the pressure change value at the outlet point and the inlet point was used for controlling the flow rate. This method of control has been very successful¹⁾.

8. Conclusions

(1) The simulation method, which applies to the Simultaneous-Difference-Equation (Eqs. (5) and (6)) as described in this paper, is a reliable method for researching an unsteady-state of gas flow in pipelines.

(2) In the simulator, the pressure values along the whole pipeline, $(P(x_i, t + \Delta t))$ should be calculated at first and if the pressure values at three points $(x_k - \Delta x, x_k, x_k + \Delta x)$ in the pipeline are changed by a very small amount (ϵ), the flow rate $Q_n(x_k, t + \Delta t)$ should not be calculated by Eq. (5) and should be fixed at the same value as $Q(x_k, t)$. Only in the case when a bigger pressure change than the value ϵ appears at one of these three points, Eq. (5) is used for correction.

(3) The suitable value of ϵ described above is about 1×10^{-5} considering the round off errors and computing time.

(4) The calculation span ' r ' should be selected depending on the velocity in the pipeline and computing time. The range of suitable values of ' r ' is from $1/2.5 v$ to $1/5 v$.

(5) It is not a serious problem that the selection of the *inside section* be a half-length of the general span Δx from the end section. (method A). However, method C is better than method A for selecting the *selected section*.

(6) When method C is used, *the selected section* can be selected at any distance between 2 and 50 meters from the end section.

(7) It is recommended that the friction factor (f_m) which is calculated from the field data under steady-state conditions, rather than some theoretical value will be used.

(8) The disturbance propagation on the pressure change (disturbance) seems clearer than that of flow rate and its oscillation is smaller.

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Nomenclature

D : Pipe diameter	[cm]
f_m : Friction coefficient	[—]
G : Gas gravity	[air=1]
g : Gravity acceleration	[(9.807)m/sec ²]
L : Total pipeline length	[km]
M : Molecular weight of air	[(28.97)kg/kg-mol]
P : Pressure	[Kg/cm ²]
P_0 : Pressure, standard condition	[(1.034)Kg/cm ²]
Q : Flow rate, flowing condition	[cm ³ /sec, m ³ /hr]

Q_n : Flow rate, standard condition	[cm ³ /sec, m ³ /hr]
R : Gas constant	[(847.8)Kg-m/kg-mol·K]
S : Cross section area	[cm ²]
t : Time	[sec]
T : Temperature	[K]
T_0 : Temperature, standard condition	[(273.2)K]
v : Velocity	[cm/sec]
x : Distance	[cm, km]
Z : Gas compressibility	[—]
Z_0 : Gas compressibility, at T_0, P_0	[—]
ρ : Density	[g/cm ³]
$(\bar{})$: Average value	[—]

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要 旨

パイプラインにおけるガスの非定常流に関するシュミレーション

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定常的にガスが流れている一本のパイプラインに対し、一端の圧力あるいは流量に外乱（昇圧・増量等）を与えた時、その外乱の影響がどのようにパイプライン内に伝達するかを、シュミレーションにより考察した。すなわち、パイプラインにおける、ガスの非定常流動を運動の式 Eq. (1) および連続の式 Eq. (2) としてとらえ、これを展開した後、フィールド・ユニットを入れて連立差分方程式 Eqs. (5), (6) としてとらえた。これを用いて単一パイプライン系に対するユニット・シュミレーターをコンピューター内に作成し、外乱発生後の流動状況を諸ケースについて調査した。

シュミレーターの構成および使用したシュミレーション方法は次の通りである。

(I) 初期条件：外乱発生前の流動状態は定常流とするという仮定より Eq. (10) を用いて求めた。なお Eq. (10) の摩擦係数 f_m は、モデルに使用した実パイプライン・データより求めた。

(II) 外乱の与え方：問題を明解にするため、外乱はパイプラインのいずれかの端に一度与え、それを保持するものとした。

(III) 境界条件：Eqs. (5), (6) を解くには4個の境界条件 $Q_n(0, t)$, $P(0, t)$, $Q_n(L, t)$, $P(L, t)$ が要求される。この内2個は与えられた外乱数値および (I) で定めた初期値として定め得るが、他の2個は Eqs. (5), (6) の解として求められるもので、前もって定めることが出来ない。

そこで (Fig. 2 参照), ①パイプライン両端（境界点）より $\Delta x/2$ はなれた所に内点をもうけ、この内点と境界点におけるガス流動は瞬時に定常流となるという仮定を立て、Eqs. (5), (6) により Δt 時間後の内点の圧力、流量をもとめ、この数値より Eqs. (8), (9) を用いてその時刻における境界点の圧力、あるいは流量を求める。(方法A) ①内点として $\Delta x/4$ を取り②と同様な方法により境界値を求める。(方法B) ③パイプラインの系外 Δx の所に架空点を定め、この点と境界点におけるガス流動が瞬時に定常流となるという仮定を立て他は②と同様とする。ただし外乱時における境界値のみは適当な点を選択点として用い方法Aを用いる。(方法C) の3方法を使用した。

(IV) シュミレーションの進め方：基本的には、Eqs. (5), (6) を用いて Δt 時間後の各点における圧力、流量を求めてい

く方法であるが、単純にこの方法をくり返すと、式の係数中にあるまるめの誤差等が系内に伝播し、長時間後の解析値は、かなり非理論的な数値を示す恐れがある。誤差の大きな伝播は Fig. 1 より明らかなように、まず1時刻先の流量にあらわれ、次いで2時刻先の圧力にあらわれるという系路をたどる。そこで、ある地点 x およびその前後 $x+\Delta x$, $x-\Delta x$ の3点の圧力が前時刻と変わらなければ、その間の流量は変わらない点に注目し、系を Δt 進める際、まず Eq. (6) により系内の圧力を計算し、前記3点の圧力変化をみて、変化量 (ε) の多い時のみ Eq. (5) を使用し、少ない時は $Q_n(x, t+\Delta t) = Q_n(x, t)$ としてシュミレーションを進めることとした。

ケース・アナリシスは、 ε はどの位の数値が良いか、境界値の計算方法は A, B, C のいずれが良いか、シュミレーションステップ $\Delta t/\Delta x$ はどの位が最適か等、最適シュミレーション方法を見つけることを目的として、パイプラインの終点圧力に外乱を与えて行い、次いで、このようにして作成されたシュミレーターの信頼性を見ることを目的として、実パイプライン挙動より得られた数値を用いてシュミレーションを行い、両者の比較を行った。

その結果 (1) シュミレーションステップは、パイプライン内の流速 (v) により異なり、シュミレーション時間等を考えた時、 $1/2.5 v \sim 1/5 v$ が最適である。(Table 1 参照) (2) 境界値の求め方は、方法 C が最適である。(Table 2 参照) (3) 前記 ε は、シュミレーション時間等も考えた時 1×10^{-6} が最適である。(Table 3 参照) (4) 方法Cにおいて外乱時における境界値を求める点は、それ程問題でないがコンピューター操作時間を考慮すべきである。(Table 4 参照) の4点が判明した。これら条件を用いた Table 4, A-18 のケースにつきシュミレーションを行った結果は Figs. 3~8 の通りであり、通常考えられるパイプライン内の非定常流動を良くあらわしていた。

次に、このようにして作成されたシュミレーターの信頼性を調査すべく、Fig. 9 に示した実パイプラインにおけるパイプライン両端の流量変化の数値を用いてシュミレーションを行った。結果は Fig. 10 に示す実線の通りである。これは点線で示した実パイプラインにおける現実の圧力変化と非常に良くあっており、本シュミレーション方法およびシュミレーターの信頼性を立証した。

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Keywords

Continuity equation, Kinetic equation, Pipeline, Simulation, Unsteady gas flow