Effect of Kinetic Energy on Pressure Loss in Natural Gas Wells

Mokhtar M. El-Gassier

Petroleum Engineering Dept., College of Engineering, King Saud University, P.O. Box 800, Riyadh 11421, Saudi Arabia

(Received December 9, 1991)

A complete analytical solution of the energy equation governing the flow of natural gases in vertical wells is presented here. The pressure drop in natural gas wells is calculated by taking into consideration the kinetic energy term in the energy equation. The parameters which affect the kinetic energy term are: gas specific gravity, gas flow rate, friction factor, bottom-hole temperature, well diameter, and well depth. The effect of each of these six parameters on the calculation of the bottom-hole pressure is investigated. The effect of kinetic energy on the magnitude of the pressure drop in the well increases with gas specific gravity, gas flow rate, well radius, and well depth. It decreases with friction factor and bottom-hole temperature. The parameters, gas specific gravity and well radius, have the greatest effect on the pressure drop in the well. A case study is presented where the combined effect of the six parameters is calculated. The case study shows that when the effects of the six parameters are taken together, the pressure drop in the well calculated with kinetic energy taken into consideration is about 0.2% higher than the pressure drop in the well when the kinetic energy term is neglected. This difference, however, can be as much as 3% in some cases of extreme wells.

1. Introduction

Calculation of the bottom-hole pressure in natural gas wells requires the solution of the general energy equation shown in Eq. (1).

\[ \nu dp + \frac{\nu}{g_e} du + \frac{g}{g_e} dL + W_f + \Delta h + W_s = 0 \]  

where:

\( \nu \): gas specific volume, ft\(^3\)/lb\(_m\)

\( p \): gas pressure, psia

\( u \): gas velocity, ft/sec

\( g_e \): conversion factor=32.17, lb\(_m\)-ft/lbf-sec\(^2\)

\( W_f \): friction work, ft-lbf/lbm

\( \Delta h \): change in enthalpy, ft-lbf/lbm

\( W_s \): shaft work, ft-lbf/lbm

Assuming an adiabatic process and no shaft work involved, the last two terms on the left hand side of Eq. (1) are zero. This reduces Eq. (1) to:

\[ \nu dp + \frac{\nu}{g_e} du + \frac{g}{g_e} dL + W_i = 0 \]  

Several researchers have solved Eq. (2) assuming the second term on the left hand side of the equation to be negligible\(^{5-9}\). In this paper, the effect of this term, which represents the change in the kinetic energy of the gas, on the pressure drop in flowing gas wells is investigated. A complete analytical solution of Eq. (2) is presented. We will also present the procedure for the solution of the equation. From the analytical solution we have identified the parameters which contribute to the kinetic energy term in the general energy equation. These parameters are: gas specific gravity, gas flow rate, friction factor, bottom-hole temperature, well diameter, and well depth.

2. Energy Equation Solution

Using the real gas law, one can calculate the specific volume and the velocity of the gas by means of Eqs. (3) and (4), respectively:

\[ \nu = \frac{zRT}{MP} \]  

\[ u = \frac{vm}{A} \]  

Introducing Eq. (3) into Eq. (4) and differentiating, assuming isothermal conditions, we get:

\[ du = \frac{RTm}{MA} d\left(\frac{z}{P}\right) \]  

1. Introduction

Calculation of the bottom-hole pressure in natural gas wells requires the solution of the general energy equation shown in Eq. (1).

\[ \nu dp + \frac{\nu}{g_e} du + \frac{g}{g_e} dL + W_i = 0 \]  

where:

\( \nu \): gas specific volume, ft\(^3\)/lb\(_m\)

\( p \): gas pressure, psia

\( u \): gas velocity, ft/sec

\( g_e \): conversion factor=32.17, lb\(_m\)-ft/lbf-sec\(^2\)

\( W_i \): friction work, ft-lbf/lbm

\( \Delta h \): change in enthalpy, ft-lbf/lbm

\( W_s \): shaft work, ft-lbf/lbm

Assuming an adiabatic process and no shaft work involved, the last two terms on the left hand side of Eq. (1) are zero. This reduces Eq. (1) to:

\[ \nu dp + \frac{\nu}{g_e} du + \frac{g}{g_e} dL + W_i = 0 \]  

Several researchers have solved Eq. (2) assuming the second term on the left hand side of the equation to be negligible\(^{5-9}\). In this paper, the effect of this term, which represents the change in the kinetic energy of the gas, on the pressure drop in flowing gas wells is investigated. A complete analytical solution of Eq. (2) is presented. We will also present the procedure for the solution of the equation. From the analytical solution we have identified the parameters which contribute to the kinetic energy term in the general energy equation. These parameters are: gas specific gravity, gas flow rate, friction factor, bottom-hole temperature, well diameter, and well depth.

2. Energy Equation Solution

Using the real gas law, one can calculate the specific volume and the velocity of the gas by means of Eqs. (3) and (4), respectively:

\[ \nu = \frac{zRT}{MP} \]  

\[ u = \frac{vm}{A} \]  

Introducing Eq. (3) into Eq. (4) and differentiating, assuming isothermal conditions, we get:

\[ du = \frac{RTm}{MA} d\left(\frac{z}{P}\right) \]
The friction work is obtained from the Fanning equation:

\[ W_f = \frac{f u^2}{2gD} dL \]  

(6)

Introducing Eqs. (3), (4), (5), and (6) into Eq. (2) and integrating from well-head to bottom-hole conditions we get:

\[
\phi = \frac{z}{P_r} \int_{P_r}^{P_2} \frac{\left( \frac{z}{P_r} \right)}{1 + B \left( \frac{z}{P_r} \right)^2} dP_r + \frac{RT_2 m^2}{g_c A^3 P_r M} \int_{P_r}^{P_2} \frac{\left( \frac{z}{P_r} \right)}{1 + B \left( \frac{z}{P_r} \right)^2} d\left( \frac{z}{P_r} \right) = \frac{g_m}{g_c RT} \int_{P_r}^{P_2} \frac{\left( \frac{z}{P_r} \right)}{1 + B \left( \frac{z}{P_r} \right)^2} dL
\]

(7)

where:

\[ P_r : \frac{P}{P_c} \]

\[ P_{r1} : \text{pseudoreduced pressure at well-head conditions} \]

\[ P_{r2} : \text{pseudoreduced pressure at bottom-hole conditions} \]

\[ z : \text{gas compressibility factor} \]

\[ R : \text{gas constant}=1.545 \text{ psfa-ft}^3/\text{lb-mol-°R} \]

\[ T_s : \text{average flowing temperature, °R} \]

\[ m : \text{mass flow rate, lbm/sec} \]

\[ A : \text{cross sectional area of pipe, ft}^2 \]

\[ P_c : \text{pseudocritical pressure of the gas, psfa} \]

\[ M : \text{molecular weight of the gas, lbm/lb-mol} \]

\[ B=8\frac{m^2 R T_s}{g_c A^3 P_r M^2} \]  

(8)

other subscripts are defined in the nomenclatures.

The gas compressibility factor is given by the equation Dranchuk et al., using the Benedict–Webb–Rubin equation (BWR):

\[
z = 1 + E_1 \rho + E_2 \rho^2 + E_3 \rho^3 + E_4 \rho^4 (1 + A_8 \rho^2) e^{-A_8 \rho^2} \]  

(9)

and

\[
\rho_1 = z P_r / (z T_s) \]

\[ E_1 = A_1 + A_2 / T_s + A_3 / T_s^3 \]

\[ E_2 = A_4 + A_5 / T_s \]

\[ E_3 = A_6 A_8 / T_s \]

\[ E_4 = A_7 / T_s^3 \]

\[ A_1=0.31506237 \]

\[ A_2=-1.04670990 \]

\[ A_3=-0.57832729 \]

\[ A_4=0.53503771 \]

\[ A_5=-0.61232032 \]

\[ A_6=-0.10488813 \]

\[ A_7=0.68157001 \]

\[ A_8=0.68446549 \]

\[ z_c=0.27 \]

Dropping subscript r for simplification, the analytical solution to Eq. (7) is:

\[
(2E_1 + 1.5E_2 \rho + 1.2E_3 \rho^4 - 2E_3 \rho^2 \gamma^2 + 6E_3 \gamma^4) \frac{\rho}{A_s} - E_3 \left[ 7 \gamma^2 + 8.5 \gamma^2 + 14 A_8 \gamma^4 \right.
\]

\[
+ 10 A_8 \rho^2 \gamma^2 + 5.5 \rho^2 + A_8 \rho^4 \right] e^{-A_8 \rho^2}
\]

\[
+ 0.5 \left( 1 - 3 E_3 \gamma^2 \right) \ln (\beta) - 2 \gamma (E_1)
\]

\[
+ 3 E_3 \gamma^4 \arctan \left( \frac{\rho}{\gamma} \right) + E_4 \gamma^2 (2 A_8^2 \gamma^4
\]

\[
+ 1.5 A_8 \gamma^2 - 1.5 e^{-A_8 \gamma^2} E_i (-A_8 \beta)
\]

\[ + \Psi_{m} |_{m} = \Gamma \]  

(10)

where:

\[
\Psi = \frac{RT_2 m^2}{2g_c A^3 P_r^3 M B} \ln \left[ 1 + B \left( \frac{z}{P_r} \right)^2 \right] \]  

(11)

\[
\Gamma = \frac{g_c}{g_c} \frac{M}{RT_s} \frac{L}{L} \]  

(12)

\[
\gamma^2 = B \left( \frac{z_c}{T_s} \right)^2 \]  

(13)

\[
\beta = \rho^2 + \gamma^2 \]  

(14)

\[
E_i(x) = -\int_{x}^{\infty} \frac{e^{-y}}{y} dy \]  

(15)

\[ \rho_1 \]  

is the value of \( \rho \) at well-head pressure.
\( \rho_2 \) is the value of \( \rho \) at bottom-hole pressure

In field units, Eqs. (8), (11), and (12) become Eqs. (16), (17), and (18), respectively:

\[
B = \frac{667}{D^2} \frac{Q_{sc}^2 T_a^2}{P_c^2} 
\]  
(16)

\[
\Psi = 1.562 \times 10^{-3} \frac{\gamma_s D}{T_a} \ln \left[ 1 + \gamma_s \left( \frac{T_a}{\rho} \right)^2 \right] 
\]  
(17)

\[
\Gamma = 0.01875 \frac{\gamma_s L}{T_a} 
\]  
(18)

where the units now are:

- \( Q_{sc} \): MMSCF/D
- \( D \): inch.
- \( P_c \): psia
- \( T_a \): °R

Eq. (10) can be written as follows:

\[
V_2 = V_1 + \Gamma 
\]  
(19)

where:

- \( V_1 \): the value of LHS of Eq. (10) at bottom-hole conditions
- \( V_2 \): the value of LHS of Eq. (10) at well-head conditions

The procedure for solving the bottom-hole pressure using Eq. (19) is similar to that given in reference 1. Figure 1 shows a flow diagram for the solution.

3. Results and Discussion

Eq. (10) gives the solution of the general energy equation governing the flow of natural gas in vertical wells. The only assumptions used in Eq. (10) are: no gain or loss of heat between the gas and its environment and no shaft work. The last term on the left hand side of Eq. (10) is the contribution of the kinetic energy to the pressure drop in the well. The parameters which affect this term and, consequently, the pressure loss are: well radius, well depth, friction factor, average gas temperature, gas specific gravity, and gas flow rate. In the following paragraphs, we will discuss the effect of each of these six parameters separately. Throughout this discussion, we will refer to a parameter called Theta \( \theta \) which is defined by

\[
\theta = \frac{\Delta P_{KE} - \Delta P_{NKE}}{\Delta P_{KE}} \times 100 
\]  
(20)

where:

- \( \Delta P_{KE} \) = pressure loss including kinetic energy
- \( \Delta P_{NKE} \) = pressure loss excluding kinetic energy

Thus, Theta represents the percentage difference between the pressure drop for the case of kinetic energy being accounted for and the pressure drop for the case in which it is ignored with the former case being the reference.

To calculate the effect of each of the six parameter individually on Theta, the bottom-hole pressure is calculated varying the value of the parameter in question while keeping the values of the other five parameters at their base values. The base values of the parameters used in this paper are:

- well radius = 3.0 inch
- well depth = 7,000 ft
- friction factor = 0.02
- bottom-hole temperature = 200°F
- gas specific gravity = 0.7
- gas flow rate = 8.5 MMSCF/D

The well-head conditions are kept at 70°F and
900 psia throughout.

Looking at Eq. (17) the gas parameters which affect the kinetic energy term ($\Psi$) are gas specific gravity and gas flow rate. Eq. (17) clearly shows the relationship between $\gamma_g$ and $\Psi$. The effect of flow rate on $\Psi$, however, is indirectly affected through parameter $B$ which is a function of flow rate, see Eq. (8). Figures 2 and 3 show the effect of gas specific gravity and gas flow rate on Theta, respectively. Theta increases with both specific gravity and flow rate. The effect of gas specific gravity, however, is greater than that of flow rate.

Figures 4 through 7 illustrate the effects of well parameters on Theta. Theta decreases with both friction factor and bottom-hole temperature. It increases (almost linearly) with both diameter and depth. Of these four parameters, the well diameter has the greatest effect on Theta. At large well diameters, Theta is as high as 0.3%. On the other hand, well depths affect Theta the least. Except for well depth, the effect of the well parameters can be predicted straightforward by referring to Eq.

Fig. 2 Effect of Gas Specific Gravity on Theta

Fig. 3 Effect of Gas Flow Rate on Theta

Fig. 4 Effect of Friction Factor on Theta

Fig. 5 Effect of Bottom-hole Temperature on Theta

Fig. 6 Effect of Well Diameter on Theta
The influence of well depth, however, is indirectly affected through the reduced density $\rho'$. To illustrate the difference in bottom-hole pressure calculated with and without taking into consideration the kinetic energy term, the following gas well is taken as an example:

- well depth = 7,000 ft
- well diameter = 3.0 inch.
- friction factor = 0.02
- bottom-hole temperature = 200°F
- well-head pressure = 900 psia
- gas specific gravity = 0.70
- gas flow rate = 8.50 MMSCF/D
- well-head temperature = 70°F
- standard temperature = 60°F
- standard pressure = 14.7 psia

bottom-hole pressure (kinetic energy included) = $1,270.33$ psia
bottom-hole pressure (kinetic energy excluded) = $1,269.40$ psia

$\Delta P_{KE} = 1,270.33 - 900.0 = 370.33$ psia
$\Delta P_{NKE} = 1,269.40 - 900.0 = 369.40$ psia

$\theta = \frac{370.33 - 369.40}{370.33} \times 100 = 0.25\%$  

For shallow wells with large diameter, high flow rate, low bottom-hole temperature, and high specific gravity gas, Theta is maximum. Take for example a well with the following data:

- well depth = 2,000 ft
- well diameter = 9.0 inch.
- friction factor = 0.01
- bottom-hole temperature = 140°F
- well-head pressure = 900 psia
- gas specific gravity = 1.1
- gas flow rate = 8.5 MMSCF/D
- well-head temperature = 70°F
- standard temperature = 60°F
- standard pressure = 14.7 psia

bottom-hole pressure (kinetic energy included) = $1,060.0633$ psia
bottom-hole pressure (kinetic energy excluded) = $1,055.0533$ psia

$\Delta P_{KE} = 1,060.0633 - 900.0 = 160.0633$ psia
$\Delta P_{NKE} = 1,055.0533 - 900.0 = 155.0533$ psia

$\theta = \frac{160.0633 - 155.0533}{160.0633} \times 100 = 3.13\%$

Table 1 shows the variation of Theta as a function of well diameter. The other five parameters are kept at values corresponding to maximum Theta.

### 4. Conclusions

A complete analytical solution of the energy equation, including the kinetic energy governing the flow of natural gases in vertical wells, is presented in this paper. Contribution of the kinetic energy to the pressure loss in the well increases with well diameter, well depth, gas specific gravity, and gas flow rate. It decreases with friction factor and bottom-hole temperature. For average gas wells, contribution of the kinetic energy to the pressure loss is about 0.2%. For shallow wells with large diameter, low bottom-hole temperature, low friction factor, high specific gravity gas, and high gas flow rate, the contribu-
tion may be as high as 3%.

Nomenclatures

\( A \): cross sectional area of flow [ft\(^2\)]

\( A_{1-8} \): the eight coefficients in BWR equation [-]

\( B \): dimensionless parameter defined by Eq. (8) [-]

\( D \): diameter of the well [ft]

\( E_{i(x)} \): exponential integral defined by Eq. (15) [-]

\( E_{1-cE4} \): temperature dependent coefficients of BWR equation [-]

\( f \): Moody friction factor [-]

\( g_c \): conversion factor = 32.17 [lbm-ft/lbf-sec\(^2\)]

\( h \): enthalpy [ft-lbf/lbm]

\( L \): depth of well [ft]

\( m \): mass flow rate of gas [lbm/sec]

\( M \): molecular weight of gas [lbm/lb-mol]

\( P \): absolute pressure of gas [psfa]

\( Q \): volumetric flow rate of gas [MMSCF/D]

\( R \): universal gas constant = 1,545 [psfa-ft\(^3\)/lb-mol-\(^o\)R]

\( T \): absolute temperature of gas [\(^o\)R]

\( u \): average velocity of gas [ft/sec]

\( v \): specific volume of gas [ft\(^3\)/lbm]

\( V \): value of integral defined by Eq. (19) [-]

\( W_f \): friction work [ft-lbr/lbm]

\( W_s \): shaft work [ft-lbr/lbm]

\( z \): gas compressibility factor [-]

\( a \): average

\( c \): critical

\( KE \): kinetic energy

\( NKE \): no kinetic energy

\( r \): reduced

\( sc \): standard conditions

\(<\text{Greeks}>\)

\( \beta \): dimensionless parameter defined by Eq. (14) [-]

\( \Gamma \): potential energy term defined by Eq. (12) [-]

\( \gamma \): dimensionless parameter defined by Eq. (13) [-]

\( \gamma_i \): specific gravity of gas [-]

\( \Delta \): change or difference defined by Eq. (20) [-]

\( \theta \): percentage difference defined by Eq. (20) [-]

\( \pi \): 3.14159...

\( \rho \): density of gas [lbm/ft\(^3\)]

\( \Psi \): kinetic energy term defined by Eq. (11) [-]

References


Keywords

Bottom hole pressure, Gas kinetic energy, Well pressure loss, Vertical gas flow, Gas well