[Regular Paper]

**Inconsistency between Fracture and Anisotropic Systems in Predicting Horizontal-sink Productivity**

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Modeling a reservoir containing unidirectional natural fractures by an equivalent anisotropic system is one of the most common methods in the realm of fractured reservoir engineering. The universal validity of such a macroscopic treatment, however, has not been quantitatively confirmed. This study primarily examined whether an equivalent anisotropic system is consistent with a fracture system through horizontal-sink productivity problems.

Numerical experiments for a horizontal sink completed parallel or perpendicular to fractures with 1000 fracture realizations were performed to compare the productivity predictions based on naïve fracture systems and equivalent anisotropic systems. The results showed the anisotropic approach yields erroneous productivities higher than the true values with mean relative errors of 11% and 13% for parallel and perpendicular completions, respectively. The conversion of fracture systems to equivalent anisotropic systems is not always appropriate for flow in the vicinity of a horizontal sink and may mislead the productivity predictions.

A pseudo-skin factor was introduced to correct the overestimation of the anisotropic systems. Mean pseudo-skin factors of 0.366 and 0.258 were required for parallel and perpendicular completions, respectively. Versatility of the correction technique requires a simple means to evaluate the pseudo-skin factor, thus a set of correlation equations with minimal information (horizontal sink length, total fracture length, and geometric mean of fracture lengths) was established. The inconsistency between fracture and anisotropic systems could be corrected by the systematic procedure proposed in this study.

1. Introduction

Horizontal completions, such as horizontal wells or hydraulically fractured wells, have widespread applications in many petroleum reservoirs, particularly in unconventional gas reservoirs with permeabilities less than 0.1 md, to accelerate production. The productivity of horizontal sinks can reach values two to five times higher than that of vertical wells because of a larger reservoir contact area. The improvement may be more pronounced in naturally fractured reservoirs as reported by Beliveau.

In designing horizontal sinks to be completed in fractured reservoirs, the impact of natural fractures on horizontal-sink productivity must be well understood. The subject of "natural fractures" has been an important research target for many years. When fractures are well developed (regularly distributed) to form a fracture network and the fracture system can be treated as a continuum model, a dual permeability or double porosity model is usually applied in simulation. These methods, however, are not appropriate for discrete fractures that are randomly distributed and not connected to each other.

This study focuses on discrete fracture systems. Review of the literature on discrete fractures reveals that the methodologies may be categorized into two distinct classes: the microscopic approach in which individual fractures are honored and the macroscopic approach in which little attention is paid to individual fractures.

Microscopic studies have been largely through physical experiments, which utilize the analogy between fluid flow through a porous medium and certain physical phenomena, such as the flow of electric current in a conductive body or the flow in a Hele-Shaw model. Although these studies provide remarkable insights into fracture systems, experimental work is limited to particular fracture-patterns that have been investigated and is not always feasible. Recently, a sound mathematical technique, the complex variable boundary element method (CVBEM), has been developed and successfully applied to study detailed flow behavior in the presence of distributed fractures. Sato and Abbaspazadeh applied the CVBEM to study the effect of fractures on vertical-well productivity, and Sato extended the...
method to horizontal-sink problems\(^{(10)}\) and developed a set of correlations to predict horizontal-sink productivity\(^{(12)}\).

A much simpler approach is the macroscopic treatment of fractures through theoretical considerations. Although the microscopic approach is more rigorous, the macroscopic approach is more flexible for use in analytical methods or simulation models. When fractures are preferentially aligned in a certain direction, the apparent permeability becomes homogeneous and anisotropic. Previous investigators\(^{(7,9,10)}\) considered certain idealized fracture patterns and derived analytically the corresponding permeability anisotropy. By considering stochastic fracture systems, Nakashima \textit{et al.}\(^{(16)}\) developed a set of correlation equations for anisotropic permeabilities, based on the periodic boundary condition method\(^{(17)}\). The periodic boundary condition has been adopted by several investigators\(^{(18,19)}\) and its effectiveness for evaluating anisotropic permeability was confirmed for fracture systems in the absence of sink/source singularities.

The qualitative representation of fracture systems as equivalent anisotropic systems is consistent with the observations that fractured reservoirs often behave like anisotropic media\(^{(20,22)}\). Elkins and Skov\(^{(20)}\) and Beliveau\(^{(21)}\) utilized pressure transient data and Beliveau \textit{et al.}\(^{(20)}\) used displacement data to estimate the anisotropy ratio equivalent to the discrete fracture system. These authors solved inverse problems based on given reservoir responses, and inferred equivalent anisotropic parameters. However, adequate reservoir responses for analyses are not always available, in particular, in the early stage of development. In addition, the anisotropic parameters obtained by inverse calculations have not been quantitatively verified against equivalent anisotropic values inferred based on given fracture parameters.

Therefore, the current question is whether an equivalent anisotropic system that is forwardly obtained by the periodic boundary condition method can be used to predict well performances (with sink/source singularities, of course) in the corresponding fracture system. This study primarily investigated this question through horizontal-sink productivity problems. Unfortunately, no experimental results or comprehensive field reports on the relationships between horizontal-sink productivities and fracture parameters are available, so numerical experiments were conducted and the outcomes analyzed by several approaches.

2. Problem Definition and Assumptions

This study examined whether an equivalent anisotropic system can be used to predict the horizontal-sink productivity in the presence of discrete fractures. In addition, a correction methodology was developed for use when the actual fracture system cannot be honored and an equivalent anisotropic system must be adopted, such as when developing analytical solutions.

The horizontal-sink productivity predicted by honoring individual fractures was compared with that predicted through an equivalent anisotropic system. For generality, the pressure drawdown \(\Delta p\) can be non-dimensionalized as
\[
p_{ho} = 2\pi k h \Delta p / \mu q \]
the inverse of which can readily be converted to the horizontal-sink productivity \(J_h\) as
\[
J_h = (2\pi k h / \mu)(1/p_{ho})
\]
Steady-state flow of single-phase fluid of viscosity \(\mu\) was considered in a horizontal plane, and a negligible gravitational effect assumed, which allowed the problem to be treated in two dimensions. Discrete fractures were assumed to be unidirectional and vertical throughout the reservoir height \(h\) and were treated as lines in a two-dimensional (2D) flow domain. This study considered only macrofractures with large widths over 100 \(\mu m\), the conductivity of which is essentially infinite. Microfractures, if present, were assumed to form a homogeneous flow network and to be a part of the homogeneous and isotropic matrix system of permeability \(k\).

The term “horizontal sink” refers to two classes of sinks: a vertical well hydraulically fractured over the entire formation height and a horizontal well with a length much greater than the reservoir thickness so that the additional pressure drop due to convergence of fluid around the well is negligible compared with the pressure drop in a horizontal plane. Pressure drops along the horizontal sinks and mechanical skin effects were assumed to be negligible.

3. Method of Solution

For steady-state fluid flow through a homogeneous isotropic porous medium, the potential function \(\Phi = (k/\mu)\phi\) satisfies the Laplace equation:
\[
\nabla^2 \Phi = 0
\]
The harmonic conjugate of \(\Phi\) is called the stream function \(\Psi\), and the complex potential \(\Omega\):
\[
\Omega = \Phi + i \Psi
\]
also satisfies the Laplace equation:
\[
\nabla^2 \Omega = 0
\]
For the complex potential \(\Omega\) analytic inside and along a simple closed boundary \((\Gamma)\), Cauchy’s integral\(^{(23)}\) holds:
\[
\Omega(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{\Omega(\zeta)}{\zeta - z} d\zeta
\]
where \(\zeta\) is on \(\Gamma\) and the contour integral is calculated by traversing \(\Gamma\) so that the domain of interest lies on the left. This equation states that the non-singular solution of \(\Omega\) interior to \(\Gamma\) is completely determined by knowing the values of \(\Omega\) on \(\Gamma\). To evaluate the con-
tour integral, the boundary is divided into discrete boundary elements. This numerical procedure is known as the complex variable boundary element method (CVBEM).

3.1. Fracture Systems

Because of the semi-analytical nature of the CVBEM, singularities caused by fractures are readily coupled with the CVBEM by the use of superposition\(^9\) and a horizontal sink that may or may not be intersected by fractures is modeled by integrating a source function along the object\(^10,12\): 

\[
\Omega(z) = \frac{1}{2\pi i} \int \frac{\Omega_{ns}(\zeta)}{\zeta - z} d\zeta + \sum_{j=1}^{n} \Omega_{fj}(z) + \Omega_{s}(z) \quad (7)
\]

where \(\Omega_{ns}\) is the non-singular solution and \(\Omega_{fj}\) and \(\Omega_s\) are singular solutions due to the \(j\)-th fracture (\(n\) fractures in total) and the horizontal sink, respectively.

Singular behavior observed near the fracture tips can be dealt with by conformal mapping of a fracture (line) onto a circle and applying the method of dipoles. The dipole is defined such that the real part of \(\Phi\) (that is, \(\Phi^r\)) is continuous and the imaginary part \(\Phi^i\) is discontinuous across the fracture. Singular solutions of fractures \(\Omega_{fj}\) and horizontal sinks \(\Omega_s\) are given in Refs.\(^9\) and \(10\).

The potential function \(\Phi\) obtained for fracture systems can be transformed to \(p\) by multiplying by \(\mu/k\). To be consistent with the definition of \(J_h\), Eq. (2), the inverse of pressure drawdown \(p_{\text{D}}\) must be defined by Eq. (1) although the system is governed by the effective (fracture and matrix) permeability rather than by the matrix permeability \(k\).

3.2. Anisotropic Systems

Suppose that an equivalent anisotropic system is obtained for a fracture system. For fluid flow with permeability anisotropy, the following equation holds:

\[
\frac{\partial}{\partial x} \left( k_x \frac{\partial \Phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial \Phi}{\partial y} \right) = 0 \quad (8)
\]

where \(k_x\) and \(k_y\) are the permeabilities in the \(x\) and \(y\) directions, respectively. A different symbol \(\overline{p}\) is used for pressure to indicate that the solution is based on an isotropic system and may be different from \(p\) based on a fracture system.

To convert Eq. (8) to the Laplace equation in its normal form, the following coordinate transformation\(^24\) is appropriate:

\[
\begin{align*}
\overline{x} &= \sqrt{k/k_x} x \\
\overline{y} &= \sqrt{k/k_y} y
\end{align*} \quad (9)
\]

where

\[
k = \sqrt{k_x k_y} \quad (10)
\]

Figure 1 shows the sequence of system transformations: a fracture system, an equivalent anisotropic system, and a transformed isotropic system. In the transformed coordinates, the \(x\)-axis length becomes \(\sqrt{k_x/k}\), times the \(y\)-axis length, but the area of the transformed system remains the same.

Applying Eq. (9) to Eq. (8) yields the steady-state flow equation:

\[
\frac{\partial}{\partial x} \left( \bar{k} \frac{\partial \overline{\Phi}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \bar{k} \frac{\partial \overline{\Phi}}{\partial y} \right) = \nabla^2 \overline{\Phi} = 0 \quad (11)
\]

where \(\overline{\Phi} = (k/\mu)\overline{p}\). Thus, an anisotropic system can be transformed into an equivalent isotropic system with an effective permeability of \(\bar{k}\) and an elongated shape, to which the CVBEM is applicable.

The potential function \(\overline{\Phi}\) obtained in the transformed coordinates can be transformed to \(\overline{p}\) by multiplying by \(\mu/\bar{k}\). As for the fracture systems, the inverse of pressure drawdown \(1/p_{\text{D}}\) for anisotropic systems is defined by Eq. (1); a matrix permeability \(k\) (rather than an effective permeability \(\bar{k}\)) must be used. In this way, \(1/p_{\text{D}}\) can be compared with \(1/p_{\text{D}}\) on the common basis of matrix permeability \(k\).

4. Numerical Experiments

Consider a horizontal sink of completion length \(L\) located in the center of a square flow domain of area \(A\). For generality, the length is scaled against the side length of the domain \(\sqrt{A}\); for instance, the horizontal sink-length is scaled as \(L_{D} = L/\sqrt{A}\). The boundary of the domain is maintained at a constant pressure. Drawdown pressures at the horizontal sink were calculated by the CVBEM for fracture and equivalent anisotropic systems and compared.

4.1. Data Setting

Sato\(^12\) performed numerical experiments to generate productivity correlations for horizontal sinks completed in fractured reservoirs. The horizontal-sink length \((L_{D})\) and the total fracture length \((\Sigma_{aD})\) were selected as the influencing properties, and a data set of 1000 \(L_{D} - \Sigma_{aD}\) pairs was prepared. The same data set is used in this study.

Assuming lognormal fracture-length distribution with the geometric mean of \(m_{aD}\), \(\Sigma_{aD}\) can be written as

\[
\Sigma_{aD} = n \exp(m_{aD} + \sigma_{cept}/2) \quad (12)
\]
where \( n_f \) is the number of fractures and \( \sigma_{\ln \alpha_f} \) is the standard deviation of \( \ln \alpha_f \). \( \Sigma_{\alpha\beta} \) is first drawn from a uniform distribution \( U(1.0, 50.0) \) and then randomly decomposed through Eq. (12) into \( m_{\alpha\beta} \) and \( n_f \), the ranges of which are assumed to be \((0.0, 0.1)\) and \((1, 500)\), respectively. \( \sigma_{\ln \alpha_f} \) is independently drawn from \( U(0, \ln(3)/3) \).

Individual fracture images were generated by an unconditional Boolean technique\(^{25}\), which randomly distributes fractures in space with lengths drawn from a lognormal distribution until the desired \( \Sigma_{\alpha\beta} \) values are met. As seen in Fig. 2, which shows sample realizations with different values of \( \Sigma_{\alpha\beta} \) and \( m_{\alpha\beta} \), unidirectional fractures were aligned parallel to the x-axis. A horizontal sink of length \( L_D \) drawn from \( U(0.001, 0.5) \) was to be completed parallel or perpendicular to those fractures.

### 4.2. Anisotropic Permeability

For a unidirectional fracture system, the permeability perpendicular to the fractures (\( k_y \) in this study) is not affected by the presence of fractures and is equal to the matrix permeability \( k \). The permeability parallel to the fractures (\( k_x \)), on the other hand, is affected by the fractures and the permeability equivalent to the fracture system is higher than \( k \), resulting in an anisotropic system.

To evaluate permeability anisotropy, Nakashima et al.\(^{16}\) applied the periodic boundary condition method\(^{17}\) to stochastic fracture systems. The CVBEM was used to calculate permeability anisotropy for 500 fracture realizations with \( 1 < \Sigma_{\alpha\beta} < 25 \) and \( 0 < m_{\alpha\beta} < 0.1 \). The outcomes were analyzed by a non-parametric regression algorithm (ACE: Alternating Conditional Expectations, the essence of which is discussed in Section 5.2., 1.), and the correlations for anisotropic permeabilities were obtained. The anisotropic permeability in the x direction (\( k_x \)) was found to be well correlated with \( \Sigma_{\alpha\beta} \) and \( m_{\alpha\beta} \) through the set of equations given in Table 1.

Although the range of \( \Sigma_{\alpha\beta} \) considered in this study \((0 < \Sigma_{\alpha\beta} < 50)\) is wider than the data set \((0 < \Sigma_{\alpha\beta} < 25)\) used in Ref. 16), no other correlation is available and it is assumed that the equations in Table 1 are applicable to the current data set. This assumption should not severely affect the study results, since \( f_x \) for \( \Sigma_{\alpha\beta} \geq 10.95 \) is linear to \( \Sigma_{\alpha\beta} \) (Table 1).

For the sample fracture realizations (A), (B), and (C) shown in Fig. 2, \( k_x/k \) values were estimated as 2.10, 7.75, and 1.64, respectively. As observed in Ref. 16), \( k_x/k \) increases with \( \Sigma_{\alpha\beta} \), and if \( \Sigma_{\alpha\beta} \) is almost the same, a larger \( m_{\alpha\beta} \) yields a higher \( k_x/k \). Figure 3 is the histogram of equivalent \( k_x/k \) values (ranging from 0.97 to 11.53) corresponding to the 1000 fracture realizations. Theoretically, \( k_x/k \) must be larger than unity, indicating that only 8 of the 1000 realizations appear to be mismeasured.

### 4.3. Results and Observations

\( p_{\beta\alpha} \) values were calculated by using the CVBEM for the 1000 fracture realizations (Section 4.1.). Figure 4 shows the histograms of 1/\( p_{\beta\alpha} \) values and Table 2 lists the summary statistics. As expected, the horizontal sinks completed perpendicular to fractures (Fig. 4 (b)) yielded higher productivities than the parallel completions (Fig. 4 (a)). According to the measures of central tendency (arithmetic mean and median), perpendicular completions achieved around 30% more productivity over parallel completions (Table 2). The measures of spread (standard deviation and interquartile range, that is, the difference between upper and lower quartiles) and the positive skewness were also larger for perpendicular completions, indicating that perpendicular completions are more sensitive to the influence of fractures.

\( \bar{p}_{\beta\alpha} \) values were evaluated through the coordinate
transformation (Section 3.2.) for the 1000 anisotropic systems (Section 4.2.) corresponding to the fracture systems (Section 4.1.). Figure 5 shows the histograms of $1/\phi_{HD}$ values and Table 3 lists the summary statistics. Although qualitative observations are similar to the fracture-system predictions ($1/\phi_{HD}$), the quantitative characteristics are not exactly the same. Compared with the fracture-system predictions (Fig. 4 and Table 2), the anisotropic system (Fig. 5 and Table 3) yielded somewhat higher productivities; the arithmetic mean and the median values were larger for the anisotropic systems than for the fracture systems. The anisotropic system predicted 11% and 13% higher mean values for parallel and perpendicular completions, respectively. The measures of spread and the positive skewness were also larger for the anisotropic systems. These observations indicate that the $1/\phi_{HD}$ values of the anisotropic systems are positively and erroneously biased from the $1/\phi_{HD}$ values of the fracture systems.

Table 2 Summary Statistics of $1/\phi_{HD}$ for Fracture Systems

<table>
<thead>
<tr>
<th></th>
<th>parallel</th>
<th>perpendicular</th>
</tr>
</thead>
<tbody>
<tr>
<td>arithmetic mean</td>
<td>0.742</td>
<td>1.044</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.309</td>
<td>0.616</td>
</tr>
<tr>
<td>skewness</td>
<td>0.514</td>
<td>1.033</td>
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<tr>
<td>lower quartile</td>
<td>0.510</td>
<td>0.581</td>
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<tr>
<td>median</td>
<td>0.704</td>
<td>0.890</td>
</tr>
<tr>
<td>upper quartile</td>
<td>0.944</td>
<td>1.388</td>
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Table 3 Summary Statistics of $1/\phi_{HD}$ for Anisotropic Systems

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<tr>
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<th>parallel</th>
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<tbody>
<tr>
<td>arithmetic mean</td>
<td>0.826</td>
<td>1.182</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.366</td>
<td>0.763</td>
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<tr>
<td>skewness</td>
<td>0.681</td>
<td>1.272</td>
</tr>
<tr>
<td>lower quartile</td>
<td>0.549</td>
<td>0.622</td>
</tr>
<tr>
<td>median</td>
<td>0.765</td>
<td>0.960</td>
</tr>
<tr>
<td>upper quartile</td>
<td>1.043</td>
<td>1.550</td>
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</table>
Figure 6 is the crossplot of $1/\phi_D$ and $1/\phi_D$, in which the identical (45-degree) line indicates total correlation between the anisotropic prediction and horizontal-sink productivity. Most of the data points, however, are above the identical line, indicating that the anisotropic system overestimates the horizontal-sink productivity. The discrepancy is greater for perpendicular completions, as seen from the summary statistics (Tables 2 and 3) and the fact that the residual sum of squares (RSS) is larger for perpendicular completions (60.36) than for parallel completions (15.68).

5. Discussion

The results shown in Section 4.3. clearly show that an equivalent anisotropic system can not always be used to predict the horizontal-sink productivity in the presence of discrete fractures. Similar observations were reported previously. Durlofsky et al. studied scaleup techniques for finite-difference simulation and found that the pressure field in the vicinity of a well cannot be expected to be linear or slowly varying, which is the basic assumption for estimating upscaled permeabilities. Sato and Abbaszadeh studied vertical-well productivities and concluded that the anisotropic system is consistent with the fracture system for regions away from the well, whereas the pressure behavior around the well is controlled by the exact geometry of fractures and the well. To model the flow behavior in the vicinity of the well, a concept of skin is of practical use.

5.1. Pseudo-skin Factor

To express the degree of near-wellbore damage (or improvement), the skin factor, $s$, is frequently used:  

$$
\Delta p_s = (q\mu/2\pi k h)s
$$

where $\Delta p_s$ is the additional pressure drop at a sink due to the skin effect. Although there is no actual damage or improvement, the same concept can be utilized to correct the overestimate provided by anisotropic systems.
A pseudo-skin factor, $\bar{s}$, is introduced as
\[ \bar{s} = (2\pi kh/q\mu)(\Delta p - \bar{\Delta \bar{p}}) \] (14)
where $\Delta p$ is the true pressure drawdown for a fracture system and $\bar{\Delta \bar{p}}$ is the estimated pressure drawdown with a transformed isotropic system. Thus, $\bar{s}$ corrects for the fracture-to-anisotropic conversion and the anisotropic-to-isotropic transformation. Note that $\bar{k}$ instead of $k$ is used in order to maintain the physical definition of skin factors as given by Eq. (13). Consequently, $\bar{s}$ evaluated in this study becomes general and can be used in any independent study. The definition of $\bar{s}$ follows
\[ p_{HD} = \bar{p}_{HD} + (k/k)\bar{s} \] (15)
which states that an additional pressure drop $(k/k)\bar{s}$ must be added to the anisotropic-system prediction $\bar{p}_{HD}$ to correct for the improper treatment of fracture systems.

Figure 7 shows the histogram of $\bar{s}$ and Table 4 lists the summary statistics. Since the anisotropic system yields higher productivities (equivalently, $\bar{p}_{HD}$ tends to be lower than $p_{HD}$), most of the $\bar{s}$ values are positive and only a few are negative: 10% and 18% for parallel and perpendicular completions, respectively. Mean values of 0.366 and 0.258 are required to correct the overestimates for parallel and perpendicular completions, respectively. Positive skewness is observed in both $\bar{s}$ distributions, but is more prominent for perpendicular completions.

5.2. $\bar{s}$ Correlation
A simple method of $\bar{s}$ evaluation is desirable for a

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<th>Table 4 Summary Statistics of $\bar{s}$</th>
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<td>median</td>
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<tr>
<td>upper quartile</td>
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</table>

Fig. 8 Transformations $g$, $f_L$, $f_m$, and $f_n$ for Parallel Completions
versatile correction methodology (\(\bar{s}\) correction for \(\bar{p}_{sh}\)). Correlation of \(\bar{s}\) with appropriate predictor variables was established using the ACE (Alternating Conditional Expectation) algorithm.

5.2.1. ACE Algorithm

The ACE algorithm, a non-parametric technique originally proposed by Breiman and Friedman\(^{27}\), does not require functional forms in advance and brings objectivity to data analysis. Its utility has been demonstrated through permeability estimations\(^{28}\), horizontal-sink productivity estimations\(^{12}\), and effective permeability estimations\(^{16}\) (as mentioned in Section 4.2.).

Given a response variable \(Y\) and predictor variables \(X_i\), the ACE algorithm tries to find the maximally correlated mean-zero functions \(g(Y)\) and \(f_i(X_i)\), subject to \(E[g^2(Y)] = 1\). Equivalently, it tries to find the transformations that minimize \(e^2 = E[(g(Y) - \sum f_i(X_i))^2]\), subject to \(E[g^2(Y)] = 1\). For a given set of \(f_i(X_i)\), the minimization of \(e^2\) with respect to \(g(Y)\) yields

\[
g(Y) = \frac{E[\sum f_i(X_i)|Y]}{||E[\sum f_i(X_i)|Y]||} \quad (16)
\]

Similarly, for a given \(g(Y)\) and a given set of \(f_i(X_i)\) with \(i \neq j\), the minimization of \(e^2\) with respect to \(f_j(X_j)\) gives

\[
f_j(X_j) = E[g(Y) - \sum_{i \neq j} f_i(X_i)|X_j] \quad (17)
\]

The single-function minimizations Eqs. (16) and (17) are iterated until one complete pass over the predictor variables fails to reduce \(e^2\). The two kinds of conditional expectations appeared in Eqs. (16) and (17) are estimated by the scatterplot smoothers\(^{27}\).

Through the iterative procedure to minimize \(e^2\), the ACE algorithm provides the regression model:

\[
Y^* = g^{-1}(\sum f_i(X_i)) \quad (18)
\]

where \(Y^*\) is the prediction of \(Y\). As Eq. (18) implies, the ACE algorithm tries to make the relationship of \(g(Y)\) to \(f_i(X_i)\) as linear as possible. The resultant transformations are useful for descriptive purposes and discovering relationships between \(Y\) and \(X_i\), as the ACE algorithm makes it easier to examine how each \(X_i\) contributes to \(Y\). This feature is utilized in analyzing developed \(\bar{s}\) correlations.

Fig. 9 Transformations \(g, f_i, f_m, \) and \(f_n\) for Perpendicular Completions

\[\sigma=0.680\]

\[\sigma=0.293\]

\[\sigma=0.408\]
5.2.2. Optimal Transformations

As the number of predictor variables increases, the accuracy of the correlation is improved, while the versatility decreases. After several attempts, a reasonable compromise can be found with the response variable of log \( \bar{s} \) against the predictor variables of \( L_D \), \( m_{an} \), and \( n \). Any data set that yields negative \( \bar{s} \) values is excluded from the ACE procedure, the impact of which is not severe as can be seen later. The correlation for log \( \bar{s} \) is thus given as

\[
\log \bar{s} = g^{-1}[f_L(L_D) + f_m(m_{an}) + f_n(n)]
\]  

(19)

Figures 8 and 9 (circular symbols) show the optimal transformations \( g, f_L, f_m, \) and \( f_n \) obtained through the ACE algorithm for parallel and perpendicular completions, respectively. The response-variable transformation \( g \) (Fig. 8 (a) and Fig. 9 (a)) is positively related to \( \log \bar{s} \); thus, larger values of \( f_L, f_m, \) and \( f_n \) yield larger values of \( \log \bar{s} \).

Figures 8 and 9 (circular symbols) show the optimal transformations \( g, f_L, f_m, \) and \( f_n \) obtained through the ACE algorithm for parallel and perpendicular completions, respectively. The response-variable transformation \( g \) (Fig. 8 (a) and Fig. 9 (a)) is positively related to \( \log \bar{s} \); thus, larger values of \( f_L, f_m, \) and \( f_n \) yield larger values of \( \log \bar{s} \).

The predictor-variable transformation \( f_L(L_D) \) (Fig. 8 (b) and Fig. 9 (b)) is negatively related to \( L_D \), which indicates that \( \log \bar{s} \) becomes smaller as \( L_D \) values increase. \( \bar{s} \) was introduced to correct for the inability of the anisotropic approach to reproduce the near-sink pressure behavior and is not necessary for regions away from the sink. As \( L_D \) increases, pressure drawdown decreases and the difference in near-sink and far-sink pressure profiles becomes smaller, which is consistent with the negative relationship between \( L_D \) and \( \log \bar{s} \). The positive curvature for \( L_D \) values around 0.05, connecting a large slope segment for \( L_D < 0.05 \) and a small slope segment for \( L_D > 0.05 \), implies that when the horizontal sink is short a small change in \( L_D \) has a pronounced influence on \( \log \bar{s} \).

In contrast, \( f_m(m_{an}) \) (Fig. 8 (c) and Fig. 9 (c)) and \( f_n(n) \) (Fig. 8 (d) and Fig. 9 (d)) have positive relationships with \( m_{an} \) and \( n \), respectively. This implies that \( \log \bar{s} \) becomes larger as the degree of fracture development increases. \( f_m(m_{an}) \) has a negative curvature for \( m_{an} \) values around 0.02, beyond which any change in \( m_{an} \) has little influence on \( \log \bar{s} \). \( f_n(n) \), on the other hand, is almost linear to \( n \), so has a continuous influence on \( \log \bar{s} \).

The standard deviation \( \sigma \) of \( f_i \) is indicated for each plot. This provides a measure of how strongly each \( f_i \) contributes to \( g \). For parallel completions, \( \sigma[f_L] \) is about 4/3 of \( \sigma[f_m] \) and 9/8 of \( \sigma[f_n] \). For perpendicular completions, \( \sigma[f_L] \) is about 7/3 of \( \sigma[f_m] \) and 5/3 of \( \sigma[f_n] \). Thus, the horizontal-sink length has the strongest influence on \( g \) for both completions, and the impact is more prominent for perpendicular completions than for parallel completions.

To make the correlation portable, the least-curve fit was applied to the transformations and the explicit functional forms sought. Table 5 (parallel completions) and Table 6 (perpendicular completions) summarize these functions and the fitted curves are shown in Figs. 8 and 9 (solid lines). Substituting the resultant functions into Eq. (19) provides the correlations for \( \log \bar{s} \). Figure 10 compares \( \log \bar{s} \) values predicted through the correlations with the true values. Considering that only three parameters are used to generate

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**Table 5** Correlation for Parallel Completions: \( g = f_L + f_m + f_n \)

<table>
<thead>
<tr>
<th>( \log \bar{s} )</th>
<th>( f_L )</th>
<th>( f_m )</th>
<th>( f_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.231ln(g + 7.296)</td>
<td>-0.5575lnL_D - 0.9386</td>
<td>-6829m_{an} - 1769m_{an}^2 + 120.3m_{an} - 2.591</td>
<td></td>
</tr>
<tr>
<td>4.682 \times 10^{-3}m - 1.801</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 6** Correlation for Perpendicular Completions: \( g = f_L + f_m + f_n \)

<table>
<thead>
<tr>
<th>( \log \bar{s} )</th>
<th>( f_L )</th>
<th>( f_m )</th>
<th>( f_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.873ln(g + 3.180) - 0.0678g - 2.818</td>
<td>-0.6681lnL_D - 1.1108</td>
<td>6743m_{an}^2 - 1468m_{an}^2 + 100.1m_{an} - 1.992</td>
<td>3.891 \times 10^{-3}m - 1.535</td>
</tr>
</tbody>
</table>

---

Fig. 10 Comparison of Predicted \( \bar{s} \) and True \( \bar{s} \) Values
the correlations, the predictions are quite satisfactory.

5.3. Productivity Prediction with $\bar{s}$

Minimal information should be assumed for the prediction methodology to be practical and versatile. Since $\Sigma_{HD}$ and $m_{an}$ are required to estimate $k_x$ (Table 1) and $L_D$ is the parameter to be optimized in designing a horizontal sink, these three parameters can be considered as the minimal information.

With $L_D$, $\Sigma_{HD}$, and $m_{an}$ being provided, $n_f$ must be determined for the log $\bar{s}$ correlations (Tables 5 and 6). Assuming $\sigma_{an} = 0$ (or neglecting $\sigma_{an}$) in Eq. (12) yields the approximation

$$n_f = \frac{\Sigma_{HD}}{m_{an}}$$

and log $\bar{s}$ can be evaluated with Eq. (19). Then, $\bar{s}$ is used to correct $1/p_{HD}$ with Eq. (15), in which $k = \sqrt{k_x k_y}$ can be evaluated knowing $\Sigma_{HD}$ and $m_{an}$ (Table 1).

Figure 11 is the crossplot of $1/p_{HD}$ with $\bar{s}$ correction and $1/p_{HD}$. Unlike the predictions without $\bar{s}$ correction (Fig. 6), the cluster is symmetric about the identical line. In addition, RSS is drastically reduced (by 81% for both completions), indicating that the correction compensates for the positive bias in the anisotropic-system predictions. Therefore, the suitability of the proposed methodology is verified despite the exclusion of negative $\bar{s}$ values in developing the correlation and the approximate evaluation of $n_f$.

Figure 12 indicates the magnitude of $\bar{s}$ correction by comparing fracture-system and anisotropic-system predictions of $1/p_{HD}$ (equivalently, $1/p_{HD}$ predictions with and without $\bar{s}$ correction) as a function of $L_D$, where $m_{an}$ is set at 0.05 and $n_f$ values of 100, 300, and 500 are considered. Although the difference between the two (that is, the magnitude of $\bar{s}$ correction) is negligible for $n_f = 100$, the difference becomes large for larger values of $n_f$ and cannot be disregarded. This is consistent with the fact that $f_n(n_f)$ has a positive relationship with $n_f$ and implies that anisotropic-system predictions without $\bar{s}$ correction may mislead the horizontal-sink design. For instance, if the design calls for $1/p_{HD}$ of 1 by completing a horizontal sink in a 1000
m \times 1000 \text{ m flow domain containing 500 fractures, the fracture-system predictions suggest that } L_0 \text{ of } 0.455 \text{ (or } L \text{ of } 455 \text{ m) and } L_0 \text{ of } 0.270 \text{ (or } L \text{ of } 270 \text{ m) are required for parallel and perpendicular completions, respectively. On the other hand, the anisotropic-system predictions erroneously suggest that the same amount of productivity can be attained by shorter lengths of } 395 \text{ m for parallel completions and } 220 \text{ m for perpendicular completions, which would result in around 10% poorer actual productivity. This demonstrates the absolute importance of appropriate prediction methods for successful horizontal-sink design.}

### 6. Conclusions

1. Numerical experiments with 1000 fracture realizations were conducted to compare horizontal-sink productivities based on naïve fracture systems and equivalent anisotropic systems.

2. Anisotropic systems tend to yield erroneous predictions higher than the actual values. The mean relative errors are 11% and 13% for parallel and perpendicular completions, respectively.

3. A pseudo-skin factor was introduced to correct the overestimation with anisotropic systems. The mean pseudo-skin factors of 0.366 and 0.258 are required for parallel and perpendicular completions, respectively.

4. A set of correlation equations was developed for a pseudo-skin factor requiring minimal information (horizontal-sink length, total fracture length, and geometric mean of fracture lengths).

5. Individual contributions of correlation parameters to the pseudo-skin factor were examined quantitatively, finding that the horizontal-sink length is the most important parameter.

6. A systematic procedure is proposed to correct the productivity predictions with anisotropic systems and is successfully verified against the predictions with fracture systems.

### Acknowledgment

A part of this work was supported by the “Grant for Natural Gas Research” of The Japan Petroleum Institute.

### Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tr>
<td>$a_0$</td>
<td>dimensionless fracture length</td>
<td>[—]</td>
</tr>
<tr>
<td>$A$</td>
<td>area of a flow domain</td>
<td>[L$^2$]</td>
</tr>
<tr>
<td>$h$</td>
<td>formation thickness</td>
<td>[L]</td>
</tr>
<tr>
<td>$i$</td>
<td>imaginary number</td>
<td>[—]</td>
</tr>
<tr>
<td>$J_s$</td>
<td>horizontal-sink productivity</td>
<td>[L$^4$/m]</td>
</tr>
<tr>
<td>$k$</td>
<td>isotropic permeability</td>
<td>[L$^2$]</td>
</tr>
<tr>
<td>$k_e$</td>
<td>effective permeability, $\sqrt{k_ik_j}$</td>
<td>[L$^2$]</td>
</tr>
<tr>
<td>$L$</td>
<td>horizontal-sink length</td>
<td>[L]</td>
</tr>
<tr>
<td>$m_{a0}$</td>
<td>geometric mean of $a_0$</td>
<td>[—]</td>
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<tr>
<td>$n_f$</td>
<td>number of fractures</td>
<td>[—]</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
<td>[m/Lt$^2$]</td>
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<td>$p_o$</td>
<td>dimensionless pressure, $2\kappa k_p/\mu$</td>
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<tr>
<td>$q$</td>
<td>sink strength</td>
<td>[L/t]</td>
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<tr>
<td>$\bar{s}$</td>
<td>pseudo-skin factor</td>
<td>[—]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>fluid viscosity</td>
<td>[m/Lt]</td>
</tr>
<tr>
<td>$\sigma_{a0}$</td>
<td>standard deviation of $\ln a_0$</td>
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<tr>
<td>$\Sigma_{fD}$</td>
<td>dimensionless total fracture length</td>
<td>[—]</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>potential function, $(k/\mu)p$</td>
<td>[L/t$^2$]</td>
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<td>$\psi$</td>
<td>stream function</td>
<td>[L/t$^3$]</td>
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<tr>
<td>$\Omega$</td>
<td>complex potential</td>
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### Operators

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<thead>
<tr>
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<tr>
<td>$\bar{E}(\cdot)$</td>
<td>mathematical expectation</td>
<td>[—]</td>
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<tr>
<td>$E(\cdot</td>
<td>\cdot)$</td>
<td>conditional expectation of $\cdot$ given $\cdot$</td>
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<tr>
<td>$g(\cdot)$</td>
<td>optimal transformation of predictor variable $i$</td>
<td>[—]</td>
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<td>$\mu(</td>
<td>\cdot</td>
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### Subscripts

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### References


要 旨

水平シンク生産性予測におけるフラクチャー型システムと異方型システムの不整合性

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② 早稲田大学理工学部, 169-8555 東京都新宿区大久保3-4-1

一方向に発達したフラクチャーを含む貯留層を異方型システムとしてモデル化する手法は、フラクチャー型貯留層工学において最も一般的に用いられている手法の一つである。しかしながら、このような巨視的取扱いの普通の正当性は、定量的に検証されているわけではない。本研究では、水平シンクの生産性予測に際して、異方型システムがフラクチャー型システムと整合性を保持するか否かを検討するものである。

フラクチャーに対して平行もしくは直交して仕上げられた水平シンクに対し、1000 個のフラクチャーシステムを用いた数値実験を実施し、フラクチャー型ならびに異方型システムに基づく生産性予測を比較した。異方型システムは真の値より高い生産性予測をすることが判明し、相対誤差は平均して平均仕上げの場合 11％、直交仕上げの場合 13％である。フラクチャー型システムの等価異方型システムへの換算は、水平シンク近傍の流動に対しては必ずしも適用ではなく、生産性予測を通うする危険性が認められる。

異方型システムにおけるこのような生産性の過大予測を補正するために、擬似スキンを導入した。平均して、0.366 および 0.285 の擬似スキンが、平行ならびに直交仕上げに対してそれぞれ必要である。この補正手法の汎用化には、擬似スキン評価のための簡便な手段が必要であることから、最小限必要な情報（水平シンクの長さ、全フラクチャーの合計長、ならびにフラクチャー長さの幾何平均値）を用いた一連の関係式を導出した。異方型システムとフラクチャー型システムの不整合性は、ここで提出した体系的処理手順により補正しうるものである。

Keywords
Fluid flow, Numerical simulation, Boundary element method, Horizontal well, Well test, Reservoir

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