Representative Elementary Volume of Naturally Fractured Reservoirs Evaluated by Flow Simulation

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The single-continuum approach employing effective permeability is one of the practical methods for simulating naturally fractured reservoirs. Sensitivities of the flow behavior to the assumed region for effective permeability calculations, and to the scale of simulation grid-blocks were studied to examine the equivalent single-continuum system. A flux-continuous full-tensor model was used to deal with permeability tensors resulting from upscaling of fractured systems.

The representative elementary volume (REV) for stochastic fracture distributions was evaluated to examine the behavior of the effective permeability with change in the area of the calculating region. The REV for effective permeability was established for fracture systems of mean fracture length \( m_0 \) 0.02, 0.06, and 0.2 within the domain of unit area. Variations in effective permeability are small even for the upscaling sizes lower than the REV in the case of short fractures with \( m_0 = 0.02 \). Therefore, homogeneous models can correctly simulate tracer test performances.

For medium and long fractures of mean length 0.06 and 0.2 respectively, effective permeability corresponding to the upscaling sizes lower than the REV indicates large fluctuations reflecting significant local heterogeneity. Local heterogeneity of small scale must be incorporated in permeability distributions to obtain good simulation results. For modeling local heterogeneity, a grid-block should be assigned a specific effective permeability tensor and should be as coarse as the order of \( m_0 \), followed by refinement of the coarse grids to avoid numerical dispersion.

Keywords
Naturally fractured reservoirs, Effective permeability, Representative elementary volume, Full tensor model

1. Introduction

Two main approaches are currently available for simulating the behavior of naturally fractured reservoirs, the dual porosity or dual permeability model, and the single continuum model with effective permeability. In the dual continuum models, matrix blocks are divided by regular fracture patterns. However, field characterization studies have shown that fracture systems are very irregular, often disconnected, and occur in swarms. Estimation of the shape factor which represents the interference between the fractures and the rock matrix is another problem with the dual continuum approach. In the single continuum models, the flow through both the fracture and matrix systems as well as the interference flow is represented by the effective permeability.

The effective permeability is estimated by solving the flow equations reflecting the coupled matrix and fracture flow. In our previous study, we applied the complex variable boundary element method (CVBEM) to semi-analytically solve the coupled potential problem for matrix and fracture flow under periodic boundary conditions. Effective permeability resulting from flow calculations is generally of tensor form. Diagonal and off-diagonal elements of the effective permeability tensors can be correlated with the distribution parameters for stochastic fracture systems.

Simulation of a naturally fractured reservoir by the single continuum approach with effective permeability introduces two different scales, i.e. upscaling size and simulation grid-block size relative to the mean fracture length, which may cause problems with correctly char-
acterizing and simulating reservoirs. Effective permeability of a fracture system is a function of the rock volume over which the flow calculations are performed\(^{12}\). Measurement of a property of heterogeneous rock generally exhibits fluctuations in value with increasing sample volume caused by the heterogeneity of the micro and macro scales. Such fluctuations tend to minimize as the sample volume is increased from the macro to mega scales. As the sample volume is further increased, a gradual change is observed reflecting the heterogeneity of the mega to giga scales. The representative elementary volume (REV) is the notion that a representative value of a rock property is to be measured with sample volumes with which measurements exhibit negligible variations\(^{13}\). Long\(^{12}\) did REV studies on fractured rock and found that a megascopic continuum could be defined when the averaging region was large enough.

The grid-block size for simulation directly affects the accuracy and efficiency of computation. In the present case, we also need to consider the proper grid-block size relative to the mean fracture length. In this regard, Lee et al.\(^4\) developed a hierarchical approach to model flow in a naturally fractured reservoir with multiple length-scale fractures. They considered three classes of fractures to contribute to flows: short disconnected fractures with \(l_f/d_f \ll 1\), medium-scale fractures with \(l_f/d_f \approx 1\), and long fractures with \(l_f/d_f \gg 1\), where \(l_f\) and \(d_f\) are the fracture length and grid-block length scales, respectively. The short-scale fractures are modeled into the effective matrix permeability by an analytical method, and then the medium-length fractures in grid-blocks are modeled to generate effective permeability tensors by the boundary element method. The long-scale fractures are accounted for explicitly as conduit or wells.

Objectives of this paper are to examine the equivalent single-continuum system for flow performance prediction of naturally fractured reservoirs by studying the sensitivities to the region for effective permeability calculations, and to the scale of simulation grid-blocks. To determine the permeability tensors for flow simulation, we developed a flux-continuous full-tensor model based on the mixed finite volume element method (MFVEM)\(^{14}\).

In this paper, fractures are assumed to be discrete and distributed regularly or randomly in two dimensions in an otherwise isotropic and homogeneous matrix rock of permeability \(k_m = 9.869 \times 10^{-4} \mu\text{m}^2\) (1 mD). Individual fractures are assumed to be infinitely thin and infinitely conductive, to be perpendicular to the bedding, and to terminate at the bedding surfaces.

2. REV for Effective Permeability

A REV study of stochastic fracture distributions was performed to evaluate how the effective permeability behaves as the area of the calculating region is changed. Fracture systems comprise discrete non-crossing fractures distributed randomly. Lengths \(l_i\) and orientations \(\theta_i\) of individual fractures are normally distributed with mean length \(m_l\) and standard deviation \(\sigma_l\), and with mean angle \(m_{\theta_i}\) and standard deviation \(\sigma_{\theta_i}\), respectively. Four fracture systems were generated for REV evaluation with parameters \(m_l = 0.02, 0.06, 0.2\) m, \(\sigma_l = 0.4m_l\), \(m_{\theta_i} = 20^\circ\) with \(\sigma_{\theta_i} = 0^\circ\), and \(m_{\theta_i} = 45^\circ\) with \(\sigma_{\theta_i} = 5^\circ\) as shown in Figs. 1-4. A square window of area \(0.01-0.95 \mu\text{m}^2\) is randomly moved within the normalized domain of \(1.0 \mu\text{m}^2\), and the effective permeability is determined for the window under the periodic boundary conditions (see Appendix A.). As the window size increases, variations of effective permeability decrease. The smallest window size to exhibit variations of less than \(\pm 5\%\) of convergence is used as the REV for convenience.

Figure 1 (a) shows the fracture system for \(m_l = 0.02\) m, \(m_{\theta_i} = 20^\circ\), and the number of fractures \(n_f = 200\).

Figures 1 (b), 1 (c), and 1 (d) display the components of effective permeability \(k_{xx}, k_{yy},\) and \(k_{xy}\) versus the window area, respectively. If the calculation window is larger than about \(0.1 \mu\text{m}^2\) or \((0.316 \mu\text{m})^2\), the values of the effective permeability are within \(\pm 5\%\) of the converging values. Even for windows smaller than \(0.1 \mu\text{m}^2\), the effective permeability components do not exhibit large variations except for a few larger values with the window area of \(0.01 \mu\text{m}^2\).

Figure 2 presents the same for \(m_l = 0.06\) m, \(m_{\theta_i} = \)
If the window size is larger than about 0.5 m$^2$ or (0.707 m)$^2$, the effective permeability components are within ±5% of the converging values. The permeability values show gradually increased variability as the window area decreases to 0.1 m$^2$, and then more variability as the window is further reduced. This variation is caused by the local heterogeneity of the fracture distribution, and may need to be considered in the flow simulation.

Figures 3 and 4 show REV evaluation for $m_m = 0.2$ m, $n_f = 100$, with $m_\theta = 20^\circ$ and $m_\theta = 45^\circ$, respectively. The window sizes of about 0.75 m$^2$ or (0.866 m)$^2$ for the former, and about 0.60 m$^2$ or (0.775 m)$^2$ for the latter are estimated as the smallest areas for ±5% deviations from the converging values. Scattering of the permeability components values indicates significant heterogeneity with smaller window size, and the asymmetric scattering generates more numbers of lower values. The reason is that fracture segments extending beyond the window sides are cut off for computing the effective permeability. The REV for $m_\theta$ of 45° is smaller than that for $m_\theta$ of 20°, because fewer fractures are cut off when $m_\theta = 45^\circ$.

To validate the converging values of effective permeability obtained by the REV study, correlation of the effective permeability with the geometric parameters is established. The effective permeability is first calculated for 500 realizations. Geometric parameter values are selected uniformly in the ranges $1 < L < 20$ m, $0 < m < 0.20$ m, $0 < \alpha < 0.08$ m, $0 < m_\theta < 90^\circ$, $0 < \phi < 10^\circ$, where $L$ is summation of $l_i$, $i = 1, \ldots, n_f$. Correlation is sought by the non-parametric regression analysis method ACE$^{15}$. Given independent variables $X_i$ and a dependent variable $Y$, ACE discovers the optimal transformation functions $f_i$ and $g$ with zero mean so that the expectation $E[(g(Y) - \Sigma f_i(X_i))^2]$ is minimized under the constraint $E[g^2(Y)] = 1$. The correlation model is

![Fig. 2 Effective Permeability vs. Window Area, $m_m = 0.06$ m, $m_\theta = 20^\circ$, $n_f = 200$](image)

![Fig. 3 Effective Permeability vs. Window Area, $m_m = 0.2$ m, $m_\theta = 20^\circ$, $n_f = 100$](image)

![Fig. 4 Effective Permeability vs. Window Area, $m_m = 0.2$ m, $m_\theta = 45^\circ$, $n_f = 100$](image)
determined as $g(Y^*) = \Sigma f_i(X_i)$ or $Y^* = g^{-1}[\Sigma f_i(X_i)]$ where $Y^*$ is an estimate of $Y$, i.e. $g$ is linearly correlated with $f_i$’s.

The components of the effective permeability, $k_{xx}$, $k_{xy}$, and $k_{yy}$, can be correlated with three parameters, $L$, $m_l$, and $m_b$, and the correlations of the natural logarithm of these permeability components with $L$, and $m_l$ are almost linear because the transformations $f(L)$, $f(m_l)$, $g(\ln k_{xx})$, $g(\ln k_{xy})$, and $g(\ln k_{yy})$ are almost linear functions except for small values of the variables. These geometric parameters have approximately equal influences on both diagonal and off-diagonal elements. Effects of the variations in length $\sigma_l$ and orientation $\sigma_\theta$ are small and can be ignored.

Effective permeability values estimated from the correlation are shown as horizontal lines in Figs. 1-4.

3. Flow Simulation with Effective Permeability

Accurate flow simulation of naturally fractured reservoirs by a single-continuum model requires appropriate gridding, and assignment of adequate effective permeability to individual grid-blocks. To analyze the effects of these factors, we conducted simulations of tracer slug tests using a full-tensor model. The full-tensor model is briefly described first.

3.1. Full-tensor Flow Model

We have developed a numerical model that incorporates full tensor permeability in each grid-block to accurately simulate the behaviors of discretely fractured media. The model is based on the flux-continuouformulation using the MFVEM. The pressure equations, expressed as two coupled first-order partial differential equations for pressure and velocity, are solved simultaneously.

We start from the mass balance equation and Darcy’s law for two-phase flow of immiscible and incompressible fluids through two-dimensional horizontal porous media. These two equations are multiplied by a scalar test function $w$ and a vector test function $v$, respectively, and then integrated over the domain $\Omega$ to obtain Eqs. (1) and (2).

\[
\int_{\Omega} \nabla \cdot w d\omega = \int_{\Omega} q_i w d\omega
\]  

(1)

\[
\int_{\Omega} (\lambda(S)k) \cdot v d\omega = -\int_{\Omega} \nabla \cdot p d\omega
\]  

(2)

Here $S = S_0$ and we define the global pressure and total velocity as

\[
p = p_0 - \int_{0}^{\xi} \frac{s_{\lambda}(\xi)}{\lambda(\xi)} d\xi
\]  

(3)

\[
u = u_w + u_o
\]  

(4)

Referring to Fig. 5 (a), Eq. (1) is written as

\[
\int_{x_{j-1/2}Y_{j+1/2}} \int_{x_{j+1/2}Y_{j+1/2}} \left( \frac{\partial q_i}{\partial x} + \frac{\partial q_i}{\partial y} \right) dxdy = \int_{x_{j-1/2}Y_{j+1/2}} \int_{x_{j+1/2}Y_{j+1/2}} q_i dxdy
\]  

(5)

Equation (2) yields the two components system, Eqs. (6) and (7), for the $x$ and $y$ directions, respectively:

\[
\int_{x_{j-1/2}Y_{j+1/2}} \int_{x_{j+1/2}Y_{j+1/2}} k_{xy} \frac{\partial u_x}{\partial y} - k_{yx} \frac{\partial u_y}{\partial x} + \frac{\partial p}{\partial x} dxdy = 0
\]  

(6)

\[
\int_{x_{j-1/2}Y_{j+1/2}} \int_{x_{j+1/2}Y_{j+1/2}} k_{yy} \frac{\partial u_y}{\partial y} - k_{yx} \frac{\partial u_x}{\partial x} + \frac{\partial p}{\partial y} dxdy = 0
\]  

(7)

The discrete version of Eq. (5) is expressed with the size of the regular grids $h$ as

\[
h(u_{x,j+1/2,j} - u_{x,j-1/2,j} + u_{x,j+1/2,j+1} - u_{x,j-1/2,j+1}) = h^2 q_{x,j}
\]  

(8)

Equations (6) and (7) are discretized as

\[
T_{x_{i+1/2,j},x_{i+1/2,j+1}} + T_{x_{i+1/2,j+1},x_{i+1/2,j}} + T_{x_{i+1/2,j},x_{i+1/2,j}+1} + T_{x_{i+1/2,j+1},x_{i+1/2,j}+1} + p_{i+1,j} - p_{i,j} = 0
\]  

(9)

where $A$, $R$, ... $G$ refer to the edges of the control volumes $Q_{i,j}$ and $Q_{i,j+1}$ as illustrated in Fig. 5 (b), and

\[
T_{y_{j+1/2,j},y_{j+1/2,j+1}} + T_{y_{j+1/2,j+1},y_{j+1/2,j}} + T_{y_{j+1/2,j},y_{j+1/2,j+1}} + T_{y_{j+1/2,j+1},y_{j+1/2,j}+1} + p_{j+1,i} - p_{j,i} = 0
\]  

(10)

where $O$, $P$, $R$, ..., $V$ refer to the edges of the control volumes $Q_{i,j}$ and $Q_{i,j+1}$.
The coefficients in Eq. (9) are given as follows:

\[ T_{i+1/2,j,t} = \frac{1}{8} h \frac{k_{y,i,j}}{(\lambda(S)\det[k])_{i,j}} \]

\[ T_{i+1/2,j,b} = T_{i+1/2,j,c} = \frac{1}{4} h \frac{k_{y,i,j}}{(\lambda(S)\det[k])_{i,j}} \]

\[ T_{i+1/2,j,d} = \frac{3}{8} h \frac{k_{y,i,j}}{(\lambda(S)\det[k])_{i,j}} + \frac{3}{8} h \frac{k_{y,i,j+1}}{(\lambda(S)\det[k])_{i,j+1}} \]

\[ T_{i+1/2,j,e} = T_{i+1/2,j,f} = \frac{1}{4} h \frac{k_{y,i,j+1}}{(\lambda(S)\det[k])_{i,j+1}} \]

\[ T_{i+1/2,j,g} = \frac{1}{8} h \frac{k_{y,i,j}}{(\lambda(S)\det[k])_{i,j}} \]

The coefficients in Eq. (10) are analogous to Eq. (11). Equations (8), (9), and (10) give rise to asymmetric linear equations to be solved for the pressures at the grid-block centers and the velocities across the edges.

\[
\begin{bmatrix}
M_{xx} & M_{xy} & N_x & U_x & P_x \\
M_{yx} & M_{yy} & N_y & U_y & P_y \\
N_x^T & N_y^T & 0 & P & R_p
\end{bmatrix} = \begin{bmatrix}
R_{xx} \\
R_{xy} \\
0
\end{bmatrix}
\]

where \( M_{xx} \) and \( M_{yy} \) are tri-diagonal matrices, \( N_x \) and \( N_y \) have \( \pm 1 \) entries, and \( M_{xy} \) and \( M_{yx} \) have four non-zero bands. \( U_x, U_y, \) and \( P \) are the vectors of unknowns. \( R_{xx}, R_{xy}, \) and \( R_p \) are associated with boundary conditions of \( u_x, u_y, \) and \( p \), respectively.

3.2. Tracer Performance

The full-tensor flow model was first examined by simulating tracer slug tests in several synthetic media. The flow domain is a square of unit area to model one quarter of a repeated five-spot pattern divided into 40 x 40 uniform grids. An injection well is located at the grid (1, 1), and a production well at (40, 40). The slug size of the tracer solution is 0.5 pore volume injected into the single-phase reservoir fluid and displaced by the same fluid, and balanced production and injection rates of fluids of unit mobility ratio are assigned to the wells.

First, the effects of permeability tensors were evaluated running the model with an isotropic and diagonal tensor \( k_{xx} = k_{yy} = k_m, k_{xy} = 0 \), an anisotropic and diagonal tensor \( k_{xx} = k_m, k_{yy} = 0.5k_m, k_{xy} = 0 \), and an isotropic full tensor \( k_{xx} = k_{yy} = k_m, k_{xy} = k_{yx} = 0.5k_m \) assigned to all the grid-blocks uniformly. Here \( k_m = 9.869 \times 10^{-4} \mu m^2 \). Figure 6 shows the tracer concentrations in the effluent versus the pore volumes injected (PVI) for three cases.

The tracer concentration shows a sharp rise after the breakthrough, which describes the flow behavior of the leading edge of the tracer slug, and then a downturn representing the flow behavior of the trailing edge of the tracer slug. Theoretically, the initial breakthrough occurs at 0.718 PVI for a five-spot pattern in homogeneous media\(^{16}\). Isotropic and diagonal tensors should generate a unique curve of concentration regardless of
permeability values, if it is plotted against PVI.

The tracer performance for the isotropic diagonal tensor yields the correct breakthrough time at 0.718 PVI, whereas the anisotropic diagonal tensor tends to delay the breakthrough and distorts the profile. On the other hand, the curve for the full tensor permeability is shifted to the left side, which shows the early breakthrough due to the off-diagonal components.

In Figs. 7 (a) and 7 (b), the effects of the grid size are evaluated for the isotropic diagonal and isotropic full tensors, respectively. Numerical dispersion is more pronounced yielding an earlier breakthrough in both cases, as the number of grid-blocks is reduced from $40 \times 40$ to $10 \times 10$ (the grid size becomes coarser).

Next, we applied the full-tensor model to tracer slug tests in two fracture systems that contain 25 thin fractures spaced regularly, parallel and perpendicular to the flow direction as depicted in Figs. 8 (a) and 9 (a), respectively. The effective permeability tensors were separately calculated by the CVBEM code and assigned to the $40 \times 40$ individual grid-blocks. Figures 8 (b) and 9 (b) show the streamlines and fronts of the tracer slug, as computed by the full-tensor model. Figures 8 (c) and 9 (c) present those computed by the CVBEM for references. Three fronts shown in each of these figures illustrate how the leading edge of the tracer slug advances with time. Flow in the CVBEM was determined by the governing equation and not restricted by the artificial grid system required in domain methods such as the present full-tensor model. Close agreements are observed between the results of two models, though Fig. 8 (c) presents more pronounced fingering than Fig. 8 (b) demonstrating the resolution limit of the CVBEM that uses finite grids.

Figures 8 (d) and 9 (d) show the tracer performances simulated by the full-tensor model and CVBEM. The breakthrough curve for the homogeneous matrix without fractures computed by the CVBEM is also shown for reference. As seen in Fig. 8 (d) for the parallel fracture case, the CVBEM and full-tensor model yield breakthrough at 0.349 and 0.604 PVI, respectively. These breakthrough times are much earlier than 0.718 PVI of the homogeneous non-fractured matrix. The early breakthrough and non-sharp peaks are due to fingering caused by the parallel fractures. Compared with the semi-analytical result of CVBEM, the full-tensor model produced a.

Fig. 8 Tracer Performance in Fractured Reservoir (parallel), (a) Fracture Distribution, (b) MFVEM, (c) CVBEM, (d) Tracer Response Curves
later breakthrough and a less rugged curve right after the breakthrough. These characteristics reflect the differences in fingering computed by the two models as depicted in Figs. 8 (b) and 8 (c). On the other hand, the fractures perpendicular to the flow direction do not cause any fingering, but only allow flow to spread laterally as shown in Fig. 9 (d). Therefore, the tracer breakthrough curves are nearly the same as those of the non-fractured system.

4. REV Evaluation by Flow Simulation

4.1. Short Fractures

Simulations of tracer slug tests were conducted to examine the sensitivity of flow performance to REV of effective permeability for the stochastic fracture systems. The full-tensor model was used to simulate flow behaviors in the four stochastic realizations depicted in Figs. 1-4. The flow domain was divided into 40 x 40 grid-blocks to which the same values of effective permeability components were equally assigned. The semi-analytical CVBEM model was also run for each case as a reference.

Figure 10 presents the tracer performances for \( m = 0.02 \) m. Figure 10 (a) displays the tracer breakthrough curve computed for \( k_{xx} = 1.069 k_m, k_{xy} = k_{yx} = 0.025 k_m, \) and \( k_{yy} = 1.009 k_m, \) where \( k_m = 9.869 \times 10^{-4} \) \( \mu \)m\(^2\). These components of the permeability tensor are the averaged values of effective permeability corresponding to \( A = 0.95 \) m\(^2\) as shown in Fig. 1. The medium in this case is modeled as homogeneous and anisotropic \( (k_{xx} \neq k_{yy}) \). The 'A_avg' curve is very close to the semi-analytical solution, which proves the validity of representing the system as a homogeneous and anisotropic medium with the effective permeability. The breakthrough occurs at about 0.718 PVI because the effects of anisotropy and the cross terms, \( k_{xy} \) and \( k_{yx} \), are very small.

Figure 10 (b) shows the curves for three different sets of permeability tensor components, 'B_max', 'B_avg', and 'B_min'. These tensor components correspond to \( B = 0.1 \) m\(^2\) as marked in Fig. 1, and their values are listed in Table 1 (a). The three curves are identical and also close to the CVBEM curve. Therefore, the REV \( (0.1 \) m\(^2\)) evaluated by Fig. 1 is also valid for flow simulation. Based on these results, it is inferred that very short fractures can be satisfactorily upscaled by homogeneous representation with
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anisotropic effective permeability. The cross terms may be ignored for very short fractures, though more studies are to be conducted for fracture distributions of higher density.

Figures 11 (a) and 11 (b) display the tracer performances for $m_l=0.06$ m. The effective permeability values for $A=0.95$ m$^2$ and $B=0.1$ m$^2$ are listed in Table 1 (b). The overall characteristics of the ‘$A_{\text{avg}}$’ curve are similar to the CVBEM solution, though the turndown decreases more slowly than CVBEM. However, the ‘$A_{\text{avg}}$’ curve exhibits some deviations as seen in Fig. 11 (a). First, the breakthrough of ‘$A_{\text{avg}}$’, which occurs at 0.65 PVI reflecting the effect of the cross terms, is slightly later than that of the semi-analytical solution. The reason is that the effects of the anisotropy ($k_{xx} = 1.669 k_m$ versus $k_{yy} = 1.089 k_m$) causing predominant flow in the horizontal direction overcome the effects of the cross terms ($k_{xy} = k_{yx} = 0.244$). Secondly, the homogeneous permeability incorporated in the full-tensor model does not explicitly reproduce the second hump that reflects flow in matrix, but only effectively shows the double-porosity behavior.

The three curves ‘$B_{\text{max}}$’, ‘$B_{\text{avg}}$’, and ‘$B_{\text{min}}$’ drawn in Fig. 11 (b) are almost the same as ‘$A_{\text{avg}}$’, but the effects of differences in anisotropy caused slightly different heights of the peak concentration. Based on these results, the homogeneous and anisotropic representation according to the REV (0.5 m$^2$) evaluated by Fig. 2 can provide acceptable flow simulation. Local heterogeneity shown as the large scattering in Fig. 2 needs to be incorporated into the model in order to further improve performances.

4.2. Long Fractures

Tracer simulations were repeated for a longer fractures of $m_l=0.2$ m with the same mean orientation angle as $m_o=20^\circ$. Figures 12 (a) and 12 (b) depict the computed performances for the effective permeability for $A=0.95$ m$^2$ and $B=0.1$ m$^2$ (see Fig. 3), respectively. Gridding is $40 \times 40$, and the same permeability is assigned to all the cells to build a homogeneous model. Compared with the semi-analytical solutions, two types of defects are observed. First, the breakthrough time is about 0.63 PVI compared with 0.718

Table 1 Effective Permeability for Stochastic Fracture Systems (units: $9.869 \times 10^{-4} \mu$m$^2$)

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<th>$m_l$</th>
<th>$m_o$</th>
<th>$n_f$</th>
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</tbody>
</table>
PVI for homogeneous and isotropic media of diagonal tensor permeability. This indicates significant effects of the off-diagonal cross terms, yet the breakthrough time is much later than the semi-analytical solution. Second, the curves are smooth and do not reflect any heterogeneous behavior.

The late breakthrough is due to the strong anisotropy ($k_{xx} = 5.241\text{km}$ and $k_{yy} = 1.582\text{km}$ for the 'A_avg' curve, also see Table 1 (c) for the B curves) and relatively small cross terms ($k_{xy} = 1.563\text{km}$ for 'A_avg'). In Fig. 12 (b), three curves, 'B_max', 'B_avg', and 'B_min' show the same breakthrough time and distinct heights of the concentration peak. The difference in height demonstrates the effects of the different degrees of anisotropy. Homogeneous modeling with effective permeability evaluated at REV does not yield satisfactory results for severe anisotropy as this case. Local heterogeneity needs to be taken into account.

The tracer simulation was repeated for mean fracture length $m_l = 0.2\text{m}$ and mean orientation angle $m_{\theta} = 45^\circ$. Figures 13 (a) and 13 (b) are the results for the effective permeability for A and B, respectively, as marked in Fig. 4. The flow systems in this case are characterized by isotropic permeability ($k_{xx} = 3.101\text{km}$ versus $k_{yy} = 3.107\text{km}$ for the 'A_avg' curve, also see Table 1 (d) for the B curves) and relatively large cross terms ($k_{xy} = 2.101\text{km}$ for 'A_avg'). Overall performance of the 'A_avg' curve agrees well with the semi-analytical solution. In Fig. 13 (b), however, the three 'B' curves line up showing the same performance with different breakthrough times. This is due to different magnitude of the cross terms with isotropic tensor components. These tracer performances demonstrate the defect of homogeneous models with effective permeability at REV, and the need to model local heterogeneity in flow simulation.
4.3. Effects of Heterogeneity

When a stochastic fracture system is modeled by a homogeneous distribution of effective permeability, the tracer performance results in a smooth curve as obtained in the previous section. The semi-analytical solution, on the other hand, generates a noisy response curve reflecting the double-porosity feature and a non-uniform distribution of fractures. Representation of fractured systems by effective permeability cannot explicitly obtain the double-porosity feature. However, the features caused by local heterogeneity can be simulated by implementing the heterogeneous distribution of effective permeability.

The question posed here is how much detail of heterogeneity should be incorporated in a description of the effective permeability distribution. We applied the full-tensor model again for four systems shown in Figs. 1-4. This time each grid-block was assigned its own effective permeability tensor to model heterogeneity. The optimum grid size relative to the mean fracture length was evaluated by analyzing the sensitivity of flow behavior to the grid size.

First, the fractured domains of $m_l=0.02$ and $0.06$ m as depicted in Figs. 1 and 2, respectively, were divided into four grid systems, $10 \times 10$, $20 \times 20$, $30 \times 30$, and $40 \times 40$. Effective permeability tensors were separately computed for the individual cells of each grid system. Then the full-tensor model was run to obtain the four performance curves as graphed in Figs. 14 and 15, which also show the semi-analytical solutions previously computed.

For the case of $m_l=0.02$ m (Fig. 14), the $30 \times 30$ and $40 \times 40$ grids regenerate satisfactory tracer performances with only slight influences exerted by local heterogeneity. This can be compared to the performance curve of the homogeneous representation of the $40 \times 40$ grid.
grid system as shown in Fig. 10 (a). The 40 × 40 (grid-block length $l_g = 0.025$ m) and 30 × 30 ($l_g = 0.033$ m) systems are equally optimized to correctly reflect heterogeneity, though the latter includes higher numerical dispersion. The 10 × 10 and 20 × 20 grid systems suffer from even higher numerical dispersion yielding smoothed curves and shifting them to the origin side.

In the case of $m_l = 0.06$ m, the heterogeneous representation can produce better tracer performances (Fig. 15) than the homogeneous description (Fig. 11). The 30 × 30 grid system also yields a good match, though $l_g < m_l$. The double-porosity behavior appearing as the second lower hump cannot be simulated by the single-porosity approach using effective permeability. The noisy responses caused by local heterogeneity disappear as the grids become coarser.

Next the longer fracture systems $m_l = 0.2$ m with $m_o = 20^\circ$ and $45^\circ$ as given by Figs. 3 and 4, respectively, were simulated by three grid systems 10 × 10, 40 × 40, and 10 × 10 with refinements. The last grid system assumed 10 × 10 grids assigned individual permeability, with each cell refined into 4 × 4 grid-blocks resulting in 40 × 40 grid-blocks. The tracer performance curves are presented in Figs. 16 and 17.

Comparing the 40 × 40 curve in Fig. 16 with the ‘$A_{avg}$’ curve in Fig. 12 (a), the former better reflects the flow behavior caused by heterogeneity, though the breakthrough time is not improved. As the grid size $l_g$ is equal to 1/40 (0.025 m), much smaller than the mean fracture length $m_l$, the effective permeability for each grid cell is not correctly evaluated, in particular the value of the cross term is too low.

The 10 × 10 curve suffers from severe numerical dispersion causing early breakthrough and smoothing the roughness reflecting heterogeneity. As demonstrated by the 10 × 10 (Refined) curve in Fig. 16, grid refinement is an effective means to improve these defects.

Figure 17 displays the results for the case of $m_l = 0.2$ m and $m_o = 45^\circ$. Observations are similar to those for Fig. 16 except that the 10 × 10 (Refined) curve could not match the early breakthrough time of the semi-analytical solution. Strong fingering along $45^\circ$ that directly affects breakthrough could not be reproduced adequately by effective permeability.

5. Conclusions

(1) REV studies for stochastic fracture distributions were performed to evaluate how the effective permeability behaves as the area of the calculating region is changed. The smallest areas for representative values of effective permeability with a criterion of ±5% variations are about 0.1 and 0.5 within the domain of unit area for the mean fracture length ($m_l$) 0.02 and 0.06, respectively. With $m_l = 0.2$, the smallest areas are about 0.75 and 0.60 for the mean orientation angles of $20^\circ$ and $45^\circ$, respectively.

(2) Using a calculating domain smaller than the REV, effective permeability exhibits more variations reflecting local heterogeneity. Variations are small for $m_l$ of 0.02, but are significant for $m_l$ of 0.06 and 0.2.

(3) Homogeneous representation with effective permeability of REV and proper gridding can simulate the tracer performance reasonably well, if the mean fracture length is about 1/50 of the domain size or less. Local heterogeneity exerts only a slight influence. For the medium $m_l$ scale of about 1/15 of the domain size, local heterogeneity needs to be incorporated in the permeability descriptions, though homogeneous modeling produces acceptable results.

(4) For the long $m_l$ of about 1/5 of the domain size, homogeneous modeling with effective permeability of REV does not yield satisfactory performances because either severe anisotropy or large cross terms in effec-
tive permeability requires local heterogeneity to be modeled.

(5) To model local heterogeneity, the size of a grid-block to which its own effective permeability tensor is assigned should be of the same order as \( m_l \), and the coarse grids should be refined to avoid numerical dispersion.

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Nomenclatures

\[
\begin{align*}
&h : \text{size of regular grids} \quad \text{[m]} \\
&k : \text{permeability tensor, } \begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} \quad \text{[m}^2]\text{]} \\
&k_{m} : \text{permeability of matrix} \quad \text{[m}^2]\text{]} \\
&k_{r\alpha} : \text{relative permeability of phase } \alpha, \text{ dimensionless} \\
&L : \text{total length of fractures, } \Sigma_l \quad \text{[m]} \\
&l_i : \text{length of } i\text{-th fracture} \quad \text{[m]} \\
&m_{l} : \text{mean fracture length} \quad \text{[m]} \\
&m_\theta : \text{mean orientation angle of fractures} \quad \text{[}^\circ]\text{]} \\
P : \text{unknown pressure vector, } (p_{1,1}, p_{1,2}, \cdots, p_{i,j}, \cdots, p_{n_x,n_y})^T \\
P_c : \text{capillary pressure, } p_0 - p_w \quad \text{[Pa]} \\
p : \text{global pressure} \quad \text{[Pa]} \\
p_{\alpha} : \text{pressure of phase } \alpha \quad \text{[Pa]} \\
q_{\alpha} : \text{source/sink term per unit volume} \quad \text{[s}^{-1}\text{]} \\
q_i : \text{total source/sink term per unit volume, } q_{\alpha} + q_{\omega} \quad \text{[s}^{-1}\text{]} \\
S_{\alpha} : \text{saturatation of phase } \alpha \quad \text{[-]} \\
T : \text{transmissibility terms given by Eq. (11)} \quad \text{[-]} \\
t : \text{time} \quad \text{[s]} \\
U_x : \text{unknown velocity vector in } x \text{ direction, } (u_{x;1,1}, u_{x;2,1}, \cdots, u_{x;i,j}, \cdots, u_{x;n_x,n_y})^T \\
U_y : \text{unknown velocity vector in } y \text{ direction, } (u_{y;1,1}, u_{y;1,2}, \cdots, u_{y;i,j}, \cdots, u_{y;n_x,n_y})^T \\
u_{\alpha} : \text{velocity vector of phase } \alpha \quad \text{[m/s]} \\
u : \text{total velocity, } u_w + u_o \quad \text{[m/s]} \\
u : \text{velocity at block sides} \quad \text{[m/s]} \\
<\text{Greeks}> \\
\lambda_{\alpha} : \text{mobility functions of phase } \alpha, k_{r\alpha}/\mu_{\alpha} \quad \text{[(Pa} \cdot \text{s})^{-1}\text{]} \\
\lambda : \text{total mobility functions, } \lambda_{\omega} + \lambda_{\alpha} \quad \text{[(Pa} \cdot \text{s})^{-1}\text{]} \\
\mu_{\alpha} : \text{viscosity of phase } \alpha \quad \text{[Pa} \cdot \text{s}\text{]} \\
\phi : \text{porosity, fraction} \quad \text{[-]} \\
\theta : \text{fracture orientation angle} \quad \text{[}^\circ]\text{]} \\
\sigma : \text{standard deviation} \quad \text{[-]} \\
\xi : \text{dummy variable for saturation in Eq. (3)} \quad \text{[-]} \\
<\text{Subscripts}> \\
i, j : \text{grid block indices} \\
l : \text{fracture length} \\
x, y : \text{number of grid blocks} \\
o : \text{oil} \\
t : \text{total} \\
w : \text{water} \\
x, y : \text{coordinate direction} \\
\alpha : \text{fluid phase} \\
\theta : \text{fracture orientation angle} \\
\end{align*}
\]

Appendix A. Effective Permeability Computation by CVBEM

The effective permeability tensor is calculated by solving the two-dimensional, single-phase, incompressible fluid flow equation subject to the periodic boundary conditions (PBC). Flow within a grid-block is described by the following equation assuming unit viscosity:

\[
\nabla \cdot (k \nabla p) = 0 \quad \text{(A-1)}
\]

The effective permeability \( k^* \) is defined with the average velocity \( (u) \) and the average pressure gradient \( G \) as follows:

\[
(u) = -k^* G \quad \text{(A-2)}
\]

where \( (u) = ((u_x),(u_y))^T \) and \( G = (1, 0)^T \) or \( (0, 1)^T \).

Equation (A-1) needs to be solved twice by applying the unit pressure drop across the grid-block in one direction only. The solution of Eq. (A-1) with the unit pressure drop in the \( x \) direction provides a pressure distribution in the grid-block, from which the velocity distribution \( u \) is computed. The average velocity through the grid-block can be obtained as follows:

\[
(u_x) = -\int_{n_1} u \cdot n_1 \, dy \quad \text{(A-3)}
\]

\[
(u_y) = -\int_{n_3} u \cdot n_3 \, dy \quad \text{(A-4)}
\]

where \( n_1 \) and \( n_3 \) are unit normal vectors on the bottom side \( n_1 \) and left side \( n_3 \) of the grid-block, respectively. These average velocities and the pressure drop \( G = (1, 0)^T \) or \( (0, 1)^T \) are substituted into Eq. (A-2) to determine two elements of the effective permeability, \( k_{xx} \) and \( k_{yy} \). Calculations are repeated with \( G = (0, 1)^T \) to obtain \( k_{xy} \) and \( k_{yx} \).

The pressure equation subject to the PBC is solved semi-analytically by the CVBEM. The complex potential is expressed as \( (\Omega = \Phi + i\Psi) \) where \( \Phi = (k/\mu)p \) and \( \Psi \) are potential and stream functions, respectively. The Laplace equation \( \nabla^2 \Omega = 0 \) is solved under the PBC, where the outside boundary is divided into segments by placing nodes. The constraints of the PBC require the relationships in \( \Phi \)'s and \( \Psi \)'s, i.e. \( \Phi \)'s at the opposite nodes to be the same in one direction, and to differ by 1 in the other direction. \( \Psi \)'s are also assumed to have the same flow rate through the opposite boundary segments.

The internal boundary condition along the fracture is based on continuity of flow such that the flow rate at any point in the fracture is equal to the net inflow at the both sides. The complex potential at an arbitrary point in the grid-block is expressed as the summation of three components, i.e. non-singular, fracture, and fracture crossing complex potentials.

\[
\Omega = \Omega_m + \sum_{i=1}^{n_f} \Omega_{fi} + \sum_{i=1}^{n_c} \Omega_{ci} \quad \text{(A-5)}
\]

where \( n_f \) and \( n_c \) are numbers of fractures and crossings.
respectively. $\Omega_{\omega}$ is given by the Cauchy’s integral. $\Omega_{\xi}$ and $\Omega_{\rho}$ are given in Reference 5).

References