Encoding field theories into gravities

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We propose a method to define a $d+1$ dimensional geometry from a $d$ dimensional quantum field theory in the $1/N$ expansion.

We consider the generic large $N$ field $\varphi^{a,\alpha}(x)$ where $x$ is $d$ dimensional space-time coordinate, $a = 1, 2, \cdots$, is the large $N$ index, while $\alpha$ represents other indices such as spinor or vector indices, so that $h_{\alpha\beta}\varphi^{a,\alpha}(x)\varphi^{b,\beta}(x)$ can be made Lorentz invariant by a constant tensor $h_{\alpha\beta}$. We denote the action of this theory $S$. We first extend the $d$ dimensional field $\varphi^{a,\alpha}(x)$ to $\phi^{a,\alpha}(t,x)$ in $d+1$ dimensions, using the gradient flow equation as

$$\frac{d}{dt}\phi^{a,\alpha}(t,x) = -g^{ab}(\phi(t,x)) \frac{\delta S}{\delta \varphi^{b,\alpha}(x)}|_{\varphi \to \phi}, \quad (1)$$

with an initial condition that $\phi^{a,\alpha}(0,x) = \varphi^{a,\alpha}(x)$, where $g^{ab}$ is the metric of the space of the large $N$ index. We propose to define a $d+1$ dimensional metric as

$$\hat{g}_{\mu\nu}(z) := g_{ab}(\phi(z))h_{\alpha\beta}\partial_\mu\phi^{a,\alpha}(z)\partial_\nu\phi^{b,\beta}(z). \quad (2)$$

If we apply this method to $O(N)$ non-linear $\sigma$ model in two dimensions, we obtain

$$ds^2 = \frac{R_0^2}{\tau^2} [d\tau^2 + (d\vec{x})^2] \quad (3)$$

in the massless limit, which describes the Euclidean AdS space. Indeed, the Einstein tensor reads

$$G_{\mu\nu} = -\Lambda_0 g_{\mu\nu}, \quad \Lambda_0 = -\frac{1}{R_0^2}, \quad (4)$$

which give the negative cosmological constant $\Lambda_0$. It is interesting to see that the AdS geometry is realized for the conformal field theory defined in the massless limit, which corresponds to the UV fixed point of the theory.