Separability criteria for two-party systems in $d=2\times2$

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The Peres criterion of the non-negativity of partial transposed density matrix gives a necessary and sufficient separability criterion in $d = 2\times2$, for example. The reconstruction of the density matrix from the measured data is, however, a very involved procedure in general. We present a practically useful criterion of the separable state $\rho = \sum_k \omega_k \rho_k$ in $d = 2\times2$ without recourse to the state reconstruction, although our criterion provides only a necessary condition of separability in general unlike Peres criterion. The equality $G(a, b) = 4[\langle \psi | P(a) \otimes P(b) | \psi \rangle - \langle \psi | P(a) \otimes 1 | \psi \rangle \langle \psi | 1 \otimes P(b) | \psi \rangle] = 0$ for any two projection operators $P(a)$ and $P(b)$ provides a necessary and sufficient separability criterion in the case of a pure state $|\psi\rangle$. The separability criterion of mixed states is given by $\text{Tr}[\rho \{ a \cdot \sigma \otimes b \cdot \sigma \}] = c \cos \phi$ with $-1/3 \leq c \leq 1/3$ for two spin systems and $4\text{Tr}[\rho \{ P(a) \otimes P(b) \}] = 1 + c \cos 2\phi$ with $-1/2 \leq c \leq 1/2$ for two photon systems, respectively, after taking angular average of $a$ and $b$ with fixed $\cos \phi = a \cdot b$. The separability criteria are shown to be very efficient using the existing experimental data of Aspect et al. in 1981 and Sakai et al. in 2006. Our criterion can also judge the inseparability of the Werner state.