Composite operator and condensate in the $SU(N)$ Yang-Mills theory with $U(N-1)$ stability group


$^A$Dep. of Physics, Graduate School of Science, Chiba University
$^B$Dep. of Physics, Graduate School of Science and Engineering, Chiba University

Within a novel reformulation of $G = SU(N)$ Yang-Mills theory [Kondo et al. '05, '08] based on the Cho-Duan-Ge-Faddeev-Niemi-Shabanov decomposition, a mass term for the coset field $X_\mu \in \text{Lie}(G/H)$, $H = U(N-1)$, would no longer break gauge-invariance. If a corresponding mass dimension-two composite operator is introduced and a condensate were to exist, many interesting conclusions such as stabilization of the Savvidy-vacuum could be drawn. For the reformulation to work, two ingredients are necessary. The color-field $n(x)$, which defines the decomposition $A_\mu = X_\mu + V_\mu \in \text{Lie}(G/H) + \text{Lie}(H)$, and the reduction condition $D_\mu [V] X^\mu = 0$, which recovers the original number of degrees of freedom. To discuss the condensate, it is sufficient to fix the color-field, such that the reduction condition actually appears as a gauge fixing like term for the coset field $X_\mu$. The residual symmetry $H = SU(N-1) \times U(1)$ is fixed in the standard Lorenz-type way and we obtain the BRST-invariant Lagrangian

$$\mathcal{L} = \mathcal{L}_{YM} + i \delta_B \delta_B \text{Tr}_{G/H} \left( X_\mu X^\mu - i \xi \bar{C} \bar{C} \right) - 2i \delta_B \text{Tr}_{SU(N-1)} \left( \bar{C} \left( \partial_\mu V^\mu + \frac{\lambda}{2} N \right) \right) - 2i \delta_B \text{Tr}_{U(1)} \left( \bar{C} \left( \partial_\mu X^\mu + \frac{\alpha}{2} N \right) \right).$$ (1)

Due to fixing the color-field, a mass term is no longer gauge-invariant. However, at least a BRST-invariant dimension-two composite operator containing the coset condensate can still be constructed,

$$\mathcal{O} = \text{Tr}_{G/H} \left( X_\mu X^\mu - 2i \xi \bar{C} \bar{C} \right).$$ (2)

Within this set up, we performed a one-loop analysis of the above written theory, obtained all renormalization group functions and proved the one-loop multiplicative renormalizability of the composite operator. However, introducing a source term $J \mathcal{O}$ for this operator will lead to new divergences quadratic in the source. This can be cured within the so-called Local Composite Operator formalism [Knecht, Verschelde '95, '01] by introducing an additional term $(\kappa J^2 + \delta \kappa J^2)$, with $\delta \kappa$ being a pure counter term and $\kappa$ being an a priori arbitrary parameter. If the generating functional is required to satisfy the renormalization group equation, it is shown that $\kappa$ must satisfy

$$\left[ 2 \varepsilon + 2 \gamma \mathcal{O} - \beta_2 \frac{\partial}{\partial g^2} - \xi \gamma \frac{\partial}{\partial \xi} \right] (\kappa + \delta \kappa) = 0,$$ (3)

and can be expanded according to $\kappa = \kappa_0 / g^2 + h \kappa_1 + \mathcal{O}(g^2, h^2)$. We then performed a Hubbard-Stratonovich transformation and calculated the one-loop effective potential for the auxiliary field $\sigma$. A non-vanishing vacuum expectation value would consequently generate a tree level mass term for the composite operator and thus a coset gluon mass $m_X$, given by $m_X^2 = g(\sigma) / k_0$. Setting $\xi = 0$ in order to implement the reduction condition in a $\delta$-function like manner, we indeed found a non-trivial minimum, inducing an RG-invariant coset gluon mass proportional to $\Lambda_{QCD}$,

$$m_X^2 = \text{const.} \times \Lambda_{QCD}^2, \quad \Lambda_{QCD} = \bar{\mu} \text{ Exp} \left[ - \int \frac{dg}{\beta_g(g')} \right].$$ (4)