It has become a standard procedure in modern logic to reject the ancient square of opposition, according to which a statement of the A form both implies its corresponding I statement and is the formal contradictory of its O, and to substitute in its place the modern revised square, on which the A remains the formal contradictory of its O but does not imply its I. It therefore came as a surprise when some fifteen years ago an article by an important philosopher appeared in which the ancient square was defended against the objections which had led to its revision. And now it seems pertinent and should serve a useful purpose to try, after this lapse of time, to evaluate the criticisms and counter criticisms of the modern claims regarding the logical relations holding between statements of the A, E, I, and O forms. In a paper entitled "A Logician's Fairy Tale"* Professor H.L.A. Hart gives a unique and not easy to assess argument against what modern logicians have taken to be a consequence of the ancient square, an argument which is claimed to be in defense of "a feature of ordinary speech against a formal logician's prejudice."** It may be observed that up to now no evidence has emerged for thinking that formal logicians have been sufficiently swayed by the argument to give up or modify their account of the square. One

** Ibid, p. 204.
of the objects of this paper is to examine certain claims about the interpretation of A, E, I, and O formulas, both on the part of Hart and on the part of P. F. Strawson in his Introduction to Logical Theory, which goes against their current interpretation. The other object will be to consider whether one of the arguments given in our Fundamentals of Symbolic Logic rests "on a misunderstanding of important features of empirical discourse."* It is important to look into the linguistic charge, especially because of the increasing role the investigation of language has been playing in recent philosophy. The following compressed version of the argument was selected for criticism:

"(I) 'Given any set of four propositions answering to the forms, A, E, I, O, it is plain that at least one of them must be true' . . . and 'the underlying assumption is such that one of the two particulars I and O must be true'. . . .

(II) 'Both I and O (where the A, E, I, O propositions are taken to be "All ogres are wicked," etc.) imply that there are ogres'.

From which the authors draw their conclusions:

(III) 'Hence given that one of the four propositions about ogres must be true and that every one of them implies that there must be ogres, then there are no conceivable circumstances under which "There are ogres" will be false. This is to say that "There are ogres" is necessarily true . . .".*

Statement (I) is declared to be profoundly misleading. The reason, to quote fully, is the following:

"For the Square is simply concerned to exhibit the logical interrelations of these four forms of statement, i.e., to show how the truth value of statements of one form is determined by the truth value of the others,

and what its sides and diagonals represent are statements of entailment or statements of logical necessity or logical impossibility. . . . But it is quite clear that the only information which such statements of entailment or logical impossibility can yield as to the truth or falsity of values of any one of the A, E, I, O forms must be hypothetical: they tell us only if O is false then I must be true, if A is true then I must be true, etc. Of course we might say, e.g., of the contradictory forms A and O, E and I, ‘One of them must be true,’ but this would be a loose idiom for ‘If either is true then the other must be false, and if either if false the other must be true.’ Accordingly instead of (I) the authors are entitled only to assert:

(I-A) ‘Given any set of propositions of the forms A, E, I, O, it is plain that if one or more are false one of them at least must be true’ and ‘the underlying assumption is such that if one of two particulars I and O is false the other must be true.’

Now if this simple and vital correction of (I) to (IA) is made then the authors’ conclusion (III) does not follow and Aristotle is not guilty of having provided a proof of the ‘existence of not only ogres but something of every mentionable kind’ and the ‘theologians and metaphysicians’ are not guilty of having neglected this proof.”**

It is by no means easy to see what the vital correction comes to. Two statements are each other’s contradictories if and only if they can be neither jointly true nor jointly false, i.e., if they are so related that the truth or falsity of either one entails the falsity or truth of the other. Thus, if p and q are each other’s contradictories, p·q and ¬p·¬q are impossible conjunctions, and p entails ¬q and ¬p entails q. It follows directly that mutually contradictory statements must, by logical necessity, have opposite truth-values; one must be true and the other false. And they cannot have opposite truth-values without having truth-
values. The plain consequence is that a pair of statements cannot both be each other's contradictories and fail to have truth-values.

Now it has to be seen that it is one thing to say, (1) if $p$ and $q$ are mutually contradictory, then one, either $p$ or $q$, must be true and the other false, and that it is quite a different thing to say, (2) if $p$ and $q$ are mutually contradictory, then if either one is true the other must be false. Or as Strawson puts it for the special case where $p$ and $q$ are A and O: “. . . the rule that A is the contradictory of O states that, *if corresponding statements of the A and O forms both have truth-values, then they must have opposite truth-values.*”

(1) implies that $p$ and $q$ are not each other's contradictories if they are not so related to each other that of necessity one is true and the other false, whereas (2) does not imply this. Instead, (2) implies that $p$ and $q$ *could* logically be each other's contradictories without either one being either true or false, i.e., without either one having a truth-value. And it is quite clear that (1) is an *a priori* truth, a truth of logic recognized by ancient as well as modern logicians, and that (2) is an absurdity. For the notion of two statements being each other's contradictories not only entails but is strictly equivalent to the notion of statements being so related to each other that they must have truth-values which are opposite, i.e., one must be true and the other false. And it is to embrace an absurdity to think that a pair of statements could contradict each other while failing to have truth-values, which is to say that they could contradict each other while failing to contradict each other. It is self-contradictory to maintain, for example, both that the statement that all Martian metaphysicians are unconscious theologians is the contradictory of the statement that some

---

Martian metaphysicians are not unconscious theologians and also that they do not have opposite truth-values. They would not, of course, have opposite truth-values if they do not have truth-values.

If this is correct, it is hard to see in what way the recommended correction is a correction. Either (IA) is precisely to the same effect as (I); or if it is not, it differs from (I) in implying that statements can be contradictory to each other without having truth-values and thus without having opposite truth-values. If (IA) and (I) are to the same effect, (IA) is, of course, not a correction of (I). And if (IA) differs from (I) in the way pointed out, much less than being a vital correction, it is not a correction at all. It is a mistake. With regard to a suitable pair of instances of the A and O forms, to say that one must be true and the other false is not a “loose” way of saying if either is true the other must be false and if either is false the other must be true. They come to the same thing precisely. They would not come to the same thing, of course, if it were logically possible for statements to be the contradictories of each other without having, of necessity, opposite truth-values, and more generally, without having truth-values at all. But this is impossible. Hence on the ancient square, according to which an A statement not only implies its I but is the formal contradictory of its O, one of the other, A or O, must be true, and the other false.

From this is follows that it is incorrect to think it possible for statements which are genuine instances of these forms to be such as to have no truth-values at all while being statements to which the square of opposition applies, i.e., while being such that A is the contradictory of O, etc. Nevertheless, according to the view under discussion the square “depicts the logical relations between sentences which hold good independently of whether or not on any particular occasion the use
made of these forms is serious statement and so true or false, or fiction and so neither.”* More concretely, it is held that fairy tale statements, e.g., the statements that all ogres are wicked and that some ogres are not wicked, are neither true nor false, although the square of opposition does, somehow, apply to them in precisely the same way it applies to A, E, I, and O statements which do have truth-values.

It might reasonably be expected that criticism of modern logicians for failing to distinguish between the serious and fictional uses of sentences of the four forms would be supported by a specification of criteria for deciding betwixt such uses. The first readers of Moll Flanders, contrary to Defoe’s intent, believed it to be a report of fact. So fiction, one may say, does not carry its identification tag: a piece of fiction could be a report of fact, and a report of fact could be a piece of fiction. Some indication of what constitutes the serious use of language, at least in some areas, is given by the statement that “... the normal use of the A, E, I, O forms outside science and mathematics is in a context where (1) members of their subject class exist and (2) the speaker believes this.”**. And the following passage, which describes the use of language in fiction indicates that the difference lies in the absence of one or both of these features.

“To make serious factually true or false statements is no doubt the most important use of intelligible descriptive sentences, but it is not the only use. For we can and do often say when a sentence has been used in our presence that the question of its truth or falsity does not arise on this occasion... One such situation is the occasion when fairy tales are told us: ogres, we know, do not exist, but the storyteller is for our

* A Logician’s Fairy Tale, p. 209.
** Ibid, p. 207.
entertainment making the peculiar use of words which we can call speaking as if they do, but since they do not exist are neither. The context of this occasion, verbal, factual (no ogres), and histrionic, makes it clear that it would be absurd to press seriously the question: 'Is it true (not just part of the story) that all ogres are wicked?' Indeed so far is it from being the case that every statement we understand must be either true or false that if we did press this question seriously that would be a sign that we had not understood the speaker's use of the sentence."*

The differences pointed out are cited in order to justify the thesis that an indicative sentence may be used intelligibly without being used to state anything which has a truth-value. But one might assent to the differences cited without assenting to the thesis. Admittedly we should consider it very unperceptive of a hearer to be concerned about the truth-value of what a storyteller said, for example, if he challenged the storyteller's assertion of a proposition of the I or O forms with the accusation, "What you've told us is false." However, the oddity of such responses on the part of a hearer need not be explained as due to their irrelevance to a statement which is neither true nor false, but rather to the hearer's apparent misunderstanding of the storyteller's intent. Outside of fiction the intent of a speaker or writer is to have his audience believe that what he asserts is true, the focus of interest being on its truth-value. To assume that the intent is the same when a fictional sentence is uttered is merely to misconstrue the intent. And an accusation of lying or a charge that the narrator did not have the facts in his possession is irrelevant because the storyteller is not concerned about whether his narrative is true or false. This unconcern does not indicate indifference to or irresponsibility about fact. Fact just is not relevant

to his purpose, which is to beguile, rather than to inform or to palm off a falsehood. The modern logician would hold that the storyteller’s intent has no bearing on the truth-value of what is narrated, and *a fortiori* on whether it is true-or-false. He would also hold that failure to satisfy the other condition for normal use, namely, that members of the subject-class exist, could only bear on the truth, or the falsity, of a general proposition but not on whether it had any truth-value whatever.

The logician could argue for his position by pointing out the unacceptable consequences of holding that in fiction, when neither of the two conditions for the normal use of sentences of the four forms is satisfied, nothing either true or false is asserted. Consider the statement, “All witches derive their powers from the devil,” made by a person who believes witches do not exist and asserts it as part of a story. The statement is accordingly neither true nor false. Suppose it is heard by a child who “takes it seriously,” i.e., believes that witches exist, and furthermore, who repeats it to another listener. We assume that the child’s use of the sentence is serious even though witches do not exist. Are we to say that the statement made by the storyteller and repeated by the child both asserts something not having a truth-value and something having a truth-value? Or must we in such cases make a decision as to whether its possession of a truth-value is determined by the belief of the narrator or of the hearer? Or, finally, are we to say that a statement made by the storyteller is a different one than that made by someone else, according as it is believed or disbelieved that members of the subject-class exist? If we consider merely the child’s use of the sentence about witches, there is the further difficulty that the child’s statement is a serious factual one, which is to say it is either true or false, and at the same time the nonexistence of witches is sufficient to make it
neither true nor false.

Inasmuch as these difficulties may be due to a general statement's failure to meet the two conditions for possessing a truth-value, namely, that its subject-class has members and that the speaker believes this, it is well to drop consideration of the combination of these conditions and attend to the single condition set out by Strawson. He writes: "... in order for any statement of one of the four forms to have a truth-value, to be true or false, it is necessary that the subject-class should have members." That is, if an instance of \( \sim(x) \cdot fx \) is true, the related A, E, I, and O statements are neither true nor false. This position also has consequences which can hardly recommend it to a logician. For one thing, it follows that whether a statement of the form A v\( \sim \)A, or of the form E v\( \sim \)E, is a tautology, and whether a statement of the form A\( \sim \)A, or of the form E\( \sim \)E, is a self-contradiction, depends not only on its logical form, but also on matter of fact. In addition to having to be of the required logical form, in order to be a tautology or a self-contradiction a statement S requires the existence of a contingent fact of the form \( (\exists x) \cdot fx \). For any statement of the forms A, E, I, or O will have a truth-value only if a fact of this form obtains. From the logical fact that A and \( \sim \)A will only contradict each other if they each have truth-values, it follows that A v\( \sim \)A will be a tautology only if each disjunct has a truth-value. Hence the condition under which A has a truth-value will be identical with the condition under which A v\( \sim \)A is a tautology, and A\( \sim \)A a self-contradiction. In order, then, to know that S is a tautology, or is a self-contradiction, a person who entertains the statement must know that there is a particular matter of fact. But

---

*Introduction to Logical Theory*, p. 177. Hart notes that certain scientific laws present an exception to this. See p. 211.
whether a statement is a tautology, and true a priori, or a self-contradiction, and false a priori, is determined by nothing but its logical form; and in order to know that it is a tautology or a self-contradiction no knowledge of empirical fact is necessary or even relevant.

For another thing, the view has the consequence that at least some tautologies entail empirical statements: a fact to the effect that A has a truth-value entails that (Ax). fx, and since that fact that A v ~A is a tautology entails that A has a truth-value, that A v ~A is a tautology entails the same thing. That is, A v ~A is a tautology only if it is true that (Ax). fx. In the Tractatus Wittgenstein wrote: "A tautology has no truth-conditions, since it is unconditionally true; and a contradiction is true on no condition."* "Tautologies and contradictions are not pictures of reality. They do not represent any possible situations. For the former admit all possible situations, and the latter none."** Obviously a tautology would not allow every possible state of affairs if its being a tautology depended on the truth of an empirical proposition, i.e., on a proposition which requires for its truth a particular state of affairs.

There is reason to think that anyone who holds that (Ax). fx is a necessary condition for a general proposition's having a truth-value also holds, implicitly if not in so many words, that it is a sufficient condition. Strawson says*** "... the question whether sentences of the A, E, I, O forms are being used to make true or false assertions does not arise except when the existential condition is fulfilled for the subject term." It is plain that if he falsity of (Ax). fx determines, not that O, for

* 4.461.

** 4.462 (Translation by D. F. Pears and B. F. McGuiness).

example, is false but that it is neither-true-nor-false, then the only remaining means of knowing that it could be true, or that it could be false, that is, that it had a truth-value, would be by getting to know that \((\exists x).\ fx\) is true. That \((\exists x).\ fx\) is true would then be equivalent to "O is either true or false," i.e., to the tautology "O v ~O"—this, of course, on the assumption that \((\exists x).\ fx\) is a sufficient as well as a necessary condition for a related set of statements on the square to have truth-values. A tautology, then, is such that it could conceivably be false. Being a tautology, i.e., a statement the falsity of which is logically impossible, is in a great many cases equivalent to being a statement the falsity of which is possible.

It might be contended that "has a truth-value" or "is either true or false" is on a footing with "is false" and "is true." Some statements, i.e., empirical statements, are such that they possess their truth-values contingently, that is, they are such that though in fact true they could possibly be false, and though in fact false they could possibly be true. If we use the phrase "denotes a property" to cover such terms as "is true" and "is false," then we should say, in this way of speaking, that the actual truth-value of, say, "The sun is shining," is a contingent property of it. Now the Strawson-Hart view appears to imply that "has a truth-value" denotes a contingent property of any set of statements on the square of opposition, on a footing with "is true" and "is false." But a statement's being either-true-or-false is not a contingent fact about it, any more than "\((\exists x).\ fx\ v ~(\exists x).\ fx\)" is itself a contingent statement. "Has a truth-value," unlike "has the truth-value truth" or "has the truth-value falsehood," is an a priori, not a contingent fact: it is a contingent fact that a statement S on the square of opposition is true, or that it is false, but it is not a contingent fact that S is true or false.
A statement of the form $S \lor \sim S$ (where $S$ is not nonsense) is not contingently true, and the assertion that "$S \lor \sim S$" is a tautology is itself not a contingent statement; it is a priori. Thus the view according to which "$S \lor \sim S$ is a tautology" is equivalent to a contingent statement of the form $(\exists x). f x$, implies that an a priori truth is equivalent to an empirical one. It may be conjectured at this point that the term "has a truth-value" has been surreptitiously classified with phrases denoting contingent properties.

To limit the discussion again to the explicitly made claim that a necessary condition for a set of statements on the square having a truth-value is that their subject-term denotes a memberd class, or what is the same thing, that the associated existential statement is true. This claim implies that being either true or false is not a defining characteristic of statements: some possible values of the function "$x$ is a statement" are either true or false, while other possible values are neither. Now statements of the A, E, I, and O forms are not, of course, the only ones whose class terms may denote an empty class. Existential statements, such as "Ogres exist," can also contain such terms. One may take it that according to Hart, any sentences used in a fictional context have no truth-values. And there would be no reason for holding that sentences of the form $(\exists x). f x$ cannot be fictional—fairy tales frequently begin with the words, "Once upon a time there was a . . ." Strawson makes clear that, like Hart, he considers the use of language in storytelling to be very different from "those uses involved in the important business of stating, or trying to state, empirical facts."* But although he makes it clear that what prevents an A, E, I, or O statement from having a truth-value is a fact to the effect that nothing answers to its subject-

term, we are not told under what conditions a statement of the form \((\exists x). fx\) has none. Were the same condition which deprives the forms on the square of a truth-value to operate in this case likewise then no existential statement could ever possibly be false. It could be true, or neither true nor false, but never false*, which is absurd. And that this follows can easily be seen. If nothing answers to its class term, which is the grammatical subject-term of its related A, E, I, O statements, it is not false, but rather is neither true nor false. Quite plainly the only circumstance in which an existential statement could, possibly, be false is one in which nothing answers to its class term. And if this circumstance, instead of making it false, makes it neither true nor false, then it could, possibly, be true but could in no theoretical circumstances be false. It is hard to imagine that formal logicians would be receptive to a view which has this consequence.** The required remedy seems plain. This is not to impose the condition that if nothing answers to its class term it is neither true nor false, and allow that under this condition it is false.

The thesis that in certain circumstances, the telling of a fairy story being one, sentences are used without their expressing anything either true or false, is supported by a general account of the relation between propositions of the square and propositions of the form \((\exists x). fx\), which

---

* This has interesting connections with Professor Karl Popper's unfalsifiability thesis.

** Strawson is aware of difficulties presented by existential statements and attempts to deal with them (pp. 190–2). Roughly, he makes use of a conventional grammatical classification of A, E, I, and O as “subject-predicate” and distinguishes these from existentials. But formal logicians who have departed from this classification are not ignorant of conventional grammar.
has its own difficulties. If "Some ogres are wicked" and "Some ogres are not wicked" are to count as neither true nor false, then it must be denied that these entail that there are ogres. In general, a statement of the form \((\exists x). f x\) is denied to be a necessary condition of the truth of I and O, so that the nonexistence of f's will not determine statements of the I and O forms to the truth-value falsity even though they are seriously made. But this misdescribes English usage. For consider such a sentence as "Some departed spirits communicate with the living," asserted by someone who uses it with all the signs of "stating, or trying to state, an empirical fact." Would it not be said that this statement was false if such spirits do not exist? Or consider the assertion of "Some articles on sale below cost are bargains." The fact that one makes a statement by means of this sentence would normally indicate that one either believes or knows it to express something true. And if nothing of the sort exists one's belief is said to be false or one's claim to knowledge incorrect. But more important than this, the statement is said to be false. That this would be said implies that "There are articles on sale below cost" is in the logical sense a necessary condition of the truth of the I statement. That is, "Some articles on sale below cost are bargains" entails "There are articles on sales below cost." Given that there are no such articles, what is asserted is false. If this is correct, then the original critique of the ancient square of opposition holds, namely, that it permits proving the existence of anything whatever. If I and O both entail a proposition of the form \((\exists x). f x\) and if the truth of A implies the truth of I, and the truth of E implies the truth of O, then the truth of A, and of E, also have this existential consequence.

It should be remarked that philosophical logicians who deny that I and O are false when their subject-classes are empty are not without
an explanation of the fact that under this circumstance the serious use of sentences of these forms is mistaken or deceitful. I and O statements, and A and E as well, are said to "presuppose," though not to entail, a statement S' of the form (\(\exists x\)). fx. S' is a necessary condition, not of the truth of A, etc., but of the truth-or-falsity of A*. Beyond this, "presupposes" has no explication. Its only function, so it would seem, is to give a semblance of sense to the claim that A, E, I, and O "have existential import."**

The view that fictional statements are neither true nor false requires some comment since philosophers are in a stalemate over it. The disagreement is obviously not to be settled empirically, by observation of the world. Observation will disclose which propositions are true and which propositions are false; but it can yield no information to the effect that certain propositions are neither true nor false. The view, then, is not arrived at empirically, by checking propositions against states of affairs. It would thus seem to be a priori, and to make an entailment claim to the effect that being a piece of fiction entails having no truth-value. Or to put it somewhat differently, being fictional and either true or false is a contradiction. It is hard to think that if this is a contradiction some philosophers would see it and others persistently fail to see it. We get a better view of the nature of the philosophical disagreement if we consider the parallel verbal claim and counterclaim: (1) the phrase "has a truth-value" applies to nothing to which "is fictional" applies, and (2) the phrase "has a truth-value" does apply to what "fictional" applies to. If someone were to spin a tale out of his imagination, and it was discovered that the story corresponded in every particular to

* Introduction to Logical Theory, p. 175.

actual persons and occurrences, we should say that his tale was true despite the fact that it was an invention rather an intended report of fact. We should say his words did actually describe what happened, although their doing so was no part of the author's knowledge or intention. They were true by mere coincidence, but true nevertheless. Thus when a novelist protects himself against possible suit by declaring that any resemblance of his work of fiction to persons or events is "purely coincidental", he acknowledges that his story may describe fact without allowing that it ceases to be fiction. He tells us, in other words, that "has a truth-value," in point of usage, applies to what "fictional" applies to. And philosophers who disagree about this know the linguistic facts as well as he does.

The explanation of the philosophical disagreement that suggests itself is that it is not a disagreement about matter of actual usage but is instead a disagreement over the introduction of an academically re-edited use of "has a truth-value." A philosopher who holds that fictional statements are neither true nor false wishes to mark a difference, which he feels to be important, between fictional statements and statements made "seriously." What he is particularly concerned to highlight is the difference in intention between using language to create fiction and using language to report or misreport fact, the difference, roughly, in the one case being to beguile and in the other to inform or deceive. And the philosophical logician marks the difference with the help of an artificially contracted use of "has a truth-value." We may say that in his language game the term applies to nonfictional statements and is withheld from fictional statements. The philosopher who insists that fictional alike with nonfictional statements have truth-values merely rejects an artificial contraction of the term "has a truth-value."
Fiction and the Square of Opposition

著者紹介：
アリス・アンブローズ（Alice Ambrose）

1906年生まれ。ミシガン大学卒業後、ウィスコンシン大学とカンタベリー大学で哲学博士、ミシガン大学で法学博士の学位をうけ、さらにウィスコンシン大学、ニューハム・カレジで研究した後（ヴィトゲンシュタインに学ぶ）、ミシガン大学講師を経て、1937年以降スミス大学で教鞭をとる。現在スミス大学教授、モリス・ラゼロウィッツ夫人。

主要著書：
*Fundamentals of Symbolic Logic*, 1948（夫君と共著）
*Logic: The Theory of Formal Inference*, 1961（夫君と共著）
*Essays in Analysis*, 1966
その他論文多数。

モリス・ラゼロウィッツ（Morris Lazerowitz）

1907年ポーランド生まれ、ミシガン大学を卒業、哲学博士号を得たのち、母校およびハーヴァード大学で研究を続けて、1938年以降スミス大学で教鞭をとり、現在スミス大学教授。1951—52年ロンドンのペドフォード・カレジ客員講師。

主要著書：
*The Structure of Metaphysics*, 1955
*Studies in Metaphilosophy*, 1964
*Metaphysics: Reading and Reappraisals* (with William E. Kennick), 1966