Fuzzy Comprehensive Evaluation of Water Quality at Monitoring Points

– Application to Lake Biwa –

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Abstract: A method for comprehensively evaluating water quality at various points in a water body has been developed based on the fuzzy reasoning. Vagueness in borders of conventional water quality classes formed for each water quality index is quantified by introducing fuzzy sets associated with the classes and nonlinear membership functions. Mamdani method (or min-max-center of gravity method) is employed in the fuzzy reasoning in order to synthesize concentration data on multiple water quality indices. The evaluation method developed in this study can represent the state of water quality at each points with a real number ranging from 0 to 100. The applicability of the method is demonstrated with the series of monthly observed data from April 1980 to March 2010 at 49 monitoring points in Lake Biwa, Japan. Temporal and spatial variations of the synthesized water quality scores at the monitoring points are presented. The application of the fuzzy comprehensive evaluation method would lead to more rational comparison of synthesized water quality at MPs in the lake and to producing useful information in decision-making on water quality management in the water body.

Keywords: Water quality; Monitoring; Comprehensive evaluation; Fuzzy reasoning; Decision-making

1 Introduction

Water quality level in a water body is generally evaluated by comparing observed value for a water quality index with its imposed standard value. The standard value is associated with one of the water quality classes assigned to the water body. Water quality classes are prepared to describe current water quality state and to use one of them as a target level of water quality management for the water body. For example, when water quality class II is assigned to a lake, the water quality standard for TN (Total Nitrogen) and TP (Total Phosphorus) concentrations are set as they should be less than 0.2mg/L and 0.01mg/L, respectively. However, the upper and/or lower limits of the class are too 'rigid', which leads to unreasonable comprehensive evaluation of observed water quality. Icaga (2007) has indicated two drawbacks in the conventional classification, which are (i) an index (e.g., TN concentration) being close or far from the limit of the class has equal importance for evaluation of observed concentration; (ii) all of the indices observed at a monitoring point may not belong to a single class. These drawbacks result in rough comprehensive evaluation of water quality. In order to overcome such drawbacks, some methods have recently been presented. Icaga (2007) has developed an index model for quality evaluation of surface water quality classification using fuzzy logic, and applied the model to the Eber Lake, Turkey. Zou *et al.* (2006) and Liu *et al.* (2010) have combined the information theory with the fuzzy mathematics to establish a fuzzy comprehensive evaluation method for water

quality assessment in Three Gorge region, China.

Controlling concentration of a specific water quality index (e.g., TN, TP, etc.) in a water body is generally impossible. A pollution index which has a specific value is usually meaningful only in the context of knowledge of natural background levels and regulations (Icaga, 2007). Instead, reducing wastewater discharge from pollutant sources and improving treatment level of wastewater in wastewater treatment plants are realistic and effective methods for water quality conservation. Therefore, comprehensive evaluation of water quality at monitoring points (MPs) in a water body aids in comparing the importance of water pollution control at the MPs with each other and in making decisions on controlling wasteloads transported to the water body.

In this study, fuzzy logic is introduced to the synthetic evaluation process as an improvement over Icaga (2007) to overcome the drawbacks of conventional water quality classification. The presented method is applied to the comprehensive evaluation of water quality at 49 monitoring points in Lake Biwa using monthly observed data of concentration of COD, DO, TN and TP.

2 Method of fuzzy comprehensive evaluation

2.1 Fuzzy reasoning

Generally fuzzy reasoning can be described mathematically as a process where a conclusion is derived from fuzzy rule(s) and a fact by inference. The fuzzy rule consists of the premise part and the consequence part. Typically it can be expressed as

Rule 1: If A_1 and B_1 then C_1 .

Rule 2: If A_2 and B_2 then C_2 .

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Rule K : If A_K and B_K then C_K .

Facts: x_0 and y_0

Conclusion: C'

where A_i , B_i and C_i ($i = 1, \dots, K$) are the fuzzy concepts that are represented with fuzzy sets defined in universal sets X , Y and Z , respectively; x_0 and y_0 are the elements of X and Y ; C' is the fuzzy concept represented by the fuzzy set in Z . Since A_i , B_i and Z are fuzzy sets, the memberships of their element vary between 0 and 1. This indicates that the borders of the sets are ambiguous and thus the reasoning is called fuzzy reasoning. Flexible and more realistic inferences, which are consistent with daily human reasoning, can be performed by fuzzy reasoning (Mizumoto, 1992).

2.2 Fuzzy reasoning for water quality assessment

Conventionally, values of water quality indices are classified to some categories, e.g., four classes. Fuzzy reasoning is applied to the classification problem of water quality data in this study. Mamdani method (min-max-center of gravity method) (Mizumoto, 1992) is employed as a methodology of fuzzy reasoning in water quality assessment.

Consider water sampled at a monitoring point in a water body of interest. Water quality indices for the sample, QP_j ($j=1, \dots, N$), are measured, where N = number of water quality indices. Based on the data, comprehensive water quality at the monitoring point is evaluated using fuzzy reasoning. Water quality classes I, II, $\dots$, K are defined. The categorization on water quality is applied to each water quality index.

Icaga (2007) has employed a triangular or a trapezoidal membership functions to characterize the fuzzy sets. However, since those functions are expressed as segmented linear ones, the application of Mamdani method fails when each of observed values of water quality indices falls in a different quality class. Therefore, in this study, the following non-linear membership functions (see Figure 1 also) replace the ones used in Icaga (2007) to represent the fuzzy sets corresponding to the quality classes, $A_{1j}, A_{2j}, \dots, A_{Kj}$ ($j = 1, 2, \dots, N$).

$$\mu_{A_{1j}}(x_j) = \begin{cases} 1 & (x_j \leq \theta_{1j}) \\ \exp \left[- \left(\frac{x_j - \theta_{1j}}{\beta_j} \right)^{p_j} \right] & (\theta_{1j} < x_j) \end{cases} \quad (1)$$

$$\mu_{A_{kj}}(x_j) = \begin{cases} \exp \left[- \left(\frac{\theta_{kj} - x_j}{\alpha_j} \right)^{p_j} \right] & (x_j \leq \theta_{kj}) \\ \exp \left[- \left(\frac{x_j - \theta_{kj}}{\beta_j} \right)^{p_j} \right] & (\theta_{kj} < x_j) \end{cases} \quad (2)$$

$$\mu_{A_{Kj}}(x_j) = \begin{cases} \exp \left[- \left(\frac{\theta_{Kj} - x_j}{\alpha_j} \right)^{p_j} \right] & (x_j \leq \theta_{Kj}) \\ 1 & (\theta_{Kj} < x_j) \end{cases} \quad (3)$$

where $k = 2, 3, \dots, K-1$, corresponding to the classes II, III, $\dots$, $K-1$, respectively; x_j = value of j th water quality index; θ_{lj} = limit of membership function ($l = 1, 2, \dots, K$); α_j and β_j = shape parameters ($\alpha_j, \beta_j > 0$); and p_j = attenuation parameter ($p_j \geq 1$). Since the membership functions are exponential ones, an observed water quality can be specified as a member of all the water quality classes.

Figure 1: Nonlinear membership functions corresponding to the first, k th and K th classes of the j th water quality index

Figure 2: Linear membership functions of the fuzzy set C_k (in case of $K = 4$)

Fuzzy rules are defined such as:

Rule 1: If $QP_1=I, QP_2=I, \dots, QP_N=I$ then $QP=I$.

Rule 2: If $QP_1=II, QP_2=II, \dots, QP_N=II$ then $QP=II$.

$\vdots$

Rule K : If $QP_1=K, QP_2=K, \dots, QP_N=K$ then $QP=K$.

where QP in the consequence part is the synthesized water quality. It is noted that the number of rule, K , is the same as that of water quality class. For example, in the Rule 2, “ $QP_1=II$ ” means “the value (e.g., concentration) of the first water quality index is a member of Class II with a membership value of $\mu_{A_{21}}$ ”. In the rule, the “and” operator is used to obtain the minimum value.

Suppose that values of indices are found $x_1, x_2, \dots, x_N$ for a water sample at a monitoring point. From this fact, fuzzy reasoning can be conducted based on Rules 1 through K . The membership function of the fuzzy set representing the reasoning result from Rule k can be written as

$$\mu_{C'_k}(z) = \mu_{A_{k1}}(x_1) \wedge \mu_{A_{k2}}(x_2) \wedge \dots \wedge \mu_{A_{kN}}(x_N) \wedge \mu_{C_k}(z) \quad (4)$$

where $\wedge$ indicates “min”; k = rule (= class) number; $\mu_{A_{k1}}, \mu_{A_{k2}}, \dots, \mu_{A_{kN}}$ = membership functions of the fuzzy sets ($A_{k1}, A_{k2}, \dots, A_{kN}$) in the premise part under Rule k ; μ_{C_k} = membership function of the fuzzy set (C_k) in the consequence part under Rule k ; and z = component of C_k and C'_k . The shapes of triangular membership functions of the fuzzy sets C_k ($k = 1, 2, \dots, K$) are shown in Figure 2. The final result of the reasoning can be expressed as a union of the fuzzy sets, i.e.,

$$C' = C'_1 \cup C'_2 \cup \dots \cup C'_K \quad (5)$$

with its membership function

$$\mu_{C'}(z) = \mu_{C'_1}(z) \vee \mu_{C'_2}(z) \vee \cdots \vee \mu_{C'_K}(z) \quad (6)$$

where $\vee$ indicates “max”. This result is defuzzificated to obtain the value ranging from 0 to 100 using the Centroid method:

$$z_0 = \frac{\sum z_i \cdot \mu_{C'}(z_i)}{\sum \mu_{C'}(z_i)} \quad (7)$$

z_0 represents the score of comprehensive water quality at the MP. The closer to 100 the z_0 value is, the worse water quality at the point is judged.

3 Application

3.1 Lake Biwa

Lake Biwa is the largest freshwater lake in Japan. The volume and the maximum depth are 27.5 billion m³ and 103.58m, respectively. Lake Biwa is separated with North and South Basins, whose water areas are 617.75 and 52.50km², respectively. The water depth of North Basin is deep (43m in average), whereas, that of South Basin is shallow (4m in average) (Soumiya, 2000).

The watershed area of Lake Biwa is 3,174km² (Figure 3). In the northern and western areas of the watershed, which are sparsely populated, mountains extend to the shore of the lake and there are few plains except around the Ado River. In the eastern area, many plains exist and agriculture (mostly rice) is a predominant land use. The southern area around the South Basin is urbanized and densely populated (Nakano *et al.*, 2008).

3.2 Determination of parameter values

The method of fuzzy comprehensive evaluation described above is applied to assess water quality at 49 MPs (Figure 4) in Lake Biwa. Water quality data, concentration of COD (Chemical Oxygen Demand), DO (Dissolved Oxygen), TN and TP at the MPs monthly observed from April 1980 to March 2010, are obtained from the Web site of Lake Biwa Environmental Research Institute. Four water quality classes are introduced to each index considering the current real categorization for lakes in Japan. The corresponding membership functions in Eqs.(1)-(3) are formulated with limit parameters shown in Table 1. Other parameters in Eqs.(1)-(3) are: $p_j = 1$; $\alpha_j = \beta_j = |\theta_{4j} - \theta_{1j}|/10$. As for the membership functions of C_k in Eq.(4), those shown in Figure 2 are used.

Table 1: Limits of membership functions for concentration of COD, DO, TN and TP (unit: mg/L)

	j	θ_{1j}	θ_{2j}	θ_{3j}	θ_{4j}
COD	1	0	2	4	6
DO	2	11.25	8.75	6.25	3.75
TN	3	0.05	0.15	0.3	0.5
TP	4	0.0025	0.0075	0.02	0.04

Figure 3: Lake Biwa and its watershed

Figure 4: Locations of monitoring points at Lake Biwa

3.3 Results and discussion

Monthly concentration of COD, DO, TN and TP observed from April 1980 to March 2010 and computed z_0 values at MP10B (see Figure 4) are shown in Figure 5. The relationship between observed water quality indices and synthesized water quality score can be seen in this figure. From Figure 5, it can be observed that relatively large range of the realized TN concentration compared with the widths of the classes for the concentration significantly affects the computed time series of z_0 . This point is clear if the graphs of 12-month

Figure 5: Time series of observed concentration ((a) COD and DO; (b) TN and TP) and computed (c) z_0 at MP 10B from April 1980 to March 2010

Table 2: Observed values of water quality indices (unit: mg/L), water quality classes (II - IV) and computed score of synthesized water quality z_0

	Data 1	Data 2	Data 3	Data 4
COD	3.6 (III)	4.7 (III)	3.1 (III)	5.8 (IV)
DO	9.2 (II)	7.4 (II)	8.8 (II)	6.9 (III)
TN	0.27 (III)	0.28 (III)	0.26 (III)	0.22 (II)
TP	0.024 (III)	0.021 (III)	0.02 (III)	0.022 (III)
z_0	52.7	61.8	48.6	62.1

moving average for TN concentration and z_0 is compared. The fuzzy comprehensive evaluation method presented here can successfully evaluate the conflicting levels of water quality indices.

The advantage of applying the developed fuzzy comprehensive evaluation method to the observed water quality data in Lake Biwa is demonstrated here. Four sets of observed data, Data 1 through 4, are summarized in Table 2. Data 1 and 2 are for April and October, 2009 at MP 6C, respectively, and Data 3 and 4 are for April and October, 2009 at MP 5C, respectively. Observed concentration, corresponding water quality classes for COD, DO, TN, TP and computed scores z_0 in Eq.(7) are shown in the table. Since the assignment of conventional water quality classes for Data 1, 2 and 3 are similar, comparing the water quality state of them is difficult. The problem can however be overcome by using z_0 values computed by the fuzzy comprehensive method presented here. The MP related to Data 3 is judged as the best of the three due to the minimum value of z_0 .

Data 4 is an example that the observed data of concentration fall in three different water quality classes, II, III and IV. It is generally difficult to properly assess the water quality at the MP associated with Data 4. Application of the fuzzy method developed by Icağa (2007), where the triangular membership functions are used, results in unreliable computed synthesized score of water quality, because for Data 4 the difference in the water quality levels of COD (Class IV) and TN (Class II) is too large. In contrast, the developed comprehensive evaluation method in this study successfully computes the synthesized water quality score z_0 due to the long tails of the exponential membership functions. The z_0 value of 62.1 for Data 4 indicates the water quality at the corresponding MP is the worst of the 4 data sets in Table 2.

12-month moving average of computed z_0 values from April 1980 to March 2010 at MPs (6B, 6C), (10B, 10C) and (18B, 18C) are shown in Figures 6, 7 and 8, respectively. These three sets of points are located at the southern, middle and northern parts of Lake Biwa (Figure 4). The z_0 values themselves range from around 40 to 60 and around 40 to 70 at MPs 6B and 6C, respectively. The moving averages of z_0 at MPs 6B and 6C shown in Figure 6 are higher than those at MPs 18B and 18C shown in Figure 8. In addition, variance of z_0 at MPs 6B and 6C is also larger than that at MPs 18B and 18C. The reasons of these are considered that the water depth at MPs 6B and 6C are much smaller and influence of inflow of wastewaters from the surrounding cities (see Figures 3 and 4) is significant at the points. Temporal variations of z_0 val-

ues at MPs 10B and 10C are moderate, which is mainly due to moderate water depth at the points. Since MPs 6C, 10C and 18C are located comparatively close to the shore of the lake, whose water depth is shallow, and affected by inflowing wastewaters more significantly than respective MPs 6B, 10B and 18B, the variance of z_0 in a year is larger.

Figure 6: 12-month moving average of z_0 at MPs 6B and 6C from April 1980 to March 2010

Figure 7: 12-month moving average of z_0 at MPs 10B and 10C from April 1980 to March 2010

Figure 8: 12-month moving average of z_0 at MPs 18B and 18C from April 1980 to March 2010

4 Conclusions

A method is presented to comprehensively evaluate the state of water quality at particular points in a lake using observed concentration data on multiple water quality indices. The method is developed based on Mamdani method in the fuzzy reasoning. The drawbacks of conventional crisp categorization in water quality assessment can be solved mathematically by the fuzzy reasoning. Furthermore the non-linear membership functions newly introduced to Mamdani method in this study has successfully improved the applicability of the method developed by Icaga (2007). The presented method is applied to the comprehensive evaluation of water quality at 49 monitoring points in Lake Biwa. The comprehensively evaluated water quality scores, z_0 , are computed with monthly observed concentration of COD, DO, TN and TP at the MPs. The obtained time series of z_0 quantify the spatial heterogeneity of water quality state in the lake, appearing in the long-term trends and magnitude of variation in z_0 .

The comprehensive evaluation of water quality at MPs conducted by the method developed here finds the comparatively important MPs where water quality should be controlled. Therefore, the results could be used for more efficient activities for water quality control, including management of pollutant sources related to the specified MPs, at the watershed level.

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