Robust Control of Reservoir Release Discharge Using Precipitation Data

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Abstract: A linear control system is developed and is applied to automatic control of reservoir release discharge using precipitation data. A nominal generator, which is an estimated runoff model from precipitation to inflow into the reservoir, is assumed to be obtained. The control system has three controllers to cancel out external disturbances, to handle wind-up phenomena in the release discharge, and to stabilize the entire system under observation errors. Demonstrative calculations are executed using flood data observed at an existing dam. The results show that three controllers mend the variation in the reservoir storage complementing each other.

1 Introduction

Automatic regulation of release discharge from a reservoir is a key subject to achieving its flexible and efficient operations. In order to keep the storage at a targeted value, the release discharge should be determined according to a well established rule. The linear control theory has been applied to a water storage model which describes reservoirs (Nakanishi *et al.*, 1997) as well as open channel networks (Unami and Kawachi, 1997).

Here, a novel control system is developed for a reservoir where precipitation data is available in terms of real time operation. The control system essentially differs from the earlier works in the following two respects. In the first sense, the control errors are taken into account to prevent wind-up phenomena, or awkward expansion of a control variable. And in the second sense, the external disturbances are related to observable references by a generator, which is also represented by a linear dynamical model and may include modeling errors. In this context, three controllers are designed; one is a feed-forward controller to cancel out the external disturbances assuming that there is no modeling error in the generator, one is a controller for anti wind-up, and the other one is a feed-back controller to stabilize the control system under observation errors and to bound variations in the storage volume of the reservoir. Thus, the system is an extension of the two-degree-of freedom system which consists of a feed-back controller and a feed-back controller (Horowitz, 1963). The robust control theory, which rigorously deals with the modeling errors as well as

the external disturbances, is applied to designing the feed-back controller. The controllers designed are tested in a reservoir using two sets of data in floods.

2 Water storage model

Throughout the mathematical description of the control system, the same notation is used for representing a variable both in the time and the frequency domains. The frequency is denoted by s .

A reservoir is modeled as the linear system given by the state equation

$$\frac{dx}{dt} = u + q \quad (1)$$

with the observation equation

$$y = x \quad (2)$$

where t = time, x = state variable which represents the storage volume deviation, u = control variable which represents the release discharge deviation, q = external disturbance, and y = observation output.

The external disturbance q , which is the external inflow discharges into the reservoir, is assumed to be generated by a generator G from an observable reference r . When the reservoir is fed by the runoff resulting from the precipitation as illustrated in Figure 1, the generator G and the reference r are a runoff model and precipitation data, respectively.

3 Errors in the model

The modeling error in the generator G and the control error are taken into account in the controller design.

The generator G , whose definite expression is hardly obtained, is perturbed from a nominal generator G_0 , which is written as

$$G_0 = \frac{k}{s + a} \quad (3)$$

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where k = load coefficient, and a = decay coefficient. In the true generator G , k and a are assumed to deviate as $k + \Delta^k$ and $a + \Delta^a$ with deviation parameters Δ^k and Δ^a whose maximum absolute values are prescribed. Then, the generator G as the transfer function from r to q is expressed as the block diagram shown in Figure 2.

The control error is mostly due to the saturation of control variables. If a gate is fully open, the discharge cannot be controlled because of free overflow. Thus, the control variable u is assumed to be multiplicatively perturbed from a nominal control variable u_1 as

where Δ^u = deviation parameter for u .

The Doyle's notation (Doyle *et al.*, 1989) is used for the system matrix. For example, $\left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array} \right]$ denotes the transfer matrix $G_{21}(sI - G_{11})^{-1}G_{12} + G_{22}$ for matrices G_{ij} ($i, j = 1, 2$) of compatible dimensions and the unit matrix I .

Let the reference r , the actual control variable u , and the observation output y be known. In the control system whose block diagram is shown in Figure 3, controllers K_G , K_C , and K_R are designed to determine the nominal control variable u_1 which stabilizes the entire system.

Figure 3: Block diagram of control system

The controller K_G is chosen as

$$K_G = -G_0 \stackrel{s}{=} \left[\begin{array}{c|c} -a & 1 \\ \hline -k & 0 \end{array} \right] \quad (5)$$

to cancel out the external disturbance if there is no error in the whole system.

Extracting the control error characteristics as an H_∞ standard problem as shown in Figure 4, the controller K_C is determined to make the H_∞ norm of the closed-loop transfer function matrix, which is equal to $\text{diag} \Delta^u$, less than unity. The class of such K_C is characterized by

$$K_C = \frac{N(s)}{\Delta^u} \quad (6)$$

where $N(s)$ = any BH_∞ transfer function matrix.

Figure 4: H_∞ standard plant for K_C

4.3 Controller K_R

The algebraic design method is employed to obtain the controller K_R . An H_∞ standard problem is formulated as expressed in Figure 5 to bound the H_∞ norm of the closed-loop transfer function from an

input disturbance vector $\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ to an error signal

vector $\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$ taking the reference r as the input

disturbance w_1 and using w^y , constant weights to scale the observation output y as the error signal z_3 . A state space representation of the generalized plant in the H_∞ standard problem is obtained as

$$\begin{bmatrix} \frac{\Delta^a}{s+a} & \frac{-\Delta^a}{s+a} & 0 & 0 \\ \frac{\Delta^k}{s+k} & \frac{-\Delta^k}{s+k} & 0 & 0 \\ 0 & \frac{-w^y \Delta^k}{s(s+a)} & \frac{w^y}{s} & \frac{-w^y}{s} \\ 0 & \frac{-\Delta^k}{s(s+a)} & \frac{1}{s} & \frac{1}{s} \end{bmatrix} = \begin{bmatrix} A_0 & B_1 & B_2 \\ C_1 & [0] & [0] \\ C_2 & [0] & [0] \end{bmatrix} \quad (7)$$

$$\text{where } A_0 = \begin{bmatrix} -a & 0 & 0 & 0 \\ 0 & -a & 0 & 0 \\ 0 & 0 & 0 & k \\ 0 & 0 & 0 & -a \end{bmatrix} = \text{state matrix,}$$

$$B_1 = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \text{disturbance distribution}$$

$$\text{matrix, } B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \text{control distribution matrix,}$$

$$C_1 = \begin{bmatrix} \Delta^a & 0 & 0 & 0 \\ 0 & \Delta^k & 0 & 0 \\ 0 & 0 & w^y & 0 \end{bmatrix} = \text{error output matrix,}$$

$$\text{and } C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} = \text{observation output matrix.}$$

Since all the components are zero matrix in the 2-2 block of the system matrix in Eqn. 7, the class of all stabilizing controllers cannot be identified as in Glover and Doyle (1988). However, Sampei *et*

Figure 5: H_∞ standard plant for K_R

al. (1990) deduce an existence theorem and take an algebraic approach similar to Zhou and Khargonekar (1988) as follows.

A strictly proper controller stabilizes the system and makes the H_∞ norm of the closed-loop transfer function less than unity if and only if the following conditions are satisfied.

1. There exists F such that there exists a positive definite solution X to the algebraic Riccati inequality

$$XA_X + A_X^T X + C_1^T C_1 + XB_1 B_1^T X < 0 \quad (8)$$

$$\text{where } A_X = A_0 + B_2 F.$$

2. There exists L such that there exists a positive definite solution Y to the algebraic Riccati inequality

$$YA_Y^T + A_Y Y + B_1 B_1^T + Y C_1^T C_1 Y < 0 \quad (9)$$

$$\text{where } A_Y = A_0 + L C_2.$$

3. The maximum singular value of (XY) is less than unity.

Moreover, one of such strictly proper controller is given by

$$K_R = \begin{bmatrix} \hat{A} & -ZY^{-1}L \\ F & 0 \end{bmatrix} \quad (10)$$

where $Z = (Y_\infty^{-1} - X_\infty)^{-1}$, and

$$\hat{A} = A_X + B_1 B_1^T X - Z(E + F^T B_2^T X - Y^{-1} L C_2) \quad (11)$$

with $E = -(X A_X + A_X^T X + C_1^T C_1 + X B_1 B_1^T X)$. Numerical solutions of the Riccati equations

$$X A_X + A_X^T X + C_1^T C_1 + X B_1 B_1^T X + \varepsilon I = 0 \quad (12)$$

and

$$Y A_Y + A_Y^T Y + B_1 B_1^T + Y C_1^T C_1 Y + \varepsilon I = 0 \quad (13)$$

determine X and Y , respectively, in practical design.

5 Application to reservoir operation in flood

The controllers are applied to determining the release discharge of a reservoir when a flood wave enters into it. Two sets of data D-1 and D-2 observed at an existing dam whose catchment area is of 32.45 km² are used for identifying the runoff model. Identification procedures using the least square method yield the runoff models $\frac{0.2012}{s + 6.899}$ and $\frac{0.0885}{s + 3.506}$ for D-1 and D-2, respectively. Then, the nominal generator G_0 is taken as $\frac{0.15}{s + 5}$ with the deviation parameters $\Delta^k = 0.10$ and $\Delta^a = 3.4$. The Bode plots of the runoff models and the nominal generator G_0 are depicted in Figure 6. Setting $\varepsilon = 10^{-2}$,

$$F = \begin{bmatrix} 0 & 0 & -5 & 0 \end{bmatrix}, \text{ and } L = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \end{bmatrix}, \text{ the con-}$$

troller K_R is obtained from Eqn.(10).

Simulation runs are executed for 5 cases which include different combination of controllers as shown in Table . The maximum release discharge is 1×10^7 m³/day and the control variable saturates when it becomes so large. Results for flood data D-1 and D-2 are depicted in Figures 7 and 8, respectively, and show that the controller K_C is necessary for settling the storage volume after the saturation of the control variable occurs. The role of the controller K_R is to stabilize the entire system.

6 Conclusions

The robust control system for operation of reservoir release discharge is designed. The system considers a generator, which is the runoff model from precipitation to inflow into the reservoir. The three controllers guarantee robustness when the generator includes the modeling errors, robustness under the control errors, and bounded variations in storage volumes in the sense of the H_∞ control problem. The specifications are verified in the simulations, which demonstrate excellent regulation performance.

The authors thank the Yasugawa land improvement association for providing the data.

Case 1	No controller
Case 2	K_G
Case 3	K_G and K_C
Case 4	K_G and K_R
Case 5	K_G , K_C , and K_R

Table 1: Combination of controllers for reservoir operation

Figure 6: Bode plots

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Figure 7: Simulation results for flood data D-1

Figure 8: Simulation results for flood data D-2