Active Sampled-data Controlled Suspension in Automobile with Vibration Manipulation Functions

- Intermittent Desired Elongation Control of Actuator -

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ABSTRACT: The intermittent feedforward function, termed vibration manipulation function (VMF), was proposed for sampled-data control of an actuator to achieve active suspension of an automobile with road profile undulation. The function can manipulate the position and velocity of a sprung mass of a 1DOF quarter car model in every arbitral operational period. Elongation or force of the actuator can be determined from VMF, which works to suppress vibration of the sprung mass without knowing road profile. Since required power of the actuator can be made smaller and intermittently-feedbacked feedforward control is much easier to be realized, proposed method could provide an alternative guiding principle for active suspension of automobile.

KEY WORDS: vehicle dynamics, electronically controlled suspension, vibration manipulation function [B1]

1. Introduction

Riding comfort is the main key technology for product differentiation strategy in deluxe cars. Although most streets have been paved in modern cities, road undulation from geographical features still causes oscillation of a sprung mass in an automobile at the natural frequency of 1~1.5Hz. Hence the most effective oscillation will be excited under the undulation of road profile in 10~15m wave-length.

To get better riding quality, technologies for active suspension or semi-active suspension have been developed by many automobile companies in several decades. Some technologies change spring constants of suspension springs, one others change dumping constants of shock absorbers, and the others adjust vehicle heights by actuators.

 Electronically controlled active suspension system has a potential to overcome limitations of passive systems. It reduces the necessity to compromise among a variety of operating conditions and among the conflicting demands to design the suspension. Such demands include providing good vibration isolation for riding comfort and keeping uninterrupted contact between tires and road to increase road-handing ability.

 Besides lowering cost and energy consumption of these mechanisms, seeking better controlling algorithms becomes important to improve the riding comfort of the car. The most well-known a guiding principle for the modern active or semi-active suspension is called Sky-Hook control theory (1). In this control algorithm an imaginary damper hanged from an imaginary roof in the sky is required to hook a sprung mass of an automobile, and actual springs, dampers and actuators will be controlled to simulate this imaginary dynamics. It has been known that good damping performance can be achieved under full bandwidth frequency under this principle. However, since there are no damper and roof in the sky and the car is placed on the ground, this algorithm is rather unreasonable. Considering merely the actual work of actuators under Sky-Hook control theory, to keep certain height from the roof in the sky, it requires the same amount of vertical displacement as the height of road undulation or disturbance. In case of steep bumpy road the displacement becomes in the scale of 10~15cm, whereas the maximum length and power of the actuator are practically limited.

 Moreover, in former studies most researchers have evaluated the performance of active suspensions from the spectrum of the sprung mass displacement. However, individual bumps are common inputs from the road profile, hence the external excitation changes intermittently over time. Therefore, suppression of residual vibration in the sprung mass from the each intermittent excitation should be the main target of active suspensions to avoid poor road-handling and bad riding comfort.

 Likewise HDD memory device (2) and cams (3), to suppress residual vibration of the sprung mass, desired trajectories of the base of the suspension need to be determined by concerning dynamical states of the vehicle. However, few analytic feedforward functions of the trajectories have been proposed to control the vibration of the sprung mass. For example the least square of jerks, called SMART control (2), has been utilized for seek scanning of a magnetic head. And profile of a polydyme cam (3) has also been proposed to suppress the vibration of a follower. Yet they merely work in limited conditions and cannot remove whole energy from the oscillator.

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Recently, we have deduced analytical intermittent function of the base trajectory, called vibration manipulation function (VMF), from the system of three vibro-impact oscillators which is designed under Grover algorithm\(^{(4)}\). The trajectories of the oscillators are determined from free motions of the system under Hamilton’s principle of least action during operational period between neighboring instants of impact. In this algorithm, arbitral energy transfer between one small oscillator and the other one in the system can be achieved from its properties \(^{(5,6)}\) during each operation. Hence VMF can be designed to remove vibration energy of the small oscillator from arbitral activating conditions.

In case the suspension of an automobile can be modeled as one-degree-of freedom (1DOF) oscillator, by enforcing displacement the position of its base along VMF, vibration of the sprung mass can be suppressed. If we could preview incoming road profile and we can well suppress the oscillation of the sprung mass with an actuator by using VMF\(^{(7)}\). However, since preview of road profile is not easy and has not been accomplished in the current technology, we should pursue control of the suspension without these information. Although unexpected road profile change can be a noise to the suspension system, we can reduce vibration in sprung mass constantly by using active suspension.

In this study, we will show the effectivity of the active sampled-data controlled suspension by using VMF with sensing the state of the sprung mass without knowing road profile. We try to suppress vibration under external noise from road undulation. In the following section, we will introduce VMF from three vibro-impact oscillators under Grover algorithm. In section 3 target suspension model of a vehicle, and in section 4 the method of impact oscillators under Grover algorithm. In the former one is called small non-impact oscillator and the latter called small impact oscillator.

Regarding the small non-impact oscillator as actual controlled oscillator and the others are virtual in controlling system, the position of large oscillator, \(Y(t)\), becomes the controlling trajectory of the base of the actual oscillator as one mass-spring system, which consists of a spring mass, a spring, a spring base and an actuator for enforced displacement of the base, as shown in Fig. 1(b).

\begin{center}
\textbf{Fig. 1} One mass-spring system extracted from three vibro-impact oscillator designed to realize Grover algorithm: (a) three vibro-impact oscillators, (b) one mass-spring oscillator.
\end{center}

We design the angular eigen-frequencies of the coupled vibrations between the large oscillator and the mass center of the small oscillators as \(\omega_0 = 2\pi (p+1/2)/\Delta t\) and \(\omega_0 = \pi \omega_0 / \Delta t\) with natural number \(p\) to realize internal resonance, where \((\omega_0 - \omega_0) \Delta t = 2\pi\alpha_0\) stands. Under these conditions, discrete dynamics of the large oscillator becomes independent from the two small oscillators in every operational period. Since interaction of the discrete dynamics merely occurs between the two small oscillators, three-body problem of chaotic oscillation is avoided.

In the multibody model of the three vibro-impact oscillators, the displacement of each oscillator is defined from the balanced position of it. The displacement and velocity of the large oscillator stand the following equation (1) in every operational period. In order to obtain VMF as enforced displacement of the base, the position and velocity of the virtual origin of the multibody model are redefined in every operational period to adjust the three oscillators to the actual controlled one.

\[
Y(t_0) = -Y(t_0 + \Delta t)
\]
\[
\dot{Y}(t_0) = V(t_0) = -V(t_0 + \Delta t) = -\dot{Y}(t_0 + \Delta t)
\]  

(1)

By enforcing displacement the spring base along \(Y(n\Delta t + t')\) in equation (2) in one operational period of \(\Delta t\), the mass will move along the trajectory of equation (3). Therefore the mass of the oscillator will be moved from \(y_0\) to \(y_{en}\) in displacement, from \(v_0\) to \(v_{en}\) in velocity in \(\Delta t\).

\[
Y(t_0 + t') = \sum p \sigma_p \Delta_p Y_p = \sum_p [\Delta_p - (2Y_{in} - (1 - \omega_0^2)(y_{en} - y_{en}) - (1 - \omega_0^2)(v_{en} + v_{en})\tan(\Delta t/2))(1 - \omega_0^2)\omega_0 \omega_0 \cos(\omega_0 t') + (2Y_{in} - (1 - \omega_0^2)(y_{en} - y_{en}) - (1 - \omega_0^2)\omega_0 \omega_0 \cos(\omega_0 t'))\Delta_p]
\]
\[ \omega^2(y_{in} + v_{en})\tan(\Delta t/2)\{1 - \omega^2\cos(\omega t')\} - 2V_{in} - (1 - \omega^2)(v_{in} + v_{en}) + (1 - \omega^2)(y_{in} + y_{en})\tan(\Delta t/2)/(1 - \omega^2)\omega\sin(\omega t') + \{2V_{in} - (1 - \omega^2)(v_{in} - v_{en}) + (1 - \omega^2)(y_{in} - y_{en})\tan(\Delta t/2)\{1 - \omega^2\\omega\sin(\omega t')\}/\{2\omega\omega\sin(\omega t') - \omega^2\} \]

where \( \sum_p \alpha_p = 1 \), \( t' \) is the time defined as \( 0 \leq t' < \Delta t \), \( t_0 \) is the starting instant of each operation and \( Y_{in} \) and \( V_{in} \) are the position and velocity of the large oscillator at the starting instant respectively. And parameter \( p \) is a natural number(5).

Since the function \( y(t_0+t') \) expresses the enforced displacement for vibration manipulation of the spring mass, we called it VMF. In case the aimed displacement of \( y_{in} \) and the aimed velocity of \( v_{in} \) are zero, the intermittent enforced displacement of the base can perfectly reduce residual vibration of the one-mass-spring oscillator.

As a caution, in the above equations, \( y(t_0+t') \) is defined as the displacement of the mass from the balanced position of the small non-impact oscillator and \( y(t_0+t') \) is defined as the displacement of the spring base from the balanced position of the large oscillator in the model of three vibro-impact oscillators. The balanced position of each element is determined from the position of each mass in the virtually defined three vibro-impact oscillators.

3. Vehicle Model

In this study we use the quarter-car model of 1DOF as shown in Fig. 2 (10). Here, the sprung mass \( m_s \) represents mass of the car body and the spring of the suspension is linear with a coefficient of spring constant, \( k \). Natural length of the spring is \( L_0 \). The natural period of the 1DOF oscillator can be expressed as \( T = 2\pi\sqrt{m_s/k} = 2\pi/\omega_s \). \( Y_s(t) \) is the height of road profile. \( Y_s(t) \) is a vertical position of the sprung mass. \( Y_{d}(t) \) is a vertical position of the suspension base. \( Y_{d}(t) \) is a vertical position of the suspension mass. \( Y_{d}(t) \) is the balanced position of the spring mass.

Here, we only discuss vertical oscillation of the sprung mass. Pitching and rolling rotations and horizontal oscillation of the body will be studied in the next chance, since plural wheels affects in these phenomena and situations are rather complicated.

In this model, between the sprung mass and the suspension base we placed an actuator, of which length, \( L_d(t) \), can be controlled electrically to realize desired evolution of its elongation, \( L_d(t)=\Delta L_d(t)\Delta t, L_d(t') \), during intermittent operation. The actuation could be realized by using hydraulic or electromagnetic device. Since connecting the actuator to the spring in series is different from a typical suspension structure, this suspension is rather difficult to be realized.

On the contrary, same effect can be achieved by applying external force to the sprung mass from unsprung mass by using electromagnetic force or hydraulic pressure as shown in Fig. 2 (b). The applied force should be equivalent to the product of the elongation and the spring constant, \( F_s(t) = k \Delta L_d(t) \). This system is much easy to be realized, since the force generating actuator can be added easily in conventional suspension structure.

Suppose that the wheel contacts to the road profile, \( Y_s(t) \), perfectly, the vertical position of the unsprung mass will be changed as time function of \( Y_s(t) \) by concerning the radius of the wheel, \( r_w \), and the travelling speed of the vehicle, \( v_r \). Road undulation during operational period of \( \Delta t \) is determined from the road profile within the travelling distance of \( d_m = v_r \Delta t \), which is around 20m in case \( \Delta t \) is 1s and \( v_r = 70 \text{km/h} \).

However, since we don’t know the change of road profile, \( Y_s(t) \) is supposed to be 0 all the time. Therefore we have to discuss the system in a local coordinate whose origin is placed on the unsprung mass, as shown in Fig. 3. In the local coordinate of inertial frame, virtual force as \(-m_s \ddot{Y}_s(t) \) is applied to the sprung mass. Fig. 3(a) expresses the system with the actuator for elongation, and Fig. 3(b) does with the actuator for applying force. Fig. 2(a) corresponds to Fig. 3(a) and Fig. 2(b) to Fig. 3(b). Since both models are equivalent dynamical systems, we mainly discuss the model with the actuator.

As shown in Fig. 3(a), vertical position of the suspension base from unsprung mass can be expressed as \( Y_{s}(t) = L_d(t) \) in the local coordinate. Moreover, the balanced position of the sprung mass is expressed as \( Y_{s}(t) = L_0 + L_a \) and position of the sprung mass is expressed as \( y_{d}(t) \) in this coordinate. Displacement of the sprung mass from its balanced position of \( Y_{d}(t) \) is expressed as \( \Delta y_{d}(t) = y_{d}(t) - Y_{d}(t) \).

Since spring constant of a wheel is mostly much harder than that of suspension, the wheel is simplified as a rigid body, which is in permanent contact with road surface. Passive dampers are also detached from this suspension.

Fig. 2 Proposed suspension system in one-degree-of-freedom quarter-car model with actuator; (a) for enforced displacement, (b) for applied force.
In every operational period, VMF can be modified from the aimed and predicted values of the sprung mass and the suspension base to define \( Y(t_0+\Delta t) \) for intermittently feedback-feedforward control. Then, the vertical displacement and velocity of the sprung mass can be sampled-data controlled with \( \Delta t \) as sampling period. Block diagram of the proposed system is shown in Fig. 4.

In the diagram a continuous plant system of the 1DOF oscillator is controlled by the actuator under VMF as continuous intermittent feedforward operation with a discrete feedback controller, since sampled-data control is a hybrid system includes digital and analogue equipment. In this control method, we need to predict the discrete displacement and velocity of the sprung mass by sensors in every operational period for feedback operations.

The proposed diagram is different from usual sampled-data control\(^{11}\), since it is based on discrete control theory besides continuous control one in former studies. Precise discussion is shown in another study\(^{12}\). This system can also be expressed in wave circuit diagram, since it is based on wave (quantum) algorithm\(^{6,9}\).

![Fig. 3 Proposed suspension system in one-degree-of freedom quarter-car model in local coordinate with actuator; (a) for enforced displacement, (b) for applied force.](image)

### 4. Sampled-data control of suspension

In this section, we will discuss the sampled-data control of the suspension with intermittent function of VMF in every operational period of \( \Delta t \). Under this control, the position and velocity of the virtual origin of the three vibro-impact oscillators are redefined in every operational period. In this system, since we don’t measure the road profile, vertical positional change of the unsprung mass, \( Y_o(t) \), becomes input noise as virtual force, \(-m_s Y_o(t_0 + \Delta t)\), in the local coordinate.

In order to reduce residual vibration of the sprung mass caused from the input noise, vertical position of the suspension base should be adjusted properly by controlling the vertical length of the actuator. To realize low power consumption active suspension, elongation of the actuator should be as small as possible. And since practical actuators have limitations in elongation, it should be adjusted properly by controlling the vertical length of the actuator under VMF as continuous intermittent feedback-feedforward control. Then, the vertical displacement and velocity of the sprung mass in every operational period, \( s(t) \), can be sampled-data controlled with \( \Delta t \) as sampling period.

The proposed diagram is different from usual sampled-data control\(^{11}\), since it is based on discrete control theory besides continuous control one in former studies. Precise discussion is shown in another study\(^{12}\). This system can also be expressed in wave circuit diagram, since it is based on wave (quantum) algorithm\(^{6,9}\).

![Fig. 4 Block diagram of the proposed sampled-data control for vibration suppression with proposed active suspension.](image)

### 5. Dynamics of suspension systems

By using the proposed model in Fig. 2(a) or Fig. 3(a) we will discuss dynamics of our suspension system. Here, the effect of wheel vibration and the radius of the wheel are ignored to make the model as simple as \( Y(t_0 + \Delta t) = Y_o(t_0 + \Delta t) + R_o \). To verify the effectiveness of the proposed method, we compared our model to a passive damping model which was shown Fig. 5.

In the passive damping model, linear spring constant and viscous coefficients are defined as \( k \) and \( c \), respectively. The spring constant, spring mass and unsprung mass are the same in our proposed model, which doesn’t have dampers but has an actuator instead. Then, the dynamic equations of the proposed and damping models can be expressed in equation (4) and (5), respectively.

\[
m_s \ddot{y}_s + k \dot{y}_s = k \Delta \alpha - m_s \ddot{y}_u
\]  

\[
m_s \ddot{y}_s + c \dot{y}_s + k \ddot{y}_s = k Y_u + c \dot{y}_u
\]
Now nondimensionalized time and mass are defined as \( t^* = t/\sqrt{m/s} \) and \( m^* = m/\sqrt{m/s} \), respectively. Then, \( \tau^* = 2\pi \). Characteristic length is unit length. Damping ratio is defined as \( \zeta = c/\omega_c \), where critical damping coefficient is \( \omega_c = 2\sqrt{m/s/c} \). Here suffix ‘*’ expresses the physical quantity is nondimensionalized.

By using above dimensionless symbols, equation (6) and (7) are obtained.

\[
\frac{d^2\Delta y^*}{dt^2} + 2\zeta \frac{d\Delta y^*}{dt} + \Delta y^* = Y^* - \Delta y^*(0 \leq t \leq 2\pi) \tag{6}
\]

\[
\frac{d^2\Delta y^*}{dt^2} + 2\zeta \frac{d\Delta y^*}{dt} + \Delta y^* = Y^* + 2\zeta \frac{d\Delta y^*}{dt} \tag{7}
\]

In case of the proposed active suspension, trajectory of the actuator elongation was decided by getting the position and velocity of the sprung mass just after the step at \( t = 0 \) as starting instant and by aiming the velocity to 0 just after the step at \( t = 2\pi \) as finishing instant. Operational period is \( 2\pi \) in this simulation.

In case road profile \( Y(t) \) was a smooth step, typical profile was given in equation (8).

\[
Y^*(t) = \begin{cases} 
-0.09 \cos\left(\frac{t}{2}\right) - 0.01 \cos\left(\frac{3t}{2}\right) & (0 \leq t \leq 2\pi) \\
0.1 & (2\pi \leq t)
\end{cases} \tag{8}
\]

The initial conditions of the sprung mass were assumed to be \( \Delta y^*(0) = 0, \Delta v_y^*(0) = 0 \) in the following simulations. The displacement of actuator, \( \Delta L_a(t_{0+t'}) \), was determined to be equal to VMF function of \( Y(t_{0+t'}) \), whose parameters were \( Y_{in} = Y(t_{0}) = \Delta L_a(t_{0}) = 0, Y_{in}^* = Y^*(t_{0}) = 0 \), \( v_{in} = v(t_{0}) = \Delta v_y(t_{0}) = 0 \), and \( v_{in}^* = v(t_{0}) = 0 \). \( \Delta L_a(t_{0+t'}) \) was decided to be given in equation (9). Where, \( y_{in}, v_{in} \) are measured displacement and velocity of sprung mass respectively at \( t = 0\).

\[
\Delta L_a(t_{0} + t') = \begin{cases} 
0.005625 \cos(0.5 t') - 0.005625 \cos(1.5 t') & (2\pi \leq t_0 + t' \leq 4\pi) \\
0.0 & (0 \leq t_0 + t' \leq 2\pi, 4\pi \leq t_0 + t')
\end{cases} \tag{9}
\]

Temporal change of \( Y(t_{0+t'}), \Delta L_a(t_{0+t'}) \) and \( \Delta Y_d(t_{0+t'}) = Y(t_{0+t'}) - \Delta L_a(t_{0+t'}) \) are shown in Fig. 6. It indicates the road profile and the displacement of the suspension base were quite similar with each other and displacement amplitude of the actuator was very small. In this simulation, amplitude of the actuator is 0.0159m, which is quite smaller than the height of the step, 0.200m.

6. Simulation

In the followings, results of numerical simulation for the proposed active suspension model with enforced displacement actuator and conventional passive suspension one will be shown. Three types of road profiles were assumed; a smooth step, a single bump and continuous bumps. Since characteristic length is merely unit length, we express displacements and positions with length unit for the convenience of practical understanding in the following simulations.

Differential equations (4) and (5) were calculated by using Runge-Kutta method with the following parameters; damping ratio of \( \zeta = 0.12 \), eigen-frequencies of \( \omega_0 = 1 \), \( \omega_s = 3/2 \) and \( \omega_0 = 1/2 \). Therefore, \( \alpha_1 = 1, \alpha_2 = 0 \) and \( \tau = 2\pi \). Since speed of sensor is very fast compared with the operational period, control delay was ignored.

6.1. Smooth step

In this section, the effect of the suspensions under a smooth step was considered. The step height is 0.200m. In case of the proposed active suspension, trajectory of the actuator elongation was decided by getting the position and velocity of the sprung mass just after the step at \( t = 0, t = 2\pi \) as starting instant and by aiming the velocity to 0 just after the step at \( t = 2\pi \) as finishing instant. Operational period is \( 2\pi \) in this simulation.

In case road profile \( Y(t) \) was a smooth step, typical profile was given in equation (8).

\[
Y^*(t) = \begin{cases} 
-0.09 \cos\left(\frac{t}{2}\right) - 0.01 \cos\left(\frac{3t}{2}\right) & (0 \leq t \leq 2\pi) \\
0.1 & (2\pi \leq t)
\end{cases} \tag{8}
\]
proposed model (blue line) shows that the residual vibration was suppressed perfectly in one period. Since the maximum amplitude of oscillation in the sprung mass was 0.0360m in the proposed suspension and that in the passive damping model was 0.0384m, degree of oscillation is almost the same each other. The proposed method would be useful practically, since even small displacement or small force of the actuator can reduce vibration drastically.

6.2. Single bump

In this section, the effect of the suspensions under a single bump was considered. In case of the proposed active suspension, trajectory of the actuator elongation was decided just after the bump at \( t_0 = 2\pi \) by getting the displacemnt and velocity of the sprung mass to be 0 at \( t_0 = 4\pi \). Operational period is \( 2\pi \) in this simulation. The single bump of the road profile was assumed to be given in equation (10) of \( Y_r(t) \) as shown in a black line of Fig. 8. The height of the bump is 0.050m. The initial conditions of the sprung mass were assumed to be \( \Delta y_s(0) = 0.0, \Delta y_s'(0) = 0.0 \) in the following simulations.

\[
Y_r(t) = \begin{cases} 
0.1 \sin \left( \frac{t}{2} \right) & (0 \leq t \leq 2\pi) \\
0.0 & (2\pi \leq t) 
\end{cases} 
\]  

(10)

Temporal change of \( Y_r(t) \) and \( \Delta L_a(t) \), are shown in Fig. 8. It can be seen that elongation of actuator is not necessary to be an opposite phase of the road profile. Maximum elongation of the actuator is 0.0417m in this case.

6.3. Comparison of number of operation

In this chapter, the effect of operational number will be considered. The road profile was assumed to be the same single bump which was given in equation (10) of \( Y_r(t) \) as previously shown in a black line of Fig. 8 or Fig. 10. Vibration control was performed at once or three times, where each operation was performed in every operational periods of \( \Delta t \).

Fig. 8 Displacement of road profile (black line), and elongation of actuator (red line).

Fig. 9 Displacement of sprung mass with passive damper \( \zeta = 0.12 \) (black line) or active suspension in our proposed model (blue line).

Fig. 10 Displacement of road profile (black line) and elongation of actuator which acts once (solid red line) or three times (dashed red line).
Here operational period is equivalent to natural period, $\Delta t = \tau = 2\pi$. For once operation performed during one operational period, the actuator expands and contracts along a solid red line in Fig. 10 after passing through the bump. And for 3 times operations during three operational periods, it expands and contracts along a dashed red line in Fig. 10. Although the maximum elongation of the actuator is 0.0417m at once, that is 0.0139m at three times. Temporal changes of displacements of the sprung mass are shown in Fig. 11 with these operations under proposed active suspension or passive damping one. It is observed that the active suspension can suppress the vibration after the bump more effectively regardless of the operational number. In case of 3 times operations of the actuator, it takes longer time to reduce the vibration compared with one operation. However, as shown in Fig. 10, by dividing the proposed control process into several steps, the amount of actuator displacement can be smaller in each step. Therefore, the active suspension can be realized with small power of the actuator. On the contrary, the maximum amplitude of oscillation, 0.156m, in the sprung mass was not changed regardless of the operational number.

6.4. Random road profile

In this section, the effect of the suspensions under a random road profile was considered. The road profile was created along ISO 8608 C class among the frequency 0.1 and 15Hz.

The random road profile is shown in a black line and temporal change of elongation in the actuator is in red line in Fig. 12. In case of the proposed active suspension, trajectory of the actuator elongation was decided by getting the position and velocity of the sprung mass at each starting instant, $t = t_0 = n\pi/2$, and by aiming the velocity to 0 at each finishing instant, $t = t_0 + \pi/2$, where $n$ is a natural number. Therefore operational period of this active suspension is $\pi/2$ in this simulation.

As the results of the simulations, amplitude of the profile is around 0.012m. As shown in this figure, the maximum elongation of the actuator becomes 0.073m, which is much larger than the amplitude of the road undulation. Since the active control works without the knowledge of road profile, larger elongation is required to suppress vibration, which is caused from input noise of road profile.

Temporal changes of displacements of the sprung mass are shown in Fig. 13 under the proposed active suspension and the conventional passive damper. The maximum amplitude of oscillation in the sprung mass was 0.015m in the proposed suspension and that in the passive damping model was 0.033m. Therefore the active suspension can suppress the vibration more effectively than passive damper. Since the vertical acceleration from random road profile keeps working as input noise to the system all the time, certain amount of vibrational energy remains in the suspension, although VMF controlled actuator tries to remove its energy.

7. Comparison with sky-hook theory

Since sky-hook theory is the guiding principle of active suspension, it might be interesting to discuss the difference between sky-hook and the present proposed methods. Since these
performances have not been compared in the above simulations, we merely discuss the conceptual difference between them.

Sky-hook theory is based on the dynamic equation of a 1DOF oscillator with sky-hook damper. High performance of vibrational suppression is guaranteed in damping properties of the 1DOF oscillator in wide frequency of road profile. It is based on classical or modern continuous control theory. Most of the active suspension is realized by controlling placements of its poles by changing damping or spring coefficients.

However, since the system is based on the continuous control theory, sampling time of feedback sensors and reacting time of actuators should be very fast to imitate analog control with digital controller. And these pole assignment methods guarantees the stability of the system after passing enough long time. Moreover, since most energy is consumed in damper, vibrational energy cannot be reused.

On the contrary, the proposed active sampled-data controlled suspension is based on the dynamics of multi-body vibro-impact oscillator, which treats both of controlled suspension in 1DOF oscillator and controlling system. High performance of vibrational suppression is guaranteed from energy transfer wave algorithm in the multiple oscillators. By modifying the equivalent VMF, it works to reduce energy of residual vibration from the suspension.

Stability of the system is guaranteed from repeatedly working of intermittent VMF to make the system still. Hence operational period is almost as long as natural period of the suspension, sampling time is very slow to realize discrete sampled-data control. Moreover, since the system does not use any damper, all vibrational energy could be transferred into the controlling system.

8. Conclusion

Without the knowledge of road profiles, active sampled-data control of suspension can be achieved with elongation of an actuator between an unsprung mass and a suspension base along VMF in every operational period. Required displacement of the actuator can be small in some cases and intermittent feed-forward control is easy to be realized. Although it is much faster to suppress residual vibration in the proposed system, the maximum amplitude of oscillation is larger than passive damping suspension in case of bumps. Though it has some weak points, proposed method has a possibility to be another guiding principle for active suspension of vehicles.

References