Inductance and Stroke Estimation Method Based on Current Slope of LRA

Gyunam Kim*1, Katsuhiro Hirata*1 (Mem.)

This study presents instantaneous inductance estimation and a sensorless stroke estimation method of a linear resonant actuator (LRA). In LRA, the stroke information is most important since the maximum distance of the mover is not mechanically constrained. In order to estimate the stroke from the voltage equation, motor parameters such as effective resistance, effective inductance and force constant are required. Especially, the inductance of LRA may change due to magnetic saturation. Therefore, it is necessary to estimate the inductance during operation and update its value continuously. This paper proposes a method to estimate the inductance during operation by using the current slope in the pulse width modulation (PWM). Furthermore, a method for estimating the stroke using estimated inductance is proposed. The effectiveness of the proposed method is validated by the results of numerical simulation.

Keywords: linear resonant actuator, inductance estimation, motor parameter, sensorless, stroke, current slope, pwm

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1. Introduction

Recently, LRA has been used in a wide range of fields such as electric shavers, electric toothbrushes, linear compressors and so on [1]. LRA is an actuator that moves at a resonant frequency determined by the mass of the moving part and the stiffness of the spring. A resonance in LRA means that the current and back electromotive force (EMF) become the same phase. Therefore, the resonant operation of LRA is similar to the operation which controls the d-axis current to zero when the maximum torque per ampere (MTPA) of the surface permanent magnet synchronous motor (SPMSM) is implemented. Moreover, when the high speed output is required in PMSM, field weakening method is used. Consequently, the d-axis current is applied in the negative direction to advance the phase of the current over that of the back-EMF. In the case of LRA, the same effect can be acquired by operating at an over-frequency which is higher than the resonant frequency. Therefore, d/q current control of PMSM is similar to frequency control of LRA and speed control of PMSM is similar to stroke control of LRA.

In LRA, the position information of the mover can be obtained by sensors such as linear variable differential transformer (LVDT) and search coil [2]. However, since the sensed method has problems related to cost, assembly tolerance and operating environment, sensorless methods of LRA are needed. Thus, a method of estimating the stroke by integrating the back-EMF from the motor parameter information in the voltage equation has been proposed [3-4]. In [3-4], the inductance is generally used as constant obtained from the static state, but the actual inductance can be changed by the magnet position and the linkage flux. Therefore, the inductance of each magnet position and current is stored as a lookup table, and a method of using it for stroke estimation has been proposed [5-6]. Nevertheless, these methods have difficulties in measuring the inductance under all conditions. As another approach, a method which detects the back-EMF is proposed [7]. In [7], since the discontinuous current section must be generated, the root mean square (RMS) value of the current is increased and the efficiency of LRA becomes low as the result.

Meanwhile, high frequency signal injection (HFSI) method is used to estimate the inductance and rotor position in PMSM [8-9]. However, since the HFSI method has a small harmonic current, it is necessary to increase the injection voltage level in order to improve precision. Moreover, a method for estimating the inductance and rotor position from the current slope that occurs during PWM control of PMSM has been proposed [10]. As the result of [10], the estimated inductance from the current slope includes the error component due to the instantaneous variation of the back-EMF.

In this study, a method to accurately estimate the inductance of LRA from the current slope of the PWM period is proposed. In addition, this study proposes a method to estimate the stroke from the estimated inductance. Lastly, we validate the effectiveness of the proposed method by numerical simulation using the results of FEA.

2. System of LRA

2.1 Mechanical model

Fig. 1 illustrates the mechanical model of LRA and the vector diagram of the force components. LRA consists of stator, mover, coil, and spring. The thrust force $F_t$ is proportional to the current $i$, and the force constant $K_f$ is determined by the design specifications. 

Correspondence: Gyunam Kim, Department of Adaptive Machine Systems, Graduate School of Engineering, Osaka University, 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan 
email: gyunam.kim @eng.eng.osaka-u.ac.jp

*1 Osaka University
of the coil turn, magnet flux, and air-gap. The thrust force is transmitted to the 1DOF m-c-k vibration system with the load $F_l$. The mechanical equations can be written as follows.

$$m_\varepsilon \frac{d^2}{dt^2} x(t) + c_\varepsilon \frac{d}{dt} x(t) + k_\varepsilon x(t) = F_\varepsilon(t) - F_l(t) \quad (1)$$

In Eq. (1), since the load $F_l$ can be converted to damping load $c_\varepsilon x$ and spring load $k_\varepsilon x$, the mechanical equation can be rewritten as follows.

$$m_\varepsilon \frac{d^2}{dt^2} x(t) + (c_\varepsilon + c_l) \frac{d}{dt} x(t) + (k_\varepsilon + k_l) x(t) = K_\varepsilon i(t) \quad (2)$$

In Eq. (2), it can be noted that the effective stiffness of the m-c-k model can vary with load component, and the resonant frequency also varies. The resonant frequency of Eq. (2) can be defined as follows.

$$f_r = \frac{1}{2\pi} \sqrt{\frac{k_\varepsilon + k_l}{m_\varepsilon}} \quad (3)$$

As shown in the vector diagram of Fig. 1, when the resonance operation is performed, the m-k composite vector becomes zero, so that the phase of the velocity component of the mover and the current component become in phase and the current becomes the minimum. From the vector diagram, the amplitude of the current can be written as follows.

$$I = \frac{1}{\cos \theta_{lx}} \frac{(c_\varepsilon + c_l) \omega X}{K_\varepsilon} \quad (4)$$

Where, $I$ is the amplitude of the current, $X$ is the stroke, which is the amplitude of the displacement, and $\theta_{lx}$ is the phase difference between the current and velocity. From Eq. (4), current is minimized and the motor efficiency is maximized in the resonance operation.

### 2.2 Electrical model

Fig. 2 shows the electrical serial circuit of LRA and the vector diagram of voltage components. The electrical equation of LRA is expressed by the following equation.

$$v_e(t) = R_e i(t) + L_e \frac{d}{dt} i(t) + v_b(t) \quad (5)$$

$$v_b(t) = K_e \frac{d}{dt} x(t) \quad (6)$$

Where, $R_e$ is effective resistance, $L_e$ is effective inductance, $v_b$ is back-EMF, and $K_e$ is back-EMF constant. When the voltage is applied to LRA at the initial position, a current is generated through the first-order delay element $R-L$, and the thrust of the mechanical equation is generated to move the mover. At the same time velocity is generated and back-EMF is generated in proportion to the velocity of the mover. In the next step, a current is generated due to the difference between the applied voltage and the back-EMF, the thrust is, then, determined and the position of the mover is determined.

### 2.3 Stroke estimation

As LRA is a reciprocating motion, the stroke information defined by the amplitude of the displacement is most important. In general, the stroke can be obtained by integrating back-EMF component of the voltage equation as follows.

$$x(t) = \frac{1}{K_e} \left( \int (v_e(t) - R_e i(t)) dt - L_e i(t) \right) \quad (7)$$

$$X = \max(x) - \min(x) \quad (8)$$

In Eq. (7), since $L_e$ can be changed by the position of mover and current, a stroke estimation error may occur. Furthermore, a phase error of the displacement used for judging the resonance condition may also occur.

### 3. Inductance Estimation

#### 3.1 Basic principle

As LRA must be capable of variable voltage and variable frequency operation, H-bridge voltage source inverter is generally used to drive LRA. It is acknowledged that the ripple current generated by PWM voltage
applied through the inverter correlates with the inductance. Fig. 3 describes the bipolar PWM voltage and current waveforms in the linear model. From Fig. 3, the inductance can be expressed as follows. 

\[ L = \frac{\Delta V}{\Delta i} = \frac{V_{dc} - R i - v_s}{\Delta i} \]  

(9)

Where, \( \Delta t \) is on-time of PWM, \( \Delta i \) is the amount of current change during on-time. While \( V_{dc} \) is measurable, back-EMF component is not measurable so that accurate estimation of inductance is impossible.

### 3.2 Proposed method for inductance estimation

In this study, the instantaneous inductance estimation method is proposed. Fig. 4 shows the current waveform of bipolar PWM switching with a triangular carrier. First, variables related to the change in the PWM period are defined as follows. 

\[ \Delta t(k) = t(k) - t(k-1) \]  

(10)

\[ \Delta i(k) = i(k) - i(k-1) \]  

(11)

\[ i_{\text{avg}}(k) = \frac{1}{2} (i(k) + i(k-1)) \]  

(12)

Moreover, during a short observation time of the PWM period, each variable can be defined as follows. 

\[ \frac{\Delta i}{\Delta t} \equiv \frac{\Delta i}{\Delta t} \]  

(13)

\[ \hat{L}(k) \equiv \hat{L}(k-1) \]  

(14)

\[ v_i(k) \equiv v_i(k-1) \]  

(15)

\[ i_{\text{avg}}(k) \equiv i_{\text{avg}}(k-1) \]  

(16)

Using the above equations, the equation for the PWM period can be expressed as follows.

\[ \hat{L}(k) \Delta t(k-1) = V_{dc} - R i_{\text{avg}}(k-1) - v_i(k-1) \]  

(17)

\[ \hat{L}(k) \Delta i(k) = V_{dc} - R i_{\text{avg}}(k) - v_i(k) \]  

(18)

From the above Eq. (17), (18), the inductance can be obtained as shown follows.

\[ \hat{L}(k) = 2V_{dc} \frac{\Delta t(k) \Delta i(k-1)}{\Delta t(k-1) \Delta t(k) \Delta i(k) \Delta t(k-1)} \]  

(19)

In the next step, to analyze the error characteristics, the error for each step is defined as follows.

\[ \Delta v_i(k) = v_i(k) - v_i(k-1) \]  

(20)

\[ \Delta i_{\text{avg}}(k) = i_{\text{avg}}(k) - i_{\text{avg}}(k-1) \]  

(21)
The inductance from \( +V_{dc} \) to \(-V_{dc} \) can be estimated as follows.
\[
\hat{L}(k) = \frac{(2V_{dc} + \Delta V_i(k) + R_i \Delta i_{avg}(k)) \Delta i(k) \Delta i(k-1)}{\Delta i(k-1) \Delta i(k) - \Delta i(k) \Delta i(k-1)} \tag{22}
\]
In the next step, the inductance from \( +V_{dc} \) to \(-V_{dc} \) can be estimated as follows.
\[
\hat{L}(k+1) = \frac{(2V_{dc} - \Delta V_i(k+1) - R_i \Delta i_{avg}(k+1)) \Delta i(k) \Delta i(k+1)}{\Delta i(k+1) \Delta i(k) - \Delta i(k) \Delta i(k+1)} \tag{23}
\]
If \( v_b \) and \( i_{avg} \) in Eq. (20), Eq. (21) take the reference in increasing direction, \( \Delta v_b \) and \( \Delta i_{avg} \) become positive components. Considering the above assumption in Eq. (22), the error component is generated in the positive direction. On the other hand, in Eq. (23), the error component occurs in the negative direction. Therefore, it is possible to estimate the inductance accurately by using the moving average as follows.
\[
\hat{L}_s(k) = \frac{1}{2} \left( \hat{L}(k-1) + \hat{L}(k) \right) \tag{24}
\]

### 3.3 Phase delay

When the inductance is estimated by the proposed method, the phase delay occurs. The phase delay of the instantaneous inductance estimated by Eq. (24) is expressed by the following expression.
\[
\theta_{delay1} = \frac{f_o}{2 f_{car}} \cdot 360 \tag{25}
\]
\[
\theta_{delay2} = \frac{f_o}{4 f_{car}} \cdot 360 \tag{26}
\]
\[
\theta_{delay3} = \frac{f_o}{4 f_{car}} \cdot 360 \tag{27}
\]
\[
\theta_{delay} = \theta_{delay1} + \theta_{delay2} + \theta_{delay3} = \frac{f_o}{f_{car}} \cdot 360 \tag{28}
\]

Where, \( f_o \) is the operating frequency and \( f_{car} \) is the carrier frequency. In Eq. (25), \( \theta_{delay1} \) is the delay occurring in Eq. (19). In Eq. (26), \( \theta_{delay2} \) is the averaged delay due to the difference between sensing point and calculation point in Fig. 4. And \( \theta_{delay3} \) in Eq. (27) is the delay due to the moving average in Eq. (24). Nonetheless, since the stroke estimation can be obtained in one cycle of the reciprocating motion, a phase delay of 90 degrees or less can be tolerated.

### 4. Stroke Estimation

The stroke can be obtained by the amplitude of the displacement. The displacement is obtained by integrating back-EMF component of the voltage equation. This method does not use current differential, so it has robust characteristics against signal noise. On the other hand, when the estimated inductance is used as a variable, the stroke can be estimated as follows.
\[
\dot{x}(t) = \frac{1}{K_e} \left( \int (v_b(t) - R_i i(t)) dt - \hat{L}_s(t) i(t) + \delta \right) \tag{29}
\]
\[
\delta = \int \left( \frac{d}{dt} \hat{L}_s(t) i(t) \right) dt \tag{30}
\]
Where, \( \delta \) is the component added due to integration by parts. It should be noted that when the inductance is used as a variable in the voltage equation, this component must be compensated. Finally, Eq. (29) can be discretized as follows.
\[
\dot{x}(k) = \frac{1}{K_e} \left[ \sum (v_b(k) - R_i i(k)) T_s \right] - \hat{L}_s(k) i(k) + \delta(k) \tag{31}
\]
\[
\delta(k) = \sum \left[ \left( \hat{L}_s(k) - \hat{L}_s(k-1) \right) i(k) \right] \tag{32}
\]
The estimated stroke also can be obtained as follows.
\[
\hat{X} = \max(\dot{x}) - \min(\dot{x}) \tag{33}
\]
5. Verification

5.1 FEA and motor parameter

Fig. 5 represents the LRA for an electric shaver designed by our laboratory [11]. FEA is performed by 3-D model using JMAG which is commercial software. Verification of the proposed method is performed by using results of FEA. Through electromagnetic analysis, the thrust of the mover and the flux of the coil can be obtained. Then, the inductance, force constant, and detent force are mapped using this analysis result. Fig. 6 shows the characteristics of the inductance obtained as a result of FEA. As shown in Fig. 6, the magnetic flux density of the stator increases in the overload region while the inductance decreases due to magnetic saturation. Therefore, since the stroke estimation error may occur especially in the overload region, the proposed method will be verified if the stroke estimation error decreases.

5.2 Numerical simulation with non-linear inductance

Fig. 7 shows the model of LRA with nonlinear parameter characteristics. The non-linear model in Fig. 7 is used as a lookup table with displacement and current input. The inductance and stroke estimator are designed in non-linear model and the simulation is performed. Table 1 signifies the main specifications of LRA used in this simulation. As shown in Table 1, the simulation was performed using inductance as a non-linear variable.

Table 1. Specification of LRA for electric shaver

<table>
<thead>
<tr>
<th>Item</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving mass</td>
<td>$m_e$</td>
<td>6.95</td>
<td>[g]</td>
</tr>
<tr>
<td>Damping coef.</td>
<td>$c_f$</td>
<td>0.14</td>
<td>[Ns/m]</td>
</tr>
<tr>
<td>Spring stiffness</td>
<td>$k_s$</td>
<td>13.3</td>
<td>[N/mm]</td>
</tr>
<tr>
<td>Force constant</td>
<td>$K_f$</td>
<td>0.66</td>
<td>[N/A]</td>
</tr>
<tr>
<td>Inductance</td>
<td>$L_e$</td>
<td>Variable</td>
<td>[H]</td>
</tr>
<tr>
<td>Resistance</td>
<td>$R_e$</td>
<td>0.23</td>
<td>[$\Omega$]</td>
</tr>
<tr>
<td>BEMF constant</td>
<td>$K_e$</td>
<td>1.32</td>
<td>[Vs/m]</td>
</tr>
<tr>
<td>Resonance Freq.</td>
<td>$F_n$</td>
<td>214</td>
<td>[Hz]</td>
</tr>
<tr>
<td>Carrier Freq.</td>
<td>$F_{\text{car}}$</td>
<td>8.96</td>
<td>[kHz]</td>
</tr>
<tr>
<td>Target stroke</td>
<td>$X$</td>
<td>3.0</td>
<td>[mm]</td>
</tr>
</tbody>
</table>

Fig. 7. LRA model with FEA results

Fig. 8. Inductance estimation results
(a) friction load 0.5[N] (b) friction load 2.0[N]

Fig. 9. Displacement estimation results
(a) friction load 0.5[N] (b) friction load 2.0[N]
5.3 Simulation result

Fig. 8 shows the inductance estimation results. The actual inductance is determined by the trajectory of current and displacement. Therefore, it can be seen that the variation of the inductance is larger in the overload condition than in the low load condition. In addition, since current ripple is generated by PWM voltage input, ripple component of actual inductance appears. On the other hand, since the estimated inductance estimates the averaged inductance during the sampling time, it is the same as the filtered waveform.

Fig. 9 implies the displacement estimation results. Since the variation of inductance is small at low load condition, displacement can be estimated well in both the proposed method and the conventional method. On the other hand, in the overload condition, the stroke estimation error occurs due to the large variation of the inductance, but the displacement error can be reduced by applying the proposed method. On the other hand, since the phase delay occurs in the process of estimating the inductance as shown in Eq. (28) and Fig. 8, the estimated position in Fig. 9 also has the same amount of phase delay.

Finally, the stroke estimation error was analyzed by extending the operating frequency and load conditions. Fig. 10 shows the results of the stroke estimation error under the conditions with the frequency range from 200Hz to 220Hz and load range from 0N and 2N. As shown in the Fig. 10, the estimation error performance of the conventional method is within -3.5% to +0.5%. In contrast, the estimation error performance of the proposed method is within -0.6% to +0.5% and good results were acquired.

6. Conclusion

This paper proposed a new method to estimate the inductance and stroke of LRA from the current slope of the PWM period. LRA for the electric shaver is used as the target model and the effectiveness of the proposed method in the non-linear model using the FEA results is verified. As a result, it was qualitatively verified that the inductance can be estimated. Furthermore, the estimation error of stroke is within -0.6% to +0.5%

References