Reciprocal Measurement of Transfer Function from Combustion to Engine Vibration *

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A method to identify the vibration transfer function from combustion pressure to engine structural vibration was developed without using complicated apparatus. In the method, which uses Maxwell's reciprocity theorem, a shaker excites structural vibration and the pressure in the combustion chamber is measured. A feasibility study on a simple structure produced transfer functions in the forward and reverse directions that coincided almost perfectly in the relevant frequency range. Transfer functions on a real inline four-cylinder gasoline engine showed reasonable peaks of structural resonances. The method is convenient for identifying peak frequencies of the transfer functions between combustion pressure and engine structural vibration responses.

Key Words: Noise, Vibration, Measurement, Engine ① ②

1. INTRODUCTION

Engine noise is one of the major noise components in the passenger compartment of an automobile. Especially, combustion noise determines the sound quality during acceleration ①. The transmission path from the excitation source of the combustion noise to the passenger's ear is illustrated as a block diagram (Fig.1). Using this diagram, noise and vibration control engineers are able to see a number of options to improve combustion noise. The engine structure is the key element, as can be seen in the diagram. Therefore, car designers need to know the transfer function from the combustion chamber to the engine structural vibration. However, it is hard to estimate the precise value of the transfer function from design parameters using finite element analysis, because vibration is transmitted through many parts, such as the piston, connecting rod and crankshaft (Fig. 2). In addition, a wide frequency range, up to 1 kHz, is a difficulty. In this study, a shaker test was conducted on a real engine using Maxwell's reciprocal theorem to obtain usable transfer functions from combustion pressure to engine structural vibration.

2. PROBLEMS IN MEASURING TRANSFER FUNCTIONS FROM COMBUSTION TO VIBRATION

In this study, a method to measure transfer functions with conventional equipment and software was developed.

2.1. Overlap in Structural Vibration Response Waveforms

The solution of the two problems—vibration in multiple parts and a wide frequency range—is to measure the transfer function on prototype engine structures, although this procedure cannot be done in the early stage of engine development. However, the overlap of vibration waves caused by combustion in different cylinders hinders the measurement of transfer functions from one combustion event to the corresponding vibration ②③ (Fig. 3).

One of the advanced techniques to overcome this overlapping is homo-morphic signal processing. The cepstrum of the vibration signal, which is obtained by the inverse Fourier transform of a spectrum, can separate repetitive signals and one-shot waveform signals ④. However, this method loses phase information because power spectrum uses only the amplitude information.

Another approach to minimize the influence of combustion in other cylinders is illustrated in Fig. 4. The estimated linear transfer function $\hat{H}(\omega)$ is expressed in the form of digital filter coefficients. If the digital filter used is a finite impulse response (FIR) filter, then linear regression can be used to minimize the error between the measured vibration signal and the filter output. The minimization algorithm generally used is the least mean square (LMS) approach ⑤. This approach is straightforward. However, the FIR filter does not necessarily correspond to its physical meaning. In contrast,
infinite impulse response (IIR) filters, which also represent the transfer function, can serve as the model model of the engine structural responses and so the physical meaning is clear. However, the algorithm to optimize IIR filters to the closest approximation of the characteristics of the engine structure is still linear and sometimes does not converge well.

\[ \frac{1}{2} \left( e^{j\omega t} + e^{-j\omega t} \right) e^{j\Omega t} = \frac{1}{2} \left( e^{j(\omega - \Omega)t} + e^{j(\omega + \Omega)t} \right) \] (2)

The right-hand side of this equation indicates that two points travel on a circle with different angular frequency absolute values of \( \omega - \Omega \) and \( \omega + \Omega \), that are lower or higher than the original exciting frequency by one crankshaft rotating order. These two different absolute frequency components cause different responses from those in non-rotating states. These two components are combined into one component when they are transmitted to bearings that support the crankshaft, and the angular frequency of the transmitted vibration returns to the original angular frequency \( \omega \).

Therefore, vibration transfer function of a structure including a rotating shaft as its major vibration transfer path can be evaluated only at a designated rotation speed. At other rotating conditions, error will occur, especially when the rotating shaft is near resonance.

2.3. Banger Test Rig

Some methods found in past studies used the so-called banger test rig \((6, 7)\). These are theoretically reasonable methods, and both precise and general information on engine vibration behavior was obtained. However, the rigs are too complicated to be used in the mass-produced vehicle development process. In addition, it is difficult to produce enough power in the high-frequency range.

Fig. 2 Generation mechanism of combustion noise

2.2. Nonlinear Character of Vibration Transmission through Rotating Shafts

In both cases of the \( \hat{R}(\omega) \) estimation, the authors were not able to obtain reasonable values for the transfer functions. The failure may be partially due to the nonlinear transmission of crankshaft vibration to the cylinder block \((3)\). This nonlinear transmission occurs as follows. Vibration along a line with angular frequency \( \omega \) can be expressed as

\[ \cos \omega t = \frac{1}{2} \left( e^{j\omega t} + e^{-j\omega t} \right) \] (1)

The two terms in the parentheses are two points on a unit circle traveling in opposite directions with angular velocity of \( \pm \omega \) \(\text{rad/s}\) respectively. If a non-rotating part such as a piston transmits this vibration to a shaft, rotating with angular velocity of \( \pm \Omega \) \(\text{rad/s}\), in its radial direction, then the two points mentioned above evaluated on a coordinate system attached to the shaft are expressed as below.

Fig. 3 Overlapping of engine vibration waves caused by successive combustions
3. APPLICATION TRIAL OF RECIPROCAL THEOREM ON A SIMPLE MODEL

3.1. Purpose of This Trial

Before applying the reciprocal theorem to a real engine structure, a feasibility study was conducted on a simple model, shown in Figure 5. The reason to perform this preliminary study is that forward and inverse direction measurements of the transmission function between combustion pressure and the corresponding vibration cannot be done on real operating engines, unlike the methods described in the previous section.

3.2. The Structure of the Simple Model

To eliminate the nonlinear effect at the boundaries, the model is made in one piece by welding without any fixing parts, such as bolts and nuts. The model is constructed as follows. A cubic combustion chamber with edges 15 mm length is attached to an aluminum plate. This combustion chamber size is chosen so that the first cavity resonance frequency resides far above the relevant frequency range of engine combustion noise, i.e., 300 Hz to 1 kHz. The dimensions of the aluminum alloy plate are 200 mm by 300 mm with a thickness of 2 mm. These dimensions are also set so that plate bending or torsion resonances fall into the frequency range where relevant engine structural vibration modes for combustion noise prevail.

A small amount of gunpowder was placed in the combustion chamber. The powder can be ignited by a hot wire. A small hole is located at the bottom of the chamber, and the opening area of the hole to the chamber can be changed by a screw to adjust the pressure decay rate after ignition. However, the maximum pressure and the increase of pressure are primarily determined by the size of the combustion chamber and gunpowder quantity. A pressure sensor is mounted so that its sensing surface is flush with the inner wall of the combustion chamber. A lightweight accelerometer is placed at multiple locations on the plate. For measurement of the inverse direction transmission function, an impedance hammer was used to supply the excitation force.

3.3. Transfer Functions Measured on the Simple Model

Typical measured transmission functions are shown in Figures 6 and 7. Figure 6 shows $H_{pc}(\omega)$, the function from the combustion pressure to the vibration acceleration, and Figure 7 shows $H_{wp}(\omega)$, the function from the applied force to the pressure in the combustion chamber. These two transfer functions are expressed in the next equations.

$$V(\omega) = H_{pc}(\omega)P(\omega)$$

$$P(\omega) = H_{wp}(\omega)F(\omega)$$

Where $V(\omega)$, $P(\omega)$ and $F(\omega)$ are the Fourier transforms of vibration $v(t)$, pressure $p(t)$ and force $f(t)$. Usually, Maxwell's reciprocal theorem is used in vibration analysis treating the relationship between forces.
and vibration, expressed in various forms as acceleration, velocity or displacement. By interchanging the excitation points belonging to the same vibration energy transfer path, we can obtain a transfer function that is hard to measure in one direction. However, in the case of combustion noise analysis, measurement of vibration at a combustion chamber is impossible even for non-running engine excited at a point on the cylinder block because of the space limitation on the combustion chamber. Vibration measurement on one spot on a combustion chamber is inadequate to obtain the transfer function between combustion pressure and the cylinder block vibration. As combustion pressure acts as a sort of internal force to the engine structure, it pushes up the cylinder head while it pushes down the piston head and expands the cylinder liner. Therefore, to evaluate the transfer function by applying an excitation force on one point on the cylinder block, we need to measure the vibration distribution normal to the wall on the combustion chamber. This is also impossible because the combustion chamber is too flat to do this measurement. Due to this reason, the authors adopted pressure measurement instead of the direct vibration measurement.

According to Gauss's theorem, the integration of vibration vector \( \mathbf{q}(x, y, z, t) \) distributed over a closed surface such as a combustion chamber wall is equal to the volumetric integral of its divergence as below.

\[
\iiint \nabla \cdot \mathbf{q} \, dV = \iiint \text{div} \mathbf{q} \, dV = 0
\]

(5)

where, \( \mathbf{n} \) is a unit vector normal to the combustion chamber surface. As the right-hand side of the equation is equal to \( \Delta V \), the increment of the combustion chamber volume, we can derive the relationship between the vibration vectors distributed over the combustion chamber wall and the pressure fluctuation inside the chamber using adiabatic change relationship as below.

\[
p = -\gamma \left( \frac{\Delta V}{V} \right) p_0 = -\left( \frac{\gamma}{\gamma} \frac{p_0}{V} \right) \iiint \nabla \cdot \mathbf{q} \, dS
\]

(6)

We can represent the pressure fluctuation with one variable \( p \), not considering its spatial variation, because the combustion chamber dimension is much smaller than the sound and vibration wavelength discussed in this study. This relationship is the basis that we can use the pressure fluctuation inside the combustion chamber as the replacement of the chamber wall vibration for the transfer function measurement by Maxwell's reciprocal theorem.

The two graphs display almost the same population of natural frequencies with damping values fairly close to each other. Numerical data of these values and the differences between them are listed in Table 1. The damping ratios in the table were calculated by so-called half-power bandwidth method. Because the physical quantity used to express the vibration is different in the two transfer functions, direct comparison of the magnitudes cannot be considered. For example, the lower the frequency becomes, the lower the magnitudes of \( H_w(\omega) \) become. This tendency is probably due to the fact that \( H_p(\omega) \) uses acceleration as the output, whereas the output of \( H_p(\omega) \) uses pressure proportional to the velocity.

![Fig. 6 Transfer function, \( H_p(\omega) \), from combustion pressure to plate vibration on the simple model.](image)

![Fig. 7 Transfer function, \( H_w(\omega) \), from plate vibration to cavity pressure on the simple model.](image)

3.4. Comparison of the Natural Modes Obtained by Measurement and Calculation

Utilizing the simplicity of the model, model measurement was compared with calculation by finite element analysis (FEA) in terms of natural frequencies and mode shapes as shown in Figure 8. Generally speaking, FEA can detect all
natural modes while experimental modal analysis sometimes misses them if the sensor locations coincide with nodal lines on the structure. The results, however, verified that all the relevant modes were found in the measured transfer functions.

Table 1 Comparison of the two measured transfer functions, $H_{p_1}(\omega)$ and $H_{p_2}(\omega)$

<table>
<thead>
<tr>
<th>Peak frequency, Hz</th>
<th>H_{p_1}(\omega)</th>
<th>Error, %</th>
<th>H_{p_2}(\omega)</th>
<th>Error, %</th>
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<tbody>
<tr>
<td>152.5</td>
<td>145</td>
<td>5.2</td>
<td>36.9</td>
<td>45.4</td>
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<td>1.1</td>
<td>29.6</td>
<td>47.1</td>
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<td>440</td>
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<td>34.2</td>
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<td>32.5</td>
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<td>39.6</td>
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</tr>
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<td>1620</td>
<td>1.2</td>
<td>20.1</td>
<td>37.7</td>
</tr>
</tbody>
</table>

5) Auxiliary equipment: Alternator, power steering pump and starter motor

![Fig. 9 Experiment apparatus for measuring transfer function on a real engine with transmission](image)

An electromagnetic shaker was attached on the bottom of the cylinder block (Fig. 10). Six excitation points denoted “a” to “f” were chosen. The excitation direction was in-line with the cylinder center lines.

![Fig. 10 Excitation points on the engine bottom](image)

4. APPLICATION OF RECIPROCAL THEOREM ON THE REAL ENGINE STRUCTURE

4.1. Experimental Apparatus

An in-line four-cylinder petroleum engine was placed on an engine test bench and connected to an eddy current dynamometer with a propeller shaft (Fig. 9).

The engine specifications are as follows:
1) Engine model: Nissan Motor SR20 DE
2) Cylinder layout: In-line 4
3) Piston displacement: 2 liter
4) Material of the engine structure: Aluminum alloy

The excitation system specifications are as follows:
1) Shaker manufacturer and model: B&K Type 4309.
2) Excitation waveform: Multiple sinusoidal waves with three frequency components with 10 Hz frequency difference between them. Only the central frequency component was used for transfer function measurement. This arrangement was used to maintain the friction between the engine parts in a dynamic friction state, even when the main frequency component attained anti-resonant frequencies. The explanations of this multi-sine described in literatures (6) do not include its true value mentioned above.
3) Frequency range and sweep rate: 25 Hz – 2 kHz, 25 Hz/s
4) Accelerometer: PCB 482A21 piezo-electric sensor
5) Pressure sensor: ACO-MIC-11 pressure microphone
6) FFT analyzer: OROS25 system
7) Experimental modal analyzing software: MeScope.
The pressure sensor (microphone) was placed on at the combustion chamber center in an adapter shown in Fig. 12.

**Fig. 12 Microphone on the combustion chamber**

4.2. Accuracy of the Measured Transfer Function

Figures 13 and 14 show a typical transfer function obtained in this experiment and the coherence function between the excitation and response. In this example, excitation was applied at point “a” and the combustion chamber pressure was measured at cylinder #1. In Figure 11, the first lateral bending mode (“1” in the figure) and the first vertical bending mode (“2”) of the entire power plant structure, consisting of the engine with the transmission, appear clearly in the low-frequency range. The first torsion mode is the peak denoted as “3.” The lateral bending of the engine is peak “4.” Higher modes represented by the peaks “5,” which correspond to the cylinder block skirt deformation and the second torsion mode, are also shown. The value of the coherence function is almost 100% everywhere except in the low-frequency range below 200 Hz and at several frequency points. From these observations, this measurement method was considered to have high potential to pick up relevant structural vibration modes related to combustion noise generation.

4.3. Examples of the Transfer Functions Obtained by This Method on the Real Engine.

Transfer functions from the combustion pressure at different cylinders to the vibration at point “c” are shown in Figure 15. From these measurements, the effect of combustion at each cylinder on engine vibration is identifiable. Though the reciprocal measurement method developed in this study can identify major natural modes, this method still suffers errors caused the non-linear transmission explained in the section 2.2. However, by knowing the crankshaft resonant frequencies at static states and the revolution speed of interest, we will be able to judge the influences of such non-linear transmission effects.

5. CONCLUSIONS

1) A method to identify the vibration transfer function from combustion pressure to engine structural vibration was developed without using complicated apparatus.
2) Maxwell’s reciprocity theorem using excitation by an electromagnetic shaker and the response measurement by a microphone at a combustion chamber produced a reasonable transfer function in the frequency range of 200 Hz to 2 kHz. This range is relevant to combustion noise.

6. ACKNOWLEDGMENTS

The authors thank Mr. Nakatani, who was a member of Prof. Ishihama’s laboratory, for his cooperation in the experiment.

**Fig. 13 Transfer function from vibration excitation to the combustion chamber pressure, \( H_{p_a}(\omega) \), on the engine**
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Fig. 15 Transfer functions from the point C in the Fig.10 to the combustion chamber pressure