Numerical Analysis of Stress and Strain on a Moldboard Plow Bottom

—Application of NISA II—

Delfin C. SUMINISTRADO*, Masayuki KOIKE*, Toshio KONAKA*

Abstract

A packaged FEM program, the NISA II, was used to analyze the distribution of stresses and strains on a moldboard plow bottom at various simulated plowing speeds. Quadrilateral thin shell elements were generated by the program based on the contour data of the moldboard and share surfaces. With the use of mathematical models, normal and tangential forces on each path of the tool surface were calculated at four plowing speeds and used as input to the FEM program. Linear static analysis of the loading conditions obtained the quantitative and qualitative behavior of the stresses and strains in the internal structure of the moldboard plow body. Points of stress concentration and large displacements were found to occur in the share. Displacements of the rear portion of the moldboard were found to increase with the speed. Graphical representations describing the distribution of the stresses and strains were also given.
I Introduction

The moldboard plow is an energy-intensive tillage tool. In performing its basic functions of cutting, loosening and inversion of the soil furrow slice, the amount of reaction it receives from the soil is unevenly distributed over the surface. The relative distribution of this load on the moldboard and share surfaces and the resulting displacement and stresses on the plow body have not been thoroughly investigated. Up until now, the design of its shape is based on a trial and error method and not on any solid theoretical background about the mechanics of soil movement on the complicated shape of the tool surface. The development of new designs are based on previous designs aided by some empirical data about plowing performance. This highly empirical method is also applied in the selection and specification of materials for the fabrication of the implement. Adequate procedures for the optimization of design in terms of the shape and material of the plow is yet to be developed.

The objective of this study was to evaluate a particular moldboard plow bottom in terms of the occurring strains and stresses in its internal structure as the soil furrow slice passes on its surface. Although the modelling procedure reported in this paper used only a small plow prototype, the theory could well be applied to plow bottoms of any size. It was with the aim of developing a method with which the selection and specification of material for the fabrication of moldboard bottoms could be optimized that this research study was pursued.

II Theoretical Background

In most of the reported works involving the determination of the total soil reaction forces occurring on the moldboard surface or on the whole plow bottom, the models lend themselves for the point-by-point or patch-by-patch computation of forces and pressure acting on each portion of the tool surface. The accuracy of the values obtained from such calculations may not have been well established yet. Nevertheless, it must be noted that as these models underwent refinements, predicted results tend to approach experimental values.

The theory of shells has been a well developed branch of mechanics and theoretical formulas to obtain analytical solutions of shell problems have been presented by many authors. A shell is a continuum which is bounded by two curved surfaces. Its shape is usually described by the middle surface and the shell thickness. The middle surface is a surface such that any normal to it intersects the two free surfaces at the same distance which is half that of its thickness. When this thickness is considerably smaller than the principal radii of curvatures of the bounding surfaces, the shell is said to be thin. The ease or difficulty in obtaining a solution for a shell problem is dependent on the geometry, loading and boundary conditions. Continuous geometry, smooth loading and idealized boundary conditions are generally required to obtain analytical solutions. Deviations result in complications which may prevent, or greatly
complicate, the attainment of an analytical solution. The only resort is by using numerical technique, the most prominent of which is the finite element method (FEM).

Undoubtedly, the complex shape of the moldboard plow bottom can also be modelled as a structure composed of thin shells. And with the availability of packaged program for FEM analysis, the numerical modelling and calculations involved can be easily performed at desired various load conditions.

In this paper, the whole procedure by which a finite element analysis of a moldboard plow as a shell structure was performed is presented. With the surface contour data of the plow bottom and the calculated loads using mathematical models both for the moldboard and the share, a linear static analysis using the packaged NISA II program was performed. The obtained results were discussed for some significant applications to plow bottom design.

III Materials and Methods

1. The computer system

The base configuration (Fig. 1) of the engineering workstation (EWS) used in this research includes 12 Mbytes of main memory, 156 Mbytes hard disk drive, 60 Mbytes cartridge tape drive and 1.2 Mbytes diskette drive. The operating system that runs in the workstation is the UTek, the Tektronix version of the UNIX operating system. The graphics display system provides extensive processing capabilities for handling both 2-D and 3-D graphics.

In addition to the UTek operating system and its standard utilities, several other software packages are available in the workstation such as the NISA family of programs.

2. NISA II

NISA (Numerically Integrated element for System Analysis) II is a general purpose finite element program which can be applied in solving various problems encountered in engineering mechanics. Its major capabilities include linear and nonlinear static, dynamic, buckling and heat transfer analysis. NISA directly interfaces with DISPLAY, another program in the NISA family, for model and input data generation and postprocessing of results.

The basic steps in the linear static analysis of the moldboard plow as a shell structure involves initially the interactive model generation using the DISPLAY-PRE, the preprocessing module of the DISPLAY program, for
NISA input deck preparation. The subsequent steps are data processing which involves various levels of data checking, followed by wavefront minimization, calculation of the stiffness matrices based on the given geometry, element types and material models, the generation of the local element vectors and, finally, the calculation of the displacements and internal forces. Following the successful run of the finite element analysis program, graphical representation of the results is obtained using the DISPLAY-POST, the post-processing module of the DISPLAY program. These steps are shown schematically in Fig. 2.

Linear static analysis is concerned with the linear behavior of elastic continua under prescribed boundary conditions and statically applied loads. This includes the calculation of displacements, strains, stresses, reactions and energy in the continua. The applied loads used in this paper are the predetermined forces and pressures on the nodes and surface of the elements comprising the shell structure.

The basic output from the linear static analysis includes the applied load vector at selected nodes, the displacement components at selected nodes, reactions at selected constrained nodes and averaged nodal stresses and stress intensities for selected nodes. Graphical representation of the results may be obtained using the DISPLAY-POST program.

3. Verification problem

To demonstrate and verify the use and applicability of the NISA II program, it was made to solve a simple shell problem and the obtained solutions were compared with results calculated using theoretical formulas. From Timoshenko and Wionowsky-Krieger⁶, the maximum displacement in a uniformly loaded plate is given by:

\[ \Delta_{\text{max}} = A_1 (q w_s^4) (1-\nu)/(Et^3) \]  

(1)

and the maximum stress is:

\[ \sigma_{\text{max}} = A_2 (q w_s^2)/t^2 \]  

(2)

where \( A_1 = .04872 \), for a square plate with simply supported edges, \( .02304 \), for a square plate with two opposite edges simply supported and two other edges clamped,
A_{2}= .2874, for a square plate with simply supported edges, 
A_{4}= .4182, for a square plate with two opposite edges simply supported and two 
other edges clamped, 
q = pressure on the plate, Pa, 
w_{p} = plate width, 1 m, 
E = modulus of elasticity of plate 
material, 
2.07 \times 10^{11} Pa 
t = plate thickness, 0.1 m, 
\nu = Poisson's ratio, 0.3.

Table 1 summarizes the results obtained in the calculation using the above 
formulas as compared with the results obtained using the NISA II program.

<table>
<thead>
<tr>
<th>Simply supported edges</th>
<th>Theory</th>
<th>NISA II</th>
<th>%error</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Maximum displacement</td>
<td>2.14x10^{-3} m</td>
<td>2.14x10^{-3} m</td>
<td>0</td>
</tr>
<tr>
<td>b. Maximum stress</td>
<td>28.74 Pa</td>
<td>29.06 Pa</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Simply supported two opposite edges and other two edges clamped

| b. Maximum displacement | 1.01x10^{-3} m | 1.01x10^{-3} m | 0 |
| b. Maximum stress | 41.82 Pa | 41.37 Pa | 1.08 |

Table 1 Theoretical and NISA II solutions for maximum displacement and stresses on a 
uniformly loaded square plate

4. The moldboard plow

A one-third sized model plow was used in this study. Three dimensional 
data of the moldboard shape were gathered using a profilograph and used 
as into the package program with which the necessary shell elements were generated. Fig. 3a shows the whole plow body divided into portions of the same 
material thickness and Fig. 3b shows the generated shell elements and the fixed nodes specified on the model as determined by measurements made on 
the plow assembly. The material and property input parameters are given 
Table 2.

5. Load on the moldboard

The trajectory of motion of the soil on the surface was first approximated by 
using equations which assume that the soil slice travels on the surface without any 
compressive strain. In the process, it was first necessary to subdivide the hypo-
Theoretical soil furrow slice into subslices of 0.01m. After obtaining the trajectory paths of each soil subslice on the surface, occurring pressure and tangential forces on each patch on the moldboard surface were calculated based on the following equations of forces:

\[ F_s = \{whD(v^2/r)\cos\tau + (F_s/r)\cos\tau + whDg \cos\rho\}ds \]  
\[ dF_s = [\mu_m(whD(v^2/r)\cos\tau + (F_s/r)\cos\tau + whDg \cos\rho) + whDg (z/s)] ds \]

where \[ ds = [1 + (dy/dz)^2 + (dx/dz)^2]^{1/2} dz \]

In the above two equations, the effect of the elasticity of the soil was too small to be considered.

Equation (4) were integrated to obtain values of the tangential force \( F_s \) (also referred to as sliding resistance) with which the normal force \( F_n \) were also calculated along the path of the soil on the moldboard. The Runge-Kutta method of numerical integration was applied to solve equation (4) at speed values of 0.1, 0.24, 0.42 and 0.63m/s. The input parameters used in evaluating the above equations are given in Table 3. The obtained values of the normal pressure and forces were the input loads applied to the various nodes and faces of the generated shell elements of the moldboard structure.

6. Load on the share

For the calculation of forces acting on the share, the following equations were used:

\[ T = \{[1/(k_s + r_k)] \cdot [F_p\{\sin\gamma \sin\varepsilon - r(\cos^2\gamma + \sin^2\gamma \cos\varepsilon) + Ca(\sin\beta + r \cos\beta \sin\gamma) + W_s + F_{ax} + rF_{ax}\}] \]  
\[ \text{Draft} = T \cdot k_1 \]  
\[ \text{Side draft} = T \cdot k_2 \]  
\[ \text{Vertical force} = T \cdot k_3 \]

where

\[ r = (\cos\beta - \mu \sin\beta) / (\sin\gamma [\sin\beta + \mu \cos\beta]) \]
\[ k_1 = \cos\theta \sin\varepsilon \sin\gamma + \sin\theta(\cos^2\gamma + \mu \sin^2\gamma \cos\varepsilon) \]
\[ k_2 = \cos^2\theta \sin\varepsilon \cos\gamma - \sin\theta(\sin\gamma \cos\gamma [1 - \cos\varepsilon]) \]
\[ k_3 = \cos^2\theta \cos\varepsilon - \sin\theta(\sin\gamma \sin\varepsilon) \]

The weight \( W_s \) is given by the formula:

\[ W_s = Dw_s h^2 g \left[ \frac{1}{\sin\gamma \sin\varepsilon} + \frac{1}{2 \tan \beta \sin \gamma} \right] \]

The forces due to acceleration are given by:

\[ F_{ax} = Dw_s \cdot v(v - v_s) / g \]
\[ F_{ay} = Dw_s \cdot vv_r / g \]
\[ F_{az} = Dw_s \cdot vv_s / g \]
Table 3 Input parameters to the equations for load on the moldboard and share

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil type</td>
<td>Clay loam</td>
</tr>
<tr>
<td>Soil bulk density, D</td>
<td>1.45 Mg/m³</td>
</tr>
<tr>
<td>Plowing depth, h</td>
<td>0.02m</td>
</tr>
<tr>
<td>Plowing width, w_s</td>
<td>0.13m</td>
</tr>
<tr>
<td>Coefficient of soil-metal of friction, μₘ</td>
<td>0.8</td>
</tr>
<tr>
<td>Coefficient of soil internal friction, μ</td>
<td>0.56</td>
</tr>
<tr>
<td>Soil cohesion, C</td>
<td>9 kPa</td>
</tr>
</tbody>
</table>

Fig. 4 Resultant displacement (red lines) superimposed on the original shape (blue lines) of the moldboard plow bottom.

Fig. 5 Resultant displacement of the rearmost nodes (Nodes 239 and 240) as affected by plowing speed.

The area a of the failure surface is given by:

\[ a = \frac{(h \cdot w_s)}{\sin \beta \cdot \sin \gamma} \]

Fig. 6 Distribution of von-Mises equivalent stresses on the plow, top surface.

Fig. 7 Distribution of von-Mises equivalent stresses on the plow, middle surface.

Fig. 8 Distribution of von-Mises equivalent stresses on the plow, bottom surface.

As input data into the NISA II program, the calculated values of reaction forces were divided proportionately among the nodes of the generated shell elements of the share.
IV Results and Discussion

The total CPU time by which the NISA II program was run was approximately an hour. From the program output, some significant values were picked up and presented in Table 4. Also, using the postprocessing module of the DISPLAY program, the DISPLAY-POST, several graphical representations were obtained as shown in Figs. 4 to 5.

In engineering design, it is vital to identify the various possible modes of failure of a component under service conditions. One such mode is excessive displacement wherein the dimensional changes of a component under load may be too great to allow the component to continue to fulfill its intended design function. Another mode of failure is yielding of the material. Presently, three yield criteria are used to provide means of identifying those critical stress states for which yielding is initiated. On the basis of these modes of failure, the NISA II results are discussed below.

1. Moldboard displacement

The values of displacements calculated by NISA II are given in terms of the three coordinates of the Cartesian system. From these values, the resultant displacement for each node is calculated using simple vector addition. Fig. 4 shows these calculated resultant displacements on the various parts of the plow bottom. Large displacements are shown to occur on the share especially at the point and at the middle portion. It is generally expected since the share receives about more than 60% of the total reaction force from the soil as previously computed by the models for forces11. The rearmost portion of the wing of the moldboard also shows large displacement. Although it receives small or no load at all, its relative position tends to magnify the displacement occurring in the portions in front of it. Fig. 5 shows the behavior of this displacement as affected by the variation of the load on the surface. Nodes 239 and 240 are the rearmost generated nodes on the plow bottom as shown in Fig. 3.

Although no criterion is yet available to have a better method of quantitative evaluation of the calculated displacements presented, their magnitudes are considerably very small and are, therefore, not expected to affect the performance of the tool.

2. Stress distribution

The NISA II program output also gives the averaged stresses for each node on each of the three surfaces of the model. From these values, the von-Mises equivalent stress for each node and for each surface was calculated using the formula9:

\[ \sigma_v = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \]

This yield criterion, also referred to as the maximum distortion energy criterion, assumes that yielding occurs when the energy density of the distortion state reaches

<table>
<thead>
<tr>
<th>Table 4 Results of the NISA II analysis of the moldboard plow body*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum deflection</td>
</tr>
<tr>
<td>Maximum von-Mises equivalent stresses</td>
</tr>
<tr>
<td>Top layer of shell</td>
</tr>
<tr>
<td>Middle layer</td>
</tr>
<tr>
<td>Bottom layer</td>
</tr>
<tr>
<td>Calculated minimum factor of safety</td>
</tr>
</tbody>
</table>

*The data are for the four load conditions. Very small differences of the maximum values were observed among the results.

**Calculated for steel with ultimate strength of 4.0x10⁸ Pa.
the value which it attains in the tensile test at the yield point\(^{19}\). Generally, it produces better agreement with experimental observations than the other yield criteria\(^{16}\) and was therefore used in this paper.

Figs. 6, 7, and 8 show the equivalent von-Mises stresses on the moldboard for the four load conditions on the top, middle and bottom surfaces of the shell elements. In all the figures, the values of the maximum stress were calculated to have very little variation as the load condition, i.e. speed, was varied. For the bottom and top surfaces, the concentration points of stress are shown to occur at the two opposite ends of the share at about the location of the fixed points. For the middle surfaces, the concentration occurs on the lower left portion of the moldboard.

As presented in Table 4, the calculated maximum stress is much below the yield strength of the material from which a very high factor of safety can be obtained. From this, it can be concluded that the material used in the fabrication of the plow bottom in this study could still be minimized while still satisfying basic strength requirements to withstand the soil reaction forces that occur on the surface.

3. Limitation of the model

The mathematical models for determining the forces and pressure on the moldboard and the share used in this paper were previously reported by Suministrado, et al.\(^{11}\) to obtain total values which approximate those of gathered by experiments. The accuracy by which they can be used in calculating the forces and pressure at each specific portion of the plow surface is, however, not very much proven yet. The procedure presented in this paper could, therefore, attain more significance as models for the reaction forces are further developed. It is in this connection, also, that comparative studies between using a computer model and an actual prototype in the laboratory will be performed by the authors in the future.

V Conclusions

The following important conclusions can be obtained from this work:
1. The packaged FEM program, NISA II, had greatly facilitated the modelling process and the calculations involved in determining the occurring strains and stresses on the moldboard plow body due to the soil reaction forces.
2. Large displacements were found to occur on the share of the plow bottom. It is generally expected since the share receives approximately more than 60% of the total soil reaction during plowing operation\(^{10}\). The displacement of the rear wing of the moldboard were also found to increase with load. In general, all the displacements were found to be very small to affect the plow performance.
3. Using the von-Mises yield criterion, the occurring stresses were found to be very small at the range of loading used in this study. Calculations have shown that the minimum factor of safety was about 150 for the material and as such could still be optimized.
4. The analysis gave not only the magnitudes of stresses and strains but also, and most importantly, their distribution on the whole structure such that locations of
stress concentration and points of maximum displacement were identified. Such results are of great importance for purposes of moldboard plow design which require the determination of appropriate bottom shape, optimum specification of material properties and construction of the moldboard assembly for adequate structural strength and stability.

References


Notations:
a the area of the failure surface, m²
A₁, A₂ derived constants for the computation of the maximum deformation and stress on a square plate
C soil cohesion, kPa
D soil density, Mg/m³
E modulus of elasticity, Pa
Fₜₓ, Fₜᵧ, Fₜₗ the forces of acceleration along the three axes, N
Fₚ the normal reaction force of the moldboard surface on the soil element, N
Fₚₜ the force acting tangentially along the direction of the soil path, N
Fₚₜ₁ the sum of all the tangential forces Fₚₜ at the front of each soil subslice on the moldboard surface, N
g gravitational acceleration, m/s²
h depth of plowing, m
q the pressure acting on the rectangular plate, Pa
r radius of curvature of the soil path, m
s distance measured along the soil path on the moldboard surface, m
t plate thickness, m
T the total reaction force received by the share, N
v plowing speed, m/s; vₓ, vᵧ, vₗ are the components of the velocity along the three axes.
w width of a furrow subslice, m
wₚ width of the rectangular plate, m
total width of furrow slice, m

weight of soil on the share, N

distance measured parallel, transverse and vertical to the direction of plow travel, respectively, m

direction cosine of the soil path with respect to the vertical axis

angle between the plane of the soil failure surface and the horizontal plane

angle between the intersection of the tangent plane and the horizontal plane and the direction of motion of the plow

the maximum displacement in a rectangular plate, m

angle between the plane tangent to a point on the moldboard surface and the horizontal plane

angle between the intersection of the tangent plane and the horizontal plane and the direction of soil particle velocity on the moldboard surface

the angle of the soil-metal friction

the coefficient of internal soil friction

the coefficient of soil-metal friction

Poisson’s ratio

angle between the principal normal to the soil path and the normal to the moldboard surface

the maximum stress occurring in a square plate, Pa

the von-Mises equivalent stress, Pa

the stresses along the three principal axes, Pa

angle between the normal to the moldboard surface and the vertical axis

angle between the plane tangent to a point on the moldboard surface and the horizontal plane

the angle of the soil-metal friction

the coefficient of internal soil friction

the coefficient of soil-metal friction

Poisson’s ratio

angle between the principal normal to the soil path and the normal to the moldboard surface

the maximum stress occurring in a square plate, Pa

the von-Mises equivalent stress, Pa

the stresses along the three principal axes, Pa

angle between the normal to the moldboard surface and the vertical axis

(原稿受理平成2年6月21日，質問期限平成3年11月30日)

コメント

[関読者のコメント]
理論計算および実験を行えば、必ず何らかの結果は出るが、その結果を実際に応用することは非常に重要なことと考える。したがって、これらの結果をプラウの設計製作にどのように反映するのか。その方法があれば教えて頂きたい。

[コメントに対する著者の見解]
エンジニアリングワークステーションを設計道具の中核とし、最適設計の知見を応用した研究技法の開発の動きが、はつ土板プラウの設計製作現場では萌芽的段階に達しているようです。本報の特色は、形態設計、材料設計に対して実用的意義のある情報を与えることができる点にあるものと考えています。最適設計については、NISA II SHAPE を用いて数値解析を進めているところでです。このような計算力学による接近で作業性能を犠牲にすることなく軽量化、低コスト化に貢献できる技術情報が得られれば、これまでの設計思想は方法論的変革を余儀なくされるようになるものと予測しています。