はつ土板ブラウ曲面部に発生する土壌反力の予測

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要 旨

はつ土板ブラウ曲面部に発生する応力予測のための数学モデルを用いて、接線方向と垂直方向の分力を求め、実験値との比較検討を行った。分力成分の実測は、試作した小型二方向分力変換器で求めた。この変換器の複数出力から、分力成分間の干渉はほとんど認められなかった。低速度域での曲面部における応力分布では、耕起速度の変化による変動は少ないことが分かった。高速度域の応力分布は理論解についてのみ検討し、その変動特性を調べた。

【キーワード】はつ土板ブラウ、反力、れき土、力変換器、応力分布、数学モデル

Prediction of Soil Reaction Force at Specific Positions on a Moldboard Plow Surface

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Abstract

Using the developed mathematical model, approximate stress distribution on moldboard portion could be calculated separately in both tangential and normal directions. The model was investigated by the use of the designed force transducer which measured three components of force simultaneously. Also, thickness of soil slice was investigated empirically. The sum of the theoretical stress was compared with that obtained by actual plowing experiments performed in the laboratory. It was concluded that stress distribution on the moldboard was relatively not sensitive to the change of plowing speed within experimental range. However, for higher speed, the tangential component showed a pronounced increase due to the increase of adhesion.

【Keywords】moldboard plow, reaction force, soil slice, force transducer, stress distribution, mathematical model

I Introduction

A tillage operation is a procedure of the breaking and loosening of soil. The soil failure mainly depends on the soil properties, cutting speed and tool geometry. The moldboard shape is one of the most widely used and important agricultural tillage tools. A considerable amount of research work has been done on it to optimize its design and uses. The recent research trend is towards the detailed theoretical analysis of the interaction of the implement shape, the soil factors and the various operating conditions for an accurate evaluation of the plowing phenomenon.

The main objective of this study was to
develop a model to predict the soil reaction force at specific positions on a moldboard plow surface. Applying the laws of mechanics on the analysis of soil movement and failure, mathematical equations were modified and established to predict the soil trajectory of motion and the soil interaction with the curved surface.

So far previous models were evaluated through the overall reaction force measurement of which draft, side draft and vertical force were taken into consideration, as reported by several researchers\(^2\)\(^-\)\(^8\). Such an approach did not provide the detailed insight of soil behavior at specific positions on the curved surface. Therefore, the triaxial stress transducers were imbedded on the surface of the moldboard to monitor the soil interaction behavior. At the same time, the reaction stress was measured to compare with the result from the calculation of the proposed model.

### II The Models

The model were formulated on the assumption that the plow bottom could be considered stationary and the furrow slice moved over it. Other assumptions will be mentioned throughout the paper as each related item is mentioned. Also, with the same assumption as that the relative speed of the slice with respect to the plow bottom is constant and equal to the speed of plowing, the following relationships were achieved. \(v'_c\) is the velocity vector of soil slice on the inclined plane with respect to the reference axis \(x'', y', z'\) which was the result of twice axis rotation by \((\pi/2 - \gamma)\) angle about \(Z\) and then \(\varepsilon\) angle about \(y'\) which is parallel to the base line of the wedge, as shown in Fig. 1. Here, \(\gamma\) was assumed equal to the angle between \(y'\) axis and \(v'_c\).

\[
\begin{align*}
v'_c &= \left[ v_{x''} v_{y'} v_{z'} \right] \\
\hat{v}'_c &= \left[ v_p \sin \gamma \quad v_p \cos \gamma \quad 0 \right] \\
\hat{v}_c &= \left[ v_p \sin \gamma \quad v_p \cos \gamma \quad 0 \right] \begin{vmatrix} \cos \varepsilon & 0 & \sin \varepsilon \\ 0 & 1 & 0 \\ -\sin \varepsilon & 0 & \cos \varepsilon \end{vmatrix} \begin{vmatrix} \sin \gamma & -\cos \gamma & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & 0 & 0 \end{vmatrix}
\end{align*}
\]

where \(\varepsilon, \gamma, v\) are rake angle, approaching and plowing speed respectively.

\[
\hat{v}'_c = \left[ \nu \left( \sin^2 \gamma \cos \varepsilon + \cos^2 \gamma \right) \right] \begin{cases} 
\nu_p \cos \gamma \sin \gamma (1 - \cos \varepsilon) \\
\nu_p \sin \gamma \sin \varepsilon \\
\nu_p \end{cases}
\]

\[
v_c = \begin{bmatrix} v_{x''} & v_{y''} & v_{z''} \end{bmatrix}
\]

However, in the real situation, when initial cutting begins, the shear failure develops progressively to a total failure ahead of the tillage tool. During cutting, the configuration of soil ahead of tillage tool and soil-tool interface changes continuously. The geometry of soil failure was reportedly varied with the rake angle such that the rising soil

![Fig. 1 Relative direction of soil particle velocity on an inclined blade](image)
slice was thicker than the plowing depth. Thus, with the assumption that the soil failure plane at soil cutting edge is normal to the plane of the soil cutting tool, then the thickness change at the soil cutting edge can be approximately computed with the following relationship, as shown in Fig. 2. $\theta_o$ is the equivalent rake angle in the case of moldboard plow by taking the effect of $\gamma_o$ angle into consideration. When $\gamma_o$ angle is right angle, the orientation of the cutting blade will become just like the ordinary inclined straight blade, as shown in Fig. 2. $D_o$ is soil slice thickness on the curve surface at the beginning of cutting edge and $D_p$ is plowing depth.

$$D_o = \frac{D_p}{\cos \theta_o} = \frac{D_p \sqrt{\sin^2 \gamma_o \sin^2 \varepsilon_o + \cos^2 \varepsilon_o}}{\cos \varepsilon_o}$$

(3)

where the subscript mark $o$ represents the property or characteristics over the position of soil cutting edge of plow share.

Therefore, velocity of soil slice was affected as shown in the below derivation. It also implied that the assumption employed in Suministrado et al. that speed of furrow slice is equal to the plowing speed was no longer valid. The term $\nu_p$ was corrected to $\nu_o$ by using the mass conservation law. $W, a, t, \rho$, and $\nu$ are mass of soil on the surface, cross-section area of soil slice, time, density and velocity respectively. Also, it was assumed that soil slice is breakable and incompressible.

$$\frac{dW}{dt} = 0 \text{ ; at any } a$$

$$pW \nu_1 = pW \nu_2$$

(4)

where the subscript mark 1 and 2 mean the property or characteristics of any two different positions along the soil path on moldboard surface.

According to Suministrado et al., plowing width had been approximately assumed equal to the soil slice width. Therefore, the velocity change at share edge expressed by equation (5) was achieved.

$$\nu_p D_p = \nu_o D_o$$

$$\nu_o = \frac{\nu_p \cos \varepsilon_o}{\sqrt{\sin^2 \gamma_o \sin^2 \varepsilon_o + \cos^2 \varepsilon_o}}$$

(5)

Then, equation (2) should be modified to the following equation of velocity vector at share edge:

$$\nu^T_o = \begin{bmatrix}
\nu_p \sin \gamma \cos \varepsilon + \cos^2 \gamma \cos \varepsilon_o \\
\nu_p \sin \gamma \sin \varepsilon \cos \varepsilon_o \\
\nu_p \sin \gamma \cos \varepsilon \cos \varepsilon_o \\
\nu_p \sin \gamma \sin \varepsilon \cos \varepsilon_o
\end{bmatrix}

(6)

Harrison found that the soil surface of slice directly above the tool is not parallel
with the plane of tool surface thus revealing that some change in the velocity of the soil occurs as the soil ascends the tool, as shown in Fig. 2. According to his results from Harrison\(^{16-17}\), the following relationship was inferred. \( \mu_m \) is soil-metal friction and \( l \) is length of inclined blade:

\[
\frac{dD}{dx'} = f(\varepsilon, \mu_m, l)
\]  

Equation (7) was taken into concern with the variation of soil slice thickness by assuming that there was no thickness change due to the motion in the \( y' \) direction. From equation (1), the velocity from share edge to moldboard end was:

\[
\nu' = \left[ \nu_p \frac{D_o}{D} \sin \gamma \cos \theta_e \quad \nu_p \cos \gamma \cos \theta_e \right] \frac{dD}{dt} = f(\varepsilon, \mu_m, l)
\]

Since \( \nu_{z'} \) is relatively small in reality, \( \frac{dD}{dt} \) was assumed as zero in this model. Therefore, the following equations describing behavior of velocity of soil slice from share edge to moldboard end were developed:

\[
\nu = \left[ \nu_p \frac{D_o}{D} \sin \gamma \cos \theta_e \quad \nu_p \cos \gamma \cos \theta_e \right] \begin{bmatrix}
\cos \varepsilon & \sin \varepsilon & -\cos \varepsilon & \cos \varepsilon & \sin \varepsilon \\
\cos \gamma & -\sin \gamma & 0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
\cos \varepsilon \sin \gamma & -\cos \varepsilon \cos \gamma & \sin \varepsilon \\
\cos \gamma & -\sin \gamma & 0 & 0 & 0
\end{bmatrix}
\]

Thus, the trajectory of motion of an ideally incompressible but breakable furrow slice on the moldboard was obtained by integrating equation (9) above.

In Fig. 3, the forces acting on a differential element of soil are shown along the \( x'', y', z' \) rotated coordinate system. With the assumption that soil failure plane was normal to the tool surface, the equations of forces \( F_{x''}, F_{y'}, F_{z'} \) are:

Along the \( z' \) direction which is normal to the moldboard surface:

\[
F_{z'} = A_{z'} + W_{z'} + C\left( \frac{dD}{dx''} \right) dx'' dy'
\]

\[
F_{z'} = \rho \left[ (\nu_{x''}^2 + \nu_{y'}^2) R + g \cos \varepsilon \right] + \frac{dD}{dx''} dx'' dy' + C\left( \frac{dD}{dx''} \right) dx'' dy'
\]

where \( A, A_{z'}, \) and \( R \) represent acceleration force, acceleration force in \( z'' \)-direction, and radius of curvature on moldboard surface respectively.

Also, it is assumed that acceleration forces in other direction due to the
change of velocity could be considered negligible and equal to zero.

Along the $y'$ direction:

$$F_{y'} = \mu_m F_x \frac{v_{y'}}{\sqrt{v_{x'}^2 + v_{y'}^2}} + C_a dx'dy' \tag{11}$$

where

$$\frac{v_{y'}}{\sqrt{v_{x'}^2 + v_{y'}^2}} = \frac{D \cos \gamma}{\sqrt{D_o^2 \sin^2 \gamma + D^2 \cos^2 \gamma}}$$

and $g, C, \mu_m,$ and $C_a$ are gravity, soil cohesion, soil-metal friction and soil adhesion respectively.

Along the $x''$ direction:

$$F_{x''} = \mu_m F_x \frac{v_{x''}}{\sqrt{v_{x'}^2 + v_{y'}^2}} + W_s + C_a dx''dy' \tag{12}$$

where

$$\frac{v_{x''}}{\sqrt{v_{x'}^2 + v_{y'}^2}} = \frac{D_o \sin \gamma}{\sqrt{D_o^2 \sin^2 \gamma + D^2 \cos^2 \gamma}}$$

Therefore, reaction forces per interacted area in three directions at specific positions were obtained.

$$P_{x'} = D \rho \left[ \frac{D_o^2 \sin^2 \gamma + \cos^2 \gamma}{\nu_p^2 \cos^2 \gamma \cos^2 \theta_e + R^2} \right] R v_p \cos^2 \theta_e + g \cos \epsilon + C_a \frac{dD}{dx''} \tag{13}$$

$$P_{y'} = \mu_m D \cos \gamma \left[ \frac{D_o^2 \sin^2 \gamma + \cos^2 \gamma}{\nu_p^2 \cos^2 \gamma \cos^2 \theta_e + R^2} \right] R v_p \cos^2 \theta_e + gD \cos \epsilon + C_a \frac{dD}{dx''} \tag{14}$$

$$P_{x''} = \mu_m D_o \sin \gamma \left[ \frac{D_o^2 \sin^2 \gamma + \cos^2 \gamma}{\nu_p^2 \cos^2 \gamma \cos^2 \theta_e + R^2} \right] R v_p \cos^2 \theta_e + gD \cos \epsilon + C_a \frac{dD}{dx''} \tag{15}$$

Before performing all the calculations involved in solving the above equations, it was first necessary to determine the parameters that define the shape, i.e. angles $\gamma$ and $\epsilon$ with assumption that each surface patch is a portion of a horizontal cylinder oriented at a certain angle from the direction of plow motion. Also, soil slice thickness $D$, which depends on other parameters such as $\epsilon$, $\mu_m$ and $l^{(16)}$, was unknown and obtained through the preliminary test because it is beyond the scope of this study to find the relationship of those parameters.

### III Laboratory Tests

1. **Materials**

A small plow with a width of cut of 0.13m was used in performing the plowing test actual value of the reaction forces. The plow was fabricated to be a one-third-sized model of a commercially available moldboard plow. However, no dimensional analysis was performed and the plow was treated as a small-sized moldboard bottom by itself such that the plowing tests and the subsequent computations performed were based on
the plow’s actual size.

Clay soil sieved and wetted with water to obtain the desired moisture content level for the tillage test, was placed in a soil bin whose length, width, and depth were 2.0, 0.9, 0.2m respectively. This soil bin was mounted on a platform which had its two pairs of wheels resting on two parallel straight tracks. A steel cable wound on a motor-driven pulley pulled the platform with the soil bin in both directions along the tracks at specified speeds during the plowing tests.

2. Fabrication of reaction force transducer

For measurement and recording of data, the plow was mounted on a test rig to which additional frame was attached to support three transducers. The detecting parts of each reaction force transducer were embedded exactly in the bore of the plow at specified positions. The reaction force transducers designed to measure three component forces simultaneously, were installed and operated by the laboratory personal computer equipped with the 4-channel sensor interface boards.

The three directional force transducer was composed of square bars which were formed as the structure shown in Fig. 4. One of two ends was designed as the detecting position, and the other end was fixed to the plow body in order to let the detecting position move freely due to the input force. The deflection was allowed to occur throughout each element of the transducer structure. The effected displacement was composed of three kinds of elements, which were axial, torsional, and bending. With the use of bending deflection and location of strain gages, the measurement of directional input force can be performed separately with the least influence to one another from each direction.

There were three combinations of strain gage attachment to detect input force in three directions which were perpendicular to one another. All combinations were in the form of the full bridge circuit for measurement of bending only.

Also, the configuration of detachable detecting part was taken into consideration concerning clearance of the bore on the the moldboard to prevent soil accumulation. Jongwatpol et al. reported that by tapering both side of clearance the tangential force measurement approached the boundary value and solved the soil clogging problem. Therefore, in this study its shape was also modified and tapered to merge with the moldboard surface properly at 0.02m of diameter.

After the fabrication of the three-directional transducers, they were calibrated and installed behind the model plow by setting their reference axes in accordance with the $x'$, $y'$, $z'$ rotated

![Fig. 4 Schematic diagram of the three-directional transducer and reference axes](image-url)
coordinate system employed in the model.

3. Methodology and laboratory experiment

Schematic diagram of experimental apparatus is shown in Fig. 5. There were nine channels of three interface boards connected to three of the installed transducers. These interface boards, which were mounted to a microcomputer (NEC PC-9801RL), were set to the sampling rate of 20ms and 200 data.

Fig. 5 Schematic diagram of laboratory apparatus

After checking the geometrical characteristics of three selected positions, plowing test under controlled laboratory conditions were performed. Before the running of each plowing test, the following preparations were made: the soil was thoroughly compacted, the model plow was allowed to make a shallow cut on the surface in order to establish both the top surface and the side boundaries of furrow slice, and the two sides of the furrow slice along the width were vertically cut at the desired plowing depth.

Plowing test were conducted at four speeds of 0.089, 0.147, 0.204, and 0.263 m/s. Also, after each run of soil bin, soil slice thickness of each position was measured by inserting a long pin into the soil slice remained on the moldboard from each reference direction. The following data were also gathered separately in the preliminary test.

(1) Apparent soil-metal coefficient of friction and adhesion
(2) Soil moisture content
(3) Bulk density of soil
(4) Soil cohesion
(5) Thickness of furrow slice at specified positions

These results are shown in Table 1, Table 2 and Fig. 7.

To measure the thickness of furrow slice, a slim steel rod 2mm of diameter was employed.

Table 1 Physical properties of soil tested

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Liquid limit</th>
<th>Plastic limit</th>
<th>Bulk density</th>
<th>Moisture content</th>
<th>Coeff. friction</th>
<th>Soil cohesion</th>
<th>Soil adhesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>55%</td>
<td>29%</td>
<td>1.259 Mg/m³</td>
<td>24.0% d.b.</td>
<td>0.71</td>
<td>800 Pa</td>
<td>120 Pa</td>
</tr>
</tbody>
</table>

Table 2 Geometrical characteristics of specific positions on the moldboard plow

<table>
<thead>
<tr>
<th>Position on the plow</th>
<th>p (°)</th>
<th>τ (°)</th>
<th>Radius of curvature (m)</th>
<th>Distance from cutting edge (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cutting edge</td>
<td>30</td>
<td>43</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>30</td>
<td>43</td>
<td>0.2</td>
<td>0.05</td>
</tr>
<tr>
<td>B</td>
<td>55</td>
<td>45</td>
<td>0.1</td>
<td>0.14</td>
</tr>
<tr>
<td>C</td>
<td>75</td>
<td>50</td>
<td>0.095</td>
<td>0.21</td>
</tr>
</tbody>
</table>
From the specified directions which are normal to the position A, B and C, such a steel rod was inserted into the furrow slice which still remained on the moldboard surface and then obtained the thickness value of furrow.

IV Results and Discussion

Results from calibration test of transducer employed at position A (nearest the share) are partly shown in Fig. 6. The combination of transducer angles were set to achieve various directions of force loading. In Fig. 6, static loads were increasingly applied along the three major axes x, y and z in order to evaluate the mutual interference by measuring how much the amount of strain in other axes was given from the loading direction. Within the loading range, 3% of mutual interference was maximally detected throughout 10 testing combinations. For other transducers in position B and C, nearly exact similar results were satisfactorily obtained. In spite of the fact that the designed transducer 0.05m small in size in which the hysteresis effect affected its accuracy, it provided the acceptable measurement during the short period of operation. Also, the maximum amount of time used in the plowing experiment was 4 s. That guarantees the applicability and validity of the designed transducer.

Fig. 7 showed soil slice behavior in terms of its thickness variation. This result could be inferred to the same conclusion as stated in the work of Harrison\(^{16}\) that plowing speed of the soil cutting blade did not affect significantly the thickness of soil slice. Rather, it was affected by the coefficient of friction on the soil cutting surface. This was observed during the preliminary test by changing the surface
with a plastic sheet or coating. Also, by varying the rake angle of the blade, a change in thickness was detected. In stress measurement of this experiment, there was only ordinary surface condition applied. The values of thickness at each position were averaged and employed in equations (13)-(15) to provide the theoretical results.

Figs. 8, 9, and 10 showed the predicted stresses at positions A, B and C in all components against plowing speed compared with those obtained by experiments. The calculated valued were obtained from equations (13)-(15) respectively. These results could be approximated by the theoretical results. Both theoretical and experimental value appeared not very sensitive to plowing speed. As observed from equations (13)-(15), the speed range of the experiment affected insignificantly the stress. In Fig. 9 at position A, the calculated y'-direction stress tended to overpredict the experimental results at position A. However, in Fig. 11 which showed the tangential stress, at position A, experimental results were satisfactorily approximated by the calculated value. This discrepancy of both values could be attributed to the geometrical error which occurred during transducer installation. Reference axes of transducer did not correspond properly to the motion of soil slice.

Also, there was another possibility that at position A, the transducer was always exposed to the soil close to the share which received higher stress. The gap between its detecting part and moldboard surface was accumulated with soil particle too much that it impeded the measurement in y'-direction which was a relatively small value. When both x" and y'-direction results were combined to the tangential results in Fig. 11, it showed relatively small influence in the overall result.

Fig. 10 showed that at position C the theoretical results were less than the experimental results. This difference could be attributed to the geometrical change of the upper part of the moldboard due to deflection during plowing operation. Also, the stress fluctuation was detected during the measurement as observed from the recorded
data in the microcomputer. Therefore, the fluctuation band of experimental results existed at all positions.

The model equations derived above were still empirically based on the relationship between soil thickness and characteristic of moldboard surface. The results in Fig. 7 were substituted in equations (13)-(15) to obtain the theoretical calculation. The detail of equation (7) is still under further consideration.

In the previous works of many researchers such as Suministrado et al.\(^7\) the experimental results of total draft of all three components were sensitive significantly to the change of plowing speed. This showed that the soil slice behaved differently on the share portion of moldboard plow. The increased draft could be attributed to the share and cutting edge which caused the soil breakage. Also, the share reportedly received approximately 60% of the overall reaction force in plowing operation. That means the change of reaction forces exerted on the share are much more severe and cause more problem of wear.

For higher plowing speed over 0.35 m/s condition, Yao et al.\(^15\) reported that adhesion increased logarithmically with the sliding speed while the frictional angle remains unchanged. Therefore, the tangential stress increased theoretically as the increasing change of plowing velocity due to adhesion whereas the normal stress was relatively not sensitive to such change. But for the low speed condition, the influence of adhesion was slightly detected.

V Conclusions

The model of stress distribution on

![Fig. 9 Comparison results of stress in y'-direction at positions A, B, and C](image)

![Fig. 10 Comparison results of normal stress on the moldboard at positions A, B, and C](image)

![Fig. 11 Comparison results of tangential stress on the moldboard at positions A, B, and C](image)
the moldboard surface developed, and three directional force transducers were fabricated, calibrated and applied to investigate the soil slice behavior in terms of stress distribution. A laboratory test was set up to verify the model as functions of speed and various geometrical parameters at specific position with the use of a model plow equipped with the transducers. The following can be made from the results of the investigations:

1) Soil slide behavior in terms of thickness does not vary due to the change of plowing speed, but the geometrical characteristics of the moldboard surface in the same soil.

2) The calculated value of force or stress in each component and at each position approximate satisfactorily the experimental results between 0.089 and 0.263 m/s plowing speed range. Also, the theoretical values were not relatively sensitive to the change of plowing speed.

3) The soil reaction force received by share were more sensitive to the change of velocity than that of moldboard.

4) The normal stress distribution were less sensitive to the change of plowing velocity than the tangential stress.

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