One-dimensional modeling of stress-induced transformation behaviour of shape memory alloy

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In this paper, the mechanical constitutive relation of Shape Memory Alloys (SMAs) during stress-induced transitions is discussed based on mixture theories. The twinned martensite variants and austenite parents are taken as two constituents with their own reorientation or transformation behavior when subjected to external loading. Corresponding internal variables for describing these transitions are newly defined. This model enables the numerical prediction of the overall mechanical behavior of SMA materials in the temperature range from M_s to M_f . The approach of mixture theory; series models are applied in the formulation of the constitutive behavior of SMAs with isostress conditions. Tensile tests have been carried out with Ti-Ni specimens in a temperature range including martensite transformation temperatures. Numerical simulation based on the proposed model accurately predicted the mechanical behavior of stress-induced transitions in the concerned temperature range.

Keywords : Shape memory alloys, Constitutive relations, Phase transformations, Mixture theories

1. INTRODUCTION

Shape memory alloys (SMAs) are known as intelligent materials with unique thermomechanical properties such as shape memory effect (SME) and pseudoelasticity (PE). Their important features in mechanical behavior; large recoverable strain and large recovery stress have attracted attention of many researchers. To develop accurate models for describing the thermomechanical behavior of SMAs needs to design of SMAs in various engineering applications and has been attracting increasing interests of researchers.

The approaches of modeling the thermomechanical behavior of SMAs have been developed to describe the complicate behavior. They can be roughly classified into microscopic view or macroscopic one.

While the microscopic thermodynamic and micromechanical models are established under some ideal assumptions and are therefore rarely available for accurate prediction of the mechanical performance of SMA materials, the phenomenological models are usually given based on the experimental phase diagrams and consequently describe the thermomechanical behavior with reasonable accuracy. One of the early attempts for the description of constitutive equation of SMAs was presented by Tanaka et al.¹ This model is derived from the thermodynamics but has a phenomenal form. It can qualitatively describe the mechanical behavior of SMAs. Tanaka's model gives a basic framework on the formulation of the constitutive behavior of SMAs. In Tanaka's and Liang's² models,

however, the effect of external stresses on the mechanical behavior with the stress-induced reorientation of martensite variants at temperatures below M_s was not considered.

For practical applications, the effect due to the mechanical reorientation from twinned martensite variants to detwinned one is necessary to be incorporated into the stress-temperature diagram. Brinson³ proposed a modified model, based on Liang's model, to enable the description of the mechanical behavior of SMAs at temperatures $T < M_s$. With the stress-temperature diagram proposed based on experimental observations by Liang⁴ and Dye,⁵ the mechanical behavior due to the reorientation was described using the internal variables, the fraction related only to the stress-induced transition and the fraction related only to the temperature-induced one. However, at temperatures $M_s > T > M_f$, the initial state of the material is a mixture of austenite parents and twinned martensite variants. The external stresses would induce both the reorientation from twinned martensite variants to detwinned martensite variants and the transition from austenite parents to detwinned one. The mechanical behavior in this temperature range results in the integration of them. Actually, such integrated behavior clearly appeared in some reported experimental results.

In the present paper, assuming that the reorientation of the martensite and the transformation from austenite to detwinned martensite are governed by their constitutive relations respectively, mixture theory will be applied to formulate the general constitutive relation. Tensile tests have been conducted using NiTi plate specimens. Also numerical simulations based on the proposed model have been conducted. The validity of the proposed model have been confirmed by the comparison between numerical results and experimental results.

II. Modeling of phase transformation

In this paper, we use basic equations from prior models by Tanaka¹ and Brinson³ but assume that the reorientation of the martensite variants and the transformation from austenite to detwinned

martensite are governed by their own constitutive relations. Then, we apply mixture theory to formulate the general constitutive relation of a mixture with austenite, twinned martensite and detwinned martensite phases.

For describing the thermomechanical behavior of SMA, a first constitutive equation correlating the applied stress σ with the strain ε , the martensite fraction ξ and the temperature T was given by Tanaka in the following form:

$$d\sigma = \frac{\partial \sigma}{\partial \varepsilon} d\varepsilon + \frac{\partial \sigma}{\partial \xi} d\xi + \frac{\partial \sigma}{\partial T} dT \quad (1)$$

$$E = \frac{\partial \sigma}{\partial \varepsilon} \quad \Omega = \frac{\partial \sigma}{\partial \xi} \quad \text{and} \quad \Theta = \frac{\partial \sigma}{\partial T} \quad (2)$$

Where E is the elastic modulus of the material, Ω the transition tensor and Θ the thermal expansion coefficient.

Based on Tanaka's model, a modified constitutive model has been given by Brinson³ to take the stress-induced transition into account,

$$(\sigma - \sigma_0) = E \times (\varepsilon - \varepsilon_0) + \Omega \times (\xi_s - \xi_{s0}) + \Theta \times (T - T_0) \quad (3)$$

where ξ_s represents the volume fraction of detwinned martensite induced by external stresses. In Equation (3), the transition tensor can be expressed by

$$\Omega(\xi) = -\varepsilon_L E(\xi) \quad (4)$$

where ε_L is a maximum residual strain.

The entire strain ε_{total} can be expressed with a summation of the elastic strain and the transition strain $\varepsilon_{tr} = \varepsilon_L \xi_s$.

$$\varepsilon_{total} = \varepsilon_{el} + \varepsilon_L \xi_s \quad (5)$$

In a stress free situation, the volume fraction of martensite depends on the temperature. Therefore, the initial martensite volume fraction can be calculated quite easily, knowing the temperature and transformation temperatures, through trigonometric equations as follows⁶

$$\xi_{Mf0} = \frac{1}{2} \left[1 + \tanh \left(\frac{2T - (Mf + Ms)}{(Mf - Ms)} \right) \right] \quad (6)$$

During the transformation, the fractions of austenite and twinned martensite transformed to detwinned martensite can be calculated through equations 9 and 10, respectively.

$$\xi_{SA} = \frac{1}{2} \left[1 + \tanh \left(\frac{2\sigma - (\sigma_{Ms} + \sigma_{Mf})}{\sigma_{Mf} - \sigma_{Ms}} \right) \right] \quad (7)$$

$$\xi_{SM} = \frac{1}{2} \left[1 + \tanh \left(\frac{2\sigma - (\sigma_{cts} + \sigma_{ctf})}{\sigma_{ctf} - \sigma_{cts}} \right) \right] \quad (8)$$

Then, we consider a mixture of twinned martensite and austenite in series at the beginning. Upon loading, the reorientation and transformation occurs independently, as Fig.1.

Fig.1. A series model of SMA during the stress-induced reorientation and transformation.

Using a series model, we have isostress conditions. As a result, we obtain the elastic strain,

$$\begin{aligned} \varepsilon_{el} = & (1 - \xi_{Mt0})(1 - \xi_{SA}) \frac{\sigma}{E_A} + \xi_{Mt0}(1 - \xi_{SM}) \frac{\sigma}{E_{Mt}} \\ & + [(1 - \xi_{Mt0})\xi_{SA} + \xi_{Mt0}\xi_{SM}] \frac{\sigma}{E_{Md}} \end{aligned} \quad (9)$$

and the transformation strain,

$$\varepsilon_{tr} = \varepsilon_{LM} \times \xi_{Mt0}\xi_{SM} + \varepsilon_{LA} \times (1 - \xi_{Mt0})\xi_{SA} \quad (10)$$

Therefore, the total strain is

$$\begin{aligned} \varepsilon = & (1 - \xi_{Mt0})(1 - \xi_{SA}) \frac{\sigma}{E_A} + \xi_{Mt0}(1 - \xi_{SM}) \frac{\sigma}{E_{Mt}} \\ & + [(1 - \xi_{Mt0})\xi_{SA} + \xi_{Mt0}\xi_{SM}] \frac{\sigma}{E_{Md}} \\ & + \varepsilon_{LM} \xi_{Mt0}\xi_{SM} + \varepsilon_{LA} (1 - \xi_{Mt0})\xi_{SA} \end{aligned} \quad (11)$$

III. Experiment

Tensile tests were performed at different temperatures on samples (thickness: 0.8mm, width: 2mm, length: 65mm) at a speed of 1mm per minute in a JTT tensile test machine. This machine allowed us to make tensile tests at controlled temperatures. Prior to tensile test, the samples were annealed (20 min at 850°C in Nitrogen and quench and then 100h at 400°C in air and quench) and, once in the machine, followed a constant thermal cycle (go to 70°C, hold 10min, go to experimental temperature, hold 20min) without load in order to have a known and reproducible structure.

The transformation temperatures were obtained through DSC measurements on a Perkin-Elmer Pyris1 as $M_s = 4.8^\circ\text{C}$, $M_f = -7.8^\circ\text{C}$. Young's modules were measured by 3 point bending tests. And other characteristics were obtained from tensile tests at different parameter temperatures. The parameters can be found in Table 1.

Table 1. Young's moduli and maximum residual strain.

Young's moduli	Maximum residual strain
$E_{mt} = 25.3\text{GPa}$	$\varepsilon_{Lr} = 0.015$
$E_{md} = 10.5\text{GPa}$	$\varepsilon_{Lm} = 0.012$
$E_r = 22\text{GPa}$	

IV. RESULTS

Once we had these parameters from the parameter temperatures (-15 °C, -10 °C, 10 °C, and 30 °C), we used them to compute stress-strain curves at test temperatures. Comparison of the obtained s-s curves and experiment is shown on Fig.2 to 4.

Fig.2 and Fig.3 show the comparison at 20°C (before M_s), and -4 °C (between M_s - M_f). In Fig.2 and Fig.3, the numerical simulation can be coincided with experimental result. But Transformation start stress is predicted lower than experimental results in Fig.4. In this model, in order to decide the transformation start and finish stress at each temperature, linear function is assumed. More tensile tests should be carried out to find more accuracy function.

Fig.2. Comparison of model and experiment at 20°C

Fig.3. Comparison of model and experiment at -4°C

Fig.4. Comparison of model and experiment at 0°C

V. Summary

The mechanical constitutive behavior of SMA materials subjected to external stresses has been modeled based on mixture theory. Two internal variables are independently defined for the stress-induced reorientation of martensite variants and the transformation from austenite to detwinned

martensite variants. This approach enables the prediction of the overall stress-strain behavior of SMA materials. Tensile tests with Ti-Ni specimens have been carried out in a temperature range including martensite transformation temperatures. Numerical simulation based on the proposed model exhibits a sufficient accuracy for predicting the mechanical behavior of SMA.

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