Calculation model of CO₂ emission by using a population balance model

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The quantitative model for CO₂ emission from passenger cars in Japan is proposed. Cars are scrapped from society according to their lifetime distribution. How much the improvement of fuel economy in order to achieve the goal by COP3 is estimated. Consequently, it turns out that it must be improved by 2.42 times of the fuel economy in 1995 by the year of 2010. Substitution scenarios to small sized cars from large sized cars are considered and it effects largely on the reduction of CO₂ emission.

Keywords: CO₂ emission, Passenger car, Population balance model, Lifetime distribution, Fuel economy

I. INTRODUCTION

The amount of CO₂ emission from passenger cars accounts for 8.5% compared to the total emission from Japan in 1994. The amount of CO₂ emission from passenger car in use occupies about 80% of its life cycle, and the CO₂ reduction in use by the improvement of fuel economy has a large effect. Recently, in order to reduce the CO₂ emission in use, high fuel economy cars such as hybrid cars and fuel cell electric cars have been developed. Also, it is desirable to substitute from a large sized cars to a small one with better fuel economy. In this study, the improvement of fuel economy to achieve the target of COP3 (6% reduction of 1990 level in 2010 for Japan), and the effect of substitution to smaller cars from large cars are investigated using a population balance model.

II. ANALYTICAL METHOD

II-A. Macro model for passenger cars in Japan

A schematic diagram of the mass flow of passenger cars in Japan is shown in Fig. 1. About 5 million units are newly registered in a year and about 0.3 million units are imported. The number of cars in use is about 50 million units. In this study, passenger cars are classified into three categories; regular car, compact car and mini car according to their displacement. Only the life cycle of passenger cars is considered in this study.
cars used in Japan is considered and imported cars are assumed to be ones manufactured in Japan. Export of used cars, recycling and reusing are not considered. Society is assumed to have a constant accumulation capacity, $V$.

II-B. Lifetime distribution

The lifetime of a car is determined by its age, $a$. Even among the same kinds of cars, the age until lifetime differs and the age distribution exists. This distribution is defined as a lifetime distribution, $g(a,t)$. Here, the scrapping rate distribution, $h(a,t)$, which determines the number of end of life vehicles (ELVs) to the number of cars in use, should be considered, and it is given by the following equation.

$$h(a,t) = \frac{g(a,t)}{1 - \int_0^t g(x,t)dx}$$  \hspace{1cm} (1)

Note that the lifetime distribution are determined at the time of registration and thus it is given as $g(a,t-a)$. However, the lifetime distribution of the passenger cars do not change so much with time and can be assumed to be determined while in use. Therefore the distribution is given as $g(a,t)$ in the equations.

II-C. Population balance model (PBM)

The ratio of the number of cars in use of age $a$ at time $t$ to $V$ is defined as a car accumulation rate, $n(a,t)$. Then the number of cars in use of age $a$ at time $t$ becomes $Vn(a,t)da$. Because cars are scrapped when its lifetime comes, the number of ELVs of age $a$ at time $t$ is obtained by $h(a,t)Vn(a,t)da$. Because of no creation and no annihilation within a society, the time change of the number of cars in use equals to the difference between the number of newly registered cars, $Q_{in}(t)$ and the number of ELVs. Then, PBM equation is as following.

$$V \frac{\partial n(a,t)}{\partial t} + V \frac{\partial}{\partial a} \left( \frac{d\alpha}{dt} n(a,t) \right) = Q_{in}(t)\delta(0,t)-h(a,t)Vn(a,t)$$  \hspace{1cm} (2)

As $n_{in}(a,t)=\delta(0,t)$ ($\delta(0,t)$ is Dirac’s delta function) and $d\alpha/dt=1$, Eq. (2) becomes as follows.

$$V \frac{\partial n(a,t)}{\partial t} + V \frac{\partial}{\partial a} \left( d\alpha \frac{d\alpha}{dt} n(a,t) \right) = Q_{in}(t)\delta(0,t)-h(a,t)Vn(a,t)$$  \hspace{1cm} (3)

By solving Eq. (3) under the initial condition $n(0,t)=Q_{in}(t)/V$ and $n(a,0)$ given by statistics, the number of cars in use, the number of ELVs and distribution of cars in use to their age are found.

II-D. Calculation of CO$_2$ emission amount

The amounts of the CO$_2$ emission at production and at scrapping are found by multiplying the numbers of newly registered cars and ELVs by inventory factors at production and scrapping, respectively. The amount of the CO$_2$ emission in use, $C_{\text{use}}(t)$, is obtained by the following equation.

$$C_{\text{use}}(t) = \int_0^t \frac{Vn(a,t)m(t)}{f(t-a)}I_{\text{gas}} d\alpha$$  \hspace{1cm} (4)

Where $f(t-a)$ is fuel economy, $m(t)$ is annual average mileage and $I_{\text{gas}}$ is inventory factor of gasoline.

III. STATISTIC AND CALCULATED VALUES

The number of newly registered cars is obtained from statistics (www.zenkeijikyo.or.jp/statistics/index.html). It is assumed to be constant after 1998 for regular cars and compact cars, and after 1999 for mini cars. The number of cars in use classified by age, which is necessary as an initial condition, is obtained from statistics. Because there is no statistics of mini car, it is estimated from the data of compact car. As a value of fuel economy, statistics data are linearized from 1980 to 2000, and after 2001 it is linearized so that it is improved by 21.4% of 1995 in 2010. Fuel economy of cars registered before 1979 equals to that of cars registered in 1980. Annual average mileage is 10,000 km after 1999 for regular cars and compact cars, and is 7,500 km for mini cars. Inventory factors are shown in Table...
1. As the inventory factor for fuel (gasoline), \(7.36 \times 10^{-4} \text{(C-ton/L)}^2\) is used.

Table 1. Inventory factors of regular car, compact car and mini car.

<table>
<thead>
<tr>
<th>Car Type</th>
<th>Production (C-ton/unit)</th>
<th>Scrapping (C-ton/unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular car</td>
<td>1.82</td>
<td>0.0931</td>
</tr>
<tr>
<td>Compact car</td>
<td>1.25</td>
<td>0.0640</td>
</tr>
<tr>
<td>Mini car</td>
<td>0.865</td>
<td>0.0443</td>
</tr>
</tbody>
</table>

III-A. Estimation of lifetime distribution

Statistics of ELVs registered in the same year are arranged by their age and they are normalized as shown with the histograms in Fig.2. The lifetime distributions are estimated based on these histograms. For regular cars and compact cars, they are approximated by two gamma distributions. For mini cars, it is estimated from the distributions of compact cars because there is no statistics. As a result of preliminary analysis using the estimated lifetime distributions, the statistics and calculation results of the number of compact cars in use were not in agreement. Therefore in the analysis, the following lifetime distributions are used. For regular cars, the average lifetime is constant and is 12.5 years. For compact cars, average lifetime varies from 10 years in 1989 to 11 years in 2000. For mini cars, average lifetime varies from 9 years to 10 years.

IV. RESULTS AND DISCUSSION

IV-A. The number of car in use

Calculation results and statistics of the number of cars in use are shown in Fig.3 (www.zenkeijikyo.or.jp/statistics/index.html). Solid curves and solid squares in the figures represent the calculation results and the statistics, respectively. Both statistics and calculation results are in agreement with each type of cars, and this ensures the validity of this model.

IV-B. The amount of \(\text{CO}_2\) emission

The calculation result of a time change of the total \(\text{CO}_2\) emission amount is shown as solid curve in Fig. 4. According to this result, it turns out that the goal of COP3 cannot be attained by the fuel
economy improvement set as the Japanese government target (1.214 times from 1995 in 2010). In order to attain it, 2.42 times improvement in fuel economy from 1995 in 2010 is necessary (broken curve in Fig. 4). For such large improvement in fuel economy of the present cars, the substitution to high fuel economy cars is needed.

The effect of the substitution of large sized cars to small ones is investigated. We analyzed two cases.

The one is the case where all of the newly registered regular cars are replaced to compact cars and the number of newly registered regular car is set to 0 after 2001, and the other is the case where mini cars are substituted for regular cars. Calculation results are shown in Fig. 5. The solid, broken and dotted curves in the figure show the results for the cases of no substitution, substitution of regular cars to compact cars and regular cars to mini cars, respectively. From these results, it turns out that the effect of substitution to small sized cars from large sized cars on the reduction of CO₂ emission is large.

**V. CONCLUSION**

A macro flow model of passenger cars in Japan by using a population balance model is proposed. The comparison between calculation results and statistics ensures the validity of this model and the amount of CO₂ emission is predicted by using the model. In order to achieve the goal of COP3, 2.42 times fuel economy improvement from 1995 in 2010 is necessary. The effect of the substitution to small sized cars from large sized ones on the reduction of CO₂ emission is large. Therefore, not only the improvement of fuel economy but also the substitution to small sized cars is necessary.

**REFERENCES**


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