DC SQUID RF AMPLIFIERS

D C S Q U I D 高周波增幅器

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ABSTRACT

The noise and signal parameters of several types of the RF amplifiers based on the different SQUIDS with integrated and hybrid input coils were studied. A new type of the multiloop dc SQUID with integrated input coil and extremely low stray capacitances was designed. The inductance of a 4-loop SQUID was 100 pH, the input coil inductance 1.3 nH and mutual inductance 300 pH. The noise temperature of untuned SQUID amplifier with hybrid input coil at 100 MHz was 1.2 ± 1 K and the power gain exceeds 20 dB. The tuned integrated 4-loop amplifier at 300 MHz has nearly 20 dB gain in 25 MHz band. For the noise calibration of such amplifiers we used SIS junctions as a shot noise source, or cooled attenuator and a room-temperature semiconductor noise source.

Key words: DC SQUID, RF amplifier, Cryogenic noise

1. SQUID-amplifier signal parameters

The dc SQUID amplifiers (SQA) may be viewed as a magnetic flux controlled device, which amplifies signal at frequencies much lower the Josephson frequency at bias point. According to Ref. 1, the SQA forward transfer impedance $t_{21} = (\partial V / \partial I_i)_{I_i} = M V_\phi$. Taking into account $V_\phi = (\partial V / \partial \Phi_e)_{I_i} \approx r/L$, where r - the SQUID resistance, L - the loop inductance, $M = \alpha(LL_i)^{1/2}$ - the mutual inductance of L and input coil L_i , one can obtain $t_{21} \approx Mr/L$ and voltage gain $K_u = t_{21}/Z_{i1} \approx \alpha^2 r / M\omega$, where $Z_{i1} \approx \omega L_i$. The dc SQUID is nonreciprocal device and $t_{12} = (\partial \Phi_i / \partial I_{ii}) = 0$. The output SQA impedance $t_{22} = (\partial V / \partial I)_{\Phi_i} = r_d$ - the dynamic resistance of SQUID. The input flux impedance $t_{11} = (\partial \Phi_i / \partial I_i)_{I_i} \approx L_i - j\omega M^2 / 2r$.

For approximate estimations if the cur-

rent gain is $K_i \approx ML$ it is possible to obtain a simple expression for the power gain $G = K_u K_i \approx \alpha^2 r / L\omega$. Another method to obtain the similar dependence was used in Ref. 2 from the expression $G = (V_0/V_i)^2 (R_i/R_d)$ in the tuned mode and optimal input resistance $G = M V_\phi / R_i R_d \approx V_\phi / \omega \approx r / L\omega$.

Numerical gain calculations of SQA without stray capacitance in Ref. 9 gives maximum power gain 34.5 dB at 36 GHz and 43 dB at 1.5 GHz. In the same work a simple expression was analytically obtained for the power gain far from the optimal bias current $G = V^2 / \omega^2 \approx (r/\omega L)^2$ which corresponds to $G = 23$ dB at 3 GHz and $G_{1.5} = 29$ dB at 1.5 GHz. This simple expression is correct for a moderate gain $G < 20$ dB, but gives underestimated about 10 times values at the gain maximum. The reason

for such discrepancy is that imaginary part of V_Φ was not accounted. The appearance of imaginary part of V_Φ is probably due to the interplay of the tuned input and output circuits through the SQUID.

As mentioned in Ref. 3 SQA may be viewed as a peculiar type of parametric amplifier in that amplification (by up conversion) and detection (down conversion) take place in the same device. For a parametric up-converter the power gain equals pump to signal frequency ratio. The pump frequency in dc SQUID with dc bias is the Josephson oscillation and the above mentioned expression $G = \alpha^2 r / L\omega$ may be explained as the ratio of frequency R/L limiting the Josephson current in the loop to the signal frequency. In the case of sufficient capacitance in the loop the limiting resonant frequency will be $(LC)^{-1/2}$. From this point of view it is useful for gain increase to reduce both inductance and stray capacitance of SQUID loop and to place the bias current point at the dynamic resistance maximum close to the voltage step corresponding the resonant frequency. If we use the limiting value of Josephson current frequency corresponding to the energy gap of Nb which is ~ 750 GHz then, in principle, it may be possible to achieve ~ 20 dB gain for the signal frequency ~ 10 GHz.

2. The noise temperature of SQA

For Johnson noise source in SQUID with spectral density $S_v(f) = 4\gamma_v kT r$, $S_i(f) = 4\gamma_i kT / r$, $S_{vi} = 4\gamma_{vi} kT$ where k - Boltzman constant, T - physical temperature, γ - constants and not too high frequencies, according to Ref. 1 the noise temperature of SQA $T_N \approx T\omega(\gamma_v \gamma_i)^{1/2} / V_\Phi$. Taking into account $\gamma_v = 8$, $\gamma_i = 5.5$, $\gamma_{vi} = 6$,⁴ one can obtain $T_N = 6.5T\omega L / r\alpha^2$, i.e. for $\omega = 10^9$, $L = 10^{-10}$ H one can obtain $T_N = 0.04$ T and $R_i^{opt} = \alpha^2 \omega L_i (\gamma_i / \gamma_v - \gamma_{vi}^2 / \gamma_v^2)^{1/2} \approx 0.3\alpha^2 \omega L_i$. It should be mentioned that in real SQUIDS the values of γ_v , γ_i , γ_{vi} and T_N may be 2-3 times higher.

The gain and noise mechanisms of SQA

were studied in Ref. 5 in the case of SQUID and input coil strong coupling and taking into account SQUID parameters mutual influence. The equations are solved for the Josephson and the signal frequency components. For the reduced SQUID inductance $L_T = (1 - \alpha^2)L$ reduced by the input circuit shielding the voltage amplitude at signal frequency is

$$V(\omega) = V_n^r(\omega) + M V_\Phi^r \frac{V_i(\omega) + M J_n^2(\omega) [R_i + 1/j\omega C_i] / L_i}{Z_\gamma(\omega) - J_\Phi^2 M^2 [R_i + 1/j\omega C_i] / L_i},$$

where L_T - total input circuit inductance and superscript r in this expression means that this value corresponds to the reduced SQUID inductance, J_n - noise current. This formula leads to the simple expression $K_u = M V_\Phi^2 / Z_T(\omega)$ and the total output noise

$$V_n = V_n^r(\omega) + M_i^2 V_\Phi^r J_n^2(\omega) [R_i + 1/j\omega C_i] / L_T Z_T(\omega)$$

where $Z_T = R_i + 1/j\omega C_i + j\omega L_T - J_\Phi^2 M^2 [R_i + 1/j\omega C_i] / L_T$,

$V_\Phi^r = \partial V / \partial \Phi$, $J_\Phi^r = \partial J / \partial \Phi$, $V_n^r(\omega)$ and $J_n^r(\omega)$ - voltage and current noise in the reduced SQUID.

In the theoretical work⁶ it was shown that the noise characteristics of the single broadband SQA is approximately an order of magnitude worse compared to an array of narrow band SQA connected in parallel and covering the same frequency band.

In contradiction with the above mentioned works is the conclusion of Ref. 7 that for the untuned SQA $T_N^{eff} = \pi f (S_v S_i - S_{vi}^2)^{1/2} / K V_\Phi$ and in the tuned mode at the resonant frequency of the input circuit the noise increases to the value $T_N^{res} = \pi f (S_v S_i)^{1/2} / K V_\Phi$.

The quantum limit of SQA at $T = 0$, according to Ref. 8, may be calculated by replacing the Johnson noise by zero fluctuations in the expression for the tuned SQA noise. The obtained value $T_N \approx hf / k \ln 2$ is just the same as for any amplifier in the quantum limit.

3. Planar integrated and hybrid single-loop dc SQA studies

At the beginning of our studies of SQA we tried to use three types of SQA, the first one was similar to Ref. 9, the second one - to Ref. 10 and the third - to Ref. 11. The first and the second contain shunted tunnel junctions and the third has the junctions with semiconductor interlayer. We studied SQUIDS with junctions situated both inside and outside the square inductive loop and used both integrated and hybrid input spiral coils. For preliminary gain and noise measurements a cooled attenuator and a room temperature signal and noise sources were used.

The output noise and voltage gain gave some interesting features. Fig. 1 shows IV curve (1), noise (2) and amplified signal (3, 4, 5) for three levels of input signal. The important feature of the IV curve is the presence of current steps at voltages corresponding to resonant frequencies of SQUID. If the bias point is placed at the maximum of dynamic resistance nearby one of these steps the amplifier gain and noise temperature may be not worth than for bias point nearby the Josephson critical current. It is possible to explain this fact, according to Ref. 12, by the presence of broad band Josephson oscillations which give rise to the noise at signal fre-

Fig. 1. For SQUID with clamp input coil, (1) - dc SQUID IV curve, (2) - output noise in 1-1000 MHz band, (3, 4, 5) - amplified signal at 100 MHz for three input signal levels differing by 10 dB versus bias voltage.

quency for the bias voltage nearby the voltage step. This noise will be sufficiently lower for the bias voltage $v > 100 \mu\text{V}$.

As it was mentioned above the efficient method to rise the gain and reduce the noise is to locate the bias point near the resonant step where the dynamic resistance exceeds the normal one and the Josephson frequency exceeds the values nearby the critical current. The amplified signal maxima (Fig. 1) at voltages far from critical current confirms this conclusion. The lower output signal amplitudes at far from I_c maxima may be explained by the impedance mismatch at the signal output. The best results for hybrid SQA with Cu spiral wire-wind coil were $G > 20 \text{ dB}$, $T_N = 1.2 \pm 1 \text{ K}$ at 100 MHz .

4. Cryogenic noise source

One of the problems of the low noise amplifier parameters measurements is the noise calibration. To measure the noise temperature Hilbert and Clarke² connected the input coil of SQUID amplifier to 50Ω resistor which was enclosed in a vacuum can. The temperature of this resistor could be regulated by means of a heater and controlled by a thermometer. In our measurements we used as the noise source a tunnel SIS junction with the bias point placed higher than the gap voltage. The noise voltage in this case, according to Ref. 13, $U_N = (4kTR + 2eIR^2)\Delta f$ is the accurate value. Between the SIS noise source and the SQUID amplifier a filter-attenuator was placed. The noise temperature at the amplifier input may be varied between 1 and 20 K. The advantages of SIS noise generator in comparison with variable temperature load noise generator are full compatibility with SQUID amplifier, the possibility of noise modulation, wide noise temperature range, small sizes.

5. Four-loop dc SQUID amplifier studies

The main disadvantages of the previous studied SQA are relatively low amplified fre-

quency and a high influence of external magnetic fields on SQA parameters. For the signal frequency increase with preservation of the low noise temperature and high gain the Josephson frequency in the SQUID loop should be increased and it means that the loop inductance and capacitance should be decreased. In the common integrated SQUID structure⁸ the stray capacitance in the loop exceeds 10 pF . The capacitance between the evaporated above the loop input coil and the coil is even more and this capacitance leads to the significant stray input-output feedback.

To eliminate these disadvantages and increase the input signal frequency and band we designed a four-loop dc SQUID with an integrated input coil in the form of rectangular turns inside the loops (see Fig. 2).

The SQUID loop inductance consists of the four parallel connected square loops of $200 \times 200 \mu\text{m}^2$ size. The input coil consists of four series connected square turns with Nb film widths $10 \mu\text{m}$. Parallel connection of the loops allows to reduce inductance of the loop and increase the resonant frequency. Series connection of the input turns allows to increase the input coil inductance and make easier the impedance matching with the input 50Ω line. The position of input turns inside the loops was chosen to reduce stray input-output capacitance.

The SQUID loop inductance in this construction is 100 pH , input coil - 1.3 nH , mutual inductance 300 pH . The stray capacitance in SQUID loop 1.8 pF , the sum junctions capacitance 0.8 pF , input coil capacitance 0.3 pH , loop-coil capacitance 2.2 pF . The Nb-AlO_x-Nb shunted tunnel junctions of $2.5 \times 2.5 \mu\text{m}^2$ area were used as Josephson junctions. The input coil resonant frequency is estimated to be 8 GHz and the loop resonant frequency 10 GHz .

Since the input coil inductance is rather small and its reactive impedance at 300 MHz is $\sim 2.5 \Omega$, then to match the input 50Ω line

Fig. 2. Four-loop planar dc SQUID with integrated input coil. (1, 3) and (2, 4) - output and input coplanar lines. (5) - parallel connected inductive SQUID loops each $200 \times 200 \mu\text{m}^2$, inside each of them single turns are arranged and connected in series to form an input coil.

and the input coil we used the resonant circuit matching.¹⁴ In the design of the matching circuit the sufficient element is series additional inductance L_s , which was in the range 5–15 nH and depends on the size of connecting leads. Scaling the input circuit elements to the input resonant circuit one may obtain the equivalent capacitance $C_r \approx$

$C_1 + C_0$ and resistance $R_r \approx R(C_1/C_0)^2$. According to Ref. 2 the optimal Q-factor is $Q \approx (1 + L_s/L_i)/\alpha^2$ which in our case gives $Q_{\text{opt}} \approx 10$ and taking into account $Q \approx \omega L_s/R_r$ one could obtain $C_1 \approx 0.2C_0$.

For Fig. 3 matching circuit parameters the dependence of the amplified noise signal on frequency is shown in Fig. 4. In the reso-

Fig. 3. SQUID amplifier connections including cooled 20 dB attenuator and resonant matching circuit. The part inside the dashed line is placed in liquid helium.

Fig. 4. Integrated 4-loop dc SQUID amplifier output signal spectral density with the matching circuit shown at Fig. 3. The input signal is white noise.

nant mode at 300 MHz the gain was $G \approx 20$ dB and the noise temperature was not worse than 4 K in 25 MHz band.

To confirm the assumption about the frequency dependence of the gain and the noise we measured the spectral density of the voltage noise at SQUID output (Fig. 5). The dependence is of $1/f$ type and corresponds to the mentioned estimations.

6. Discussion

The review of theoretical works in sections 1.1 and 1.2 shows that at present there is the lack of common point of view on asymptotic parameters of SQA. The mentioned expressions were obtained from different models and differs by order of magnitude. Nevertheless even the most pessimistic estimations give the noise temperature values several times lower than the physical temperature at 10^2 MHz band.

Obtained in the multiloop SQA gain at

frequency 300 MHz which is higher than the single -loop signal frequency 100 MHz and the presented noise dependence confirms the conclusion that to improve the SQA parameters the loop inductance and capacitance should be lowered.

The nearest competitor of SQA is cooled HEMT amplifiers which have $T_N \sim (1-3)$ K at $f \sim (1-3)$ GHz, but have rather high power dissipation of tens milliwatts and nearly achieved their asymptotic parameters. On the other hand SQA has more than one order reserve in the margins of gain, noise temperature and signal frequency even on the basis of the present technology. The sufficient advantages of SQA are extremely low power dissipation of several picowatt, small size $\sim 1 \text{ mm}^2$ and full electrical and temperature compatibility with the superconducting sensitive devices such as Josephson and SIS mixers.

The SQA has good prospects in using

Fig. 5. Internal SQUID output noise frequency dependence. The solid line oscillation is connected with the SQUID output and 50 Ω cable mismatch.

high T_c superconductors. According to the above estimations at 77 K HTSC SQA should have noise temperature lower 10 K at $f \sim 10^2$ MHz. In contrast to the SQUID magnetometer the SQA does not need superconductivity in the input coil and HTSC SQA may be realized on the basis of the present thin HTSC films and Josephson tunnel junctions technologies.

Concerning HTSC SQA asymptotic margins they are even better than of low-temperature SQA because the energy gap voltage of YBaCuO $V_\Delta \approx 25$ mV and the Nb - only 2.8 mV, so at the same temperatures the gain may be nearly an order higher and noise temperature lower in HTSC SQA. If we compare HTSC SQA at 77 K and LTSC SQA at 4.2 K the noise temperature ratio will be $T_L^N/T_H^N = (T_L/T_H) \cdot (\Delta_H/\Delta_L) \approx 0.5$ which means that the noise temperature of HTSC SQA at 77 K may be only twice higher LTSC SQA noise temperature at 4.2 K. However realization of such values will be possible only when submicrometer HTSC structures will be available.

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