Selection using an Auxiliary Measurement¹⁾

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Selection is one of the most important tools to change the genetic composition of the population concerned so as to increase the proportion of more desirable genotype in it. Hence, the method of selection has been investigated by many workers (LUSH 1945, LE ROY 1960).

Recently, selection for the first character (X_1) taking account of the information of the second character (X_2) was investigated eminently by RENDEL (1954), OSBORNE (1957) and PURSER (1960). They considered the second character as the correction (or auxiliary) component of the first character to be improved and tried to increase the genetic gain in the first character by the use of such selection index as $I = X_1 - bX_2$.

Present study was carried out to confirm theoretically the value of the index selection mentioned above.

Theory

For the sake of computation, only additive genetic variance and a constant linear genetic correlation between two characters were assumed.

A method of increasing the genetic gain for a single character (X_1) by means of correction for linear phenotypic regression on a second character (X_2) was discussed by OSBORNE (1957). According to him, the ratio (E) of the genetic gain ($\Delta G(X_1/I)$) in the first character, when selection was based on the selection index written as $I = X_1 - bX_2$, to the gain ($\Delta G(X_1)$) by the direct selection for the first character is

$$E = \frac{\Delta G(X_1/I)}{\Delta G(X_1)} = \frac{R_{G_1 I}}{h_1} = \frac{1 - r_P r_G h_2/h_1}{\sqrt{1 - r_P^2}}$$

where r_P , r_G and h_1^2 stand for phenotypic, genetic correlation and heritability of the first character respectively.

Thus the condition for the superiority of the selection index to direct selection ($E > 1$) is given from

$$r_P^2(1 + r_G^2 h_2^2/h_1^2) > 2r_P r_G h_2/h_1.$$

If $r_P > 0$, the condition is $r_P > 2k/(1 + k^2)$ and if $r_P < 0$, the condition $r_P < 2k/(1 + k^2)$ where $k = r_G h_2/h_1$. The superiority by the use of the selection index is graphically

Fig. 1. Diagram showing the conditions for superior index selection using $I = X_1 - bX_2$ to direct selection where X_1 , X_2 and b stand for the trait to be improved, auxiliary trait and phenotypic regression of X_1 on X_2 . Shaded area shows the condition for superiority of the index

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shown in Fig. 1. When the absolute value of k is nearly unit, the index selection is by no means superior to the direct selection. With increasing k , the area showing superior index selection becomes wider.

Following the OSBORNE's work, PURSER (1960) gave a general solution for this problem. He defined b in the index as a constant and showed the ratio mentioned above as

$$E = \frac{\Delta G(X_1/I)}{\Delta G(X_1)} = \left[1 - br_P \frac{\sigma_{G_2}}{\sigma_{G_1}} \right] / \sqrt{\left[1 - 2br_P \frac{\sigma_2}{\sigma_1} + b^2 \frac{\sigma_2^2}{\sigma_1^2} \right]}$$

where σ^2 and σ_G^2 represent phenotypic and genetic variance respectively, subscript refers to the character concerned. And he reported: "for $E > 1$ we find b lies between limits $0 < b < A$ where

$$A = 2 \left[r_P \frac{\sigma_1}{\sigma_2} - r_G \frac{\sigma_1^2}{\sigma_2^2} \frac{\sigma_{G_2}}{\sigma_{G_1}} \right] / \left[1 - r_G^2 \frac{\sigma_1^2}{\sigma_2^2} \frac{\sigma_{G_2}^2}{\sigma_{G_1}^2} \right].$$

However, the condition for $E > 1$ will be given strictly as follows: The value of b can be positive or negative. If $b > 0$, the conditions for $0 < b < A$ are 1) $h_1^2/h_2^2 > r_G^2$ 2) phenotypic regression of X_2 on $X_1 >$ genotypic regression of X_2 on X_1 .

Nextly, the use of the best selection index written as $I = b_1 X_M + b_2 X_L$ where X_L and X_M stand for the character to be improved and an auxiliary character respectively, will be examined. The best index will be one for which genotype of the character to be improved and I are maximally correlated. The problem is to find the weighting factors b_1 and b_2 . We assume here such situation that positive weight is given to the measurement of the character to be improved (X_L). In such case, negative attention should be paid for the auxiliary character (X_M) if the condition for $E > 1$ is provided by $b > 0$ in PURSER's work. If the condition for $E > 1$ is given by $b < 0$ in his work, b_1 should be positive as a weighting factor of the auxiliary character (X_M). The value of b_1 is given to be

$$b_1 = \frac{\sigma_{G_M G_L} \sigma_L^2 - \sigma_{G_L}^2 \sigma_{M L}}{(1 - r_P^2) \sigma_M^2 \sigma_L^2},$$

denominator being always positive. Thus the condition for $b_1 < 0$ is $r_G < (h_L/h_M) r_P$ when phenotypic and genotypic correlation are positive. Fig. 2 shows the condition of superior index selection ($I = b_1 X_M + b_2 X_L$) to direct selection. The condition giving $b_1 < 0$ represents the superior index selection when negative attention to an auxiliary character brings about superiority of the other form of index selection ($I = X_1 - b X_2$). As clearly shown, the condition of superior index selection becomes limited when the ratio ($K = h_L^2/h_M^2$) gets smaller.

Fig. 2. Diagram showing the conditions for a case of superior index selection using $I = b_1 X_M + b_2 X_L$, where K represents h_L^2/h_M^2 .

Discussion

Productivity of animals has been analysed from the standpoint of physiology and biochemistry. The trends of recently developed investigation are based on the implication

that a productivity will be improved genetically through the selection for the constitutive character of the productivity to be improved. As the first step to improve dairy performance, NAITO *et al.* (1955) and NAGAI (1960) carried out the investigation on the sensitivity of mammary gland to hormones concerned and the amount of hormones as the constitutive characters of dairy performance. KUSHNER (1941) studied on the relationship between the biochemical constituents of blood in larger animals and their productivity, suggesting possibilities of identifying desirable genotype based on the simple index. In poultry, the investigation of the physiological basis of the ovulatory cycle has not yet been attacked from the genetic standpoint, but the prospect of a profitable combination of endocrinology and biochemistry with physiological and statistical genetics is clear (LERNER 1958).

In such a progressive situation, however, preliminary comparison of efficiency of the indirect selection with that of direct selection for productivity is needed. LERNER (1958) is referable to this point and here the condition for superior indirect selection to direct selection is diagrammed as shown in Fig. 3. It is clear that the condition for superior indirect selection becomes strict when the heritability (h^2_M) of the constitutional character becomes low and that the use of indirect selection is limited only when genetic correlation between two characters is high to some extent.

On the other hand, the selection index method described before can be used in some cases when an indirect selection method is not preferable in con-

sideration of the expected genetic gain of the character to be improved. For example, PURSER (1960) computed the expected gain in body weight by the use of cannon length as a correction component based on the data of the Scottish Blackface sheep where heritability of body weight and cannon length were 0.25 and 0.15 respectively, and genetic and phenotypic correlation being 0.23 and 0.73. In this case, indirect selection is not effective as clearly shown in Fig. 3 and the use of selection index is advisable. In practice, furthermore, it is necessary to compute the extent of superiority by the superior index selection.

It is interesting to examine the merit of the selection index to improve one character using the several measurements of other characters.

Summary

The condition for superior index selection using $I = X_1 - bX_2$ to direct selection was theoretically confirmed. When 1) $h_1^2/h_2^2 > r^2_G$ and 2) phenotypic regression of X_2 on X_1 > genotypic regression of X_2 on X_1 are satisfied, a constant b lies between limits $0 < b < A$ where

$$A = 2 \left[r_P \frac{\sigma_1}{\sigma_2} - r_G \frac{\sigma_1^2}{\sigma_2^2} \frac{\sigma_{G_2}}{\sigma_{G_1}} \right] / \left[1 - r_G^2 \frac{\sigma_1^2}{\sigma_2^2} \frac{\sigma_{G_2}^2}{\sigma_{G_1}^2} \right]$$

Fig. 3. Shaded area showing the condition ($h_M r_G > h_L$) for superior indirect selection to direct selection where h_L^2 , h_M^2 and r_G stand for the trait to be improved, auxiliary trait and genetic correlation

as given by PURSER (1960).

Further, in those conditions above, the conditions for the superior index selection using $I = b_1 X_M + b_2 X_L$ to direct selection were examined.

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選抜に際しての補助的形質

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選抜指数 $I = X_1 - bX_2$ (X_1 : 改良しようとする形質, X_2 : 補助的形質, b : 恒数) を用いた選抜が, 改良しようとする形質 (X_1) に対する直接的選抜にまさる条件を理論的にしらべた. その結果 (1) $h_1^2/h_2^2 > r_G^2$ (2) X_2 の X_1 に対する表現型回帰 $> X_2$ の X_1 に対する遺伝子型回帰の場合には, 恒数 b は Purser (1960) による

如く $0 < b < A$ となり

$$A = 2 \left[r_P \frac{\sigma_1}{\sigma_2} - r_G \frac{\sigma_1^2}{\sigma_2^2} \frac{\sigma_{G_2}}{\sigma_{G_1}} \right] / \left[1 - r_G \frac{\sigma_1^2}{\sigma_2^2} \frac{\sigma_{G_2}^2}{\sigma_{G_1}^2} \right]$$

であつた.

更に, 上記の条件のとき, 最良の選抜指数 $I = b_1 X_M + b_2 X_L$ が直接選抜にまさる条件をしらべた.