RECOMBINATION EFFECT ON HOMOZYGOSITY BY DESCENT IN FULL-SIBBING

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Synopsis. Assuming a pedigree of full-sibbing, recombination effect on homozygosity by descent is studied numerically. Taking into consideration two linked loci, formulae for generalized inbreeding coefficient and coefficient of parentage are shown. Under several initial conditions, resulting degrees of homozygosity by descent are calculated. Considerable effect of recombination is observed on homozygosity by descent in many cases.

Introduction

Degrees of homozygosity by descent is expressed by inbreeding coefficient defined by MALÉCOT (1948) for a single locus. When more than one loci are linked on chromosomes, the quantity needs to be generalized. This generalization have been tried by SCHNELL (1961) and SHIKATA (1962), independently. Using the generalization, some regular systems of mating have been studied analytically (SHIKATA, 1965 and 1966). However, the last two works have contained a mistake in formulation and at the same time no quantitative study have been made. In the following sections, revised formulae for generalized inbreeding coefficient of parentage will be presented for full-sibbing, and their behaviors will be studied numerically.

Formulae of Generalized Inbreeding Coefficient and Coefficient of Parentage

Consider the pedigree in Fig. 1. I's are individuals, F's are generalized inbreeding coefficients, and R's and S's are generalized coefficients of parentage. We assume that I_M and I_F are not related. This is not a necessary requirement for deriving formulae for the generalized quantities. However, for the sake of simplicity, the assumption is convenient. As a consequence of the assumption,

$$F_{1,1}(00) = F_{1,2}(00) = 1. \quad (1)$$

Also,

$$R_M = m_0 F_M, \quad (2)$$

$$R_F = m_0 F_F, \quad (3)$$

Fig. 1. Pedigree of full-sibbing.

$$S_{1,1} = m_0 F_{1,1}, \quad (4)$$

$$S_{1,2} = m_0 F_{1,2}, \quad (5)$$

$$R_1(11) = p_1^2(R_M(11) + R_F(11)) + \frac{1}{4}p_2(R_M(1.)R_F(.1) + R_F(1.)R_M(.1)), \quad (6)$$

$$R_1(10) = p_1^2(R_M(10) + R_F(10)) + p_1p_2(R_M(1.) + R_F(1.)) + \frac{1}{4}p_2(R_M(1.)(1 + R_F(.0)) + R_F(1.)(1 + R_M(.0))), \quad (7)$$

$$R_1(01) = p_1^2(R_M(01) + R_F(01)) + p_1p_2(R_M(.1) + R_F(.1)) + \frac{1}{4}p_2((R_M(0.) + 1)R_F(.1) + (1 + R_F(0.))R_M(.1)), \quad (8)$$

and

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Fig. 2. $F_2(11)$ versus p_2 in log-log scale. Fig. 3. $F_2(10)$ versus p_2 in log-log scale.Table 1. Values of components of initial F

1	1.0000-00	0.0000-40	0.0000-40	0.0000-40	2.0000-00	0.0000-40
2	7.5000-01	2.5000-01	0.0000-40	0.0000-40	1.7500-00	1.8750-01
3	7.5000-01	0.0000-40	0.0000-40	2.5000-01	1.5000-00	7.5000-01
4	5.0000-01	5.0000-01	0.0000-40	0.0000-40	1.5000-00	2.5000-01
5	5.0000-01	0.0000-40	0.0000-40	5.0000-01	1.0000-00	1.0000-00
6	2.5000-01	7.5000-01	0.0000-40	0.0000-40	1.2500-00	1.8750-01
7	2.5000-01	0.0000-40	0.0000-40	7.5000-01	5.0000-01	7.5000-01
8	0.0000-40	1.0000-00	0.0000-40	0.0000-40	1.0000-00	0.0000-40
9	0.0000-40	0.0000-40	0.0000-40	1.0000-00	0.0000-40	0.0000-40

$$R_1(00) = p_1^2(R_M(00) + R_F(00) + 2) + p_1 p_2(R_M(0.) + R_M(.0) + R_F(.0) + R_F(0.)) + \frac{1}{2} p_2(R_M(0.) + 1)(1 + R_F(.0)) + (1 + R_F(0.))(R_M(.0) + 1). \quad (9)$$

For $j=1, 2, 3, \dots$, one obtains that

$$F_{j+1} = R_j, \quad (10)$$

$$S_{j+1} = m_0 F_{j+1}, \quad (11)$$

$$R_{j+1}(11) = 2p_1^2(S_j(11) + R_j(11)) + 2p_1 p_2(S_j(1.)R_j(.1) + R_j(1.)S_j(.1)) + \frac{1}{2} p_2(S_j(1.) + R_j(1.))(S_j(.1) + R_j(.1)), \quad (12)$$

$$R_{j+1}(10) = 2p_1^2(S_j(10) + R_j(10)) + 2p_1 p_2(S_j(1.)R_j(.0) + R_j(1.)S_j(.0)) + \frac{1}{2} p_2(S_j(1.) + R_j(1.))(S_j(.0) + R_j(.0)), \quad (13)$$

$$R_{j+1}(01) = 2p_1^2(S_j(01) + R_j(01)) + 2p_1 p_2(S_j(0.)R_j(.1) + R_j(0.)S_j(.1)) + \frac{1}{2} p_2(S_j(0.) + R_j(0.))(S_j(.1) + R_j(.1)), \quad (14)$$

and

$$R_{j+1}(00) = 2p_1^2(S_j(00) + R_j(00)) + 2p_1 p_2(S_j(0.)R_j(.0) + R_j(0.)S_j(.0)) + \frac{1}{2} p_2(S_j(0.) + R_j(0.))(S_j(.0) + R_j(.0)). \quad (15)$$

Here, $2p_2$ is the recombination probability and $2p_1 (=1-2p_2)$ is the probability of no recombination.

Weighted mean M of F is given by

$$M = 2F(11) + F(10) + F(01). \quad (16)$$

Variance V of F is

$$V = F(11)(2-M)^2 + (F(10) + F(01))(1-M)^2 + F(00)(0-M)^2. \quad (17)$$

For details of the derivation of the above formulae, SHIKATA (1968) may be referred.

Numerical Computations

By using the formulae in the preceding section, numerical computations are carried out. As the initial conditions, nine cases of F have been used, which are numbered and tabulated in Table 1. The columns in the table are, from the left to the right, $F(11)$, $F(10)$, $F(01)$, $F(00)$, M , and V . These values of F are to be used as initial values of F_M or F_F .

Seven values of recombination probability have been chosen, *i. e.*, $p_2 = 0.01, 0.05, 0.10, 0.15, 0.25, 0.40$, and 0.50 . Table 2 is a result of the computation. The table contains many small blocks

Table 2. Values of components of F_j , M , and V , for $j=2, 3, \dots, 10$ and $p_2=0.01$

0.010

1 1					
4.8520-01	1.4800-02	1.4800-02	4.8520-01	1.0000-00	9.7040-01
4.7358-01	2.6419-02	2.6419-02	4.7358-01	1.0000-00	9.4716-01
5.9461-01	3.0394-02	3.0394-02	3.4461-01	1.2500-00	8.7671-01
6.5352-01	3.3980-02	3.3980-02	2.7852-01	1.3750-00	7.9141-01
7.1564-01	3.4360-02	3.4360-02	2.1564-01	1.5000-00	6.8128-01
7.6314-01	3.3733-02	3.3733-02	1.6939-01	1.5937-00	5.8000-01
8.0400-01	3.1939-02	3.1939-02	1.3212-01	1.6719-00	4.8471-01
8.3758-01	2.9610-02	2.9610-02	1.0320-01	1.7344-00	4.0147-01
8.6564-01	2.6941-02	2.6941-02	8.0481-02	1.7852-00	3.2965-01
1 2					
4.5487-01	4.5125-02	1.3875-02	4.8612-01	9.6875-01	9.4002-01
4.5871-01	4.1290-02	2.5665-02	4.7433-01	9.8437-01	9.3280-01
5.7979-01	4.5212-02	2.9587-02	3.4541-01	1.2344-00	8.7027-01
6.4250-01	4.4997-02	3.3278-02	2.7922-01	1.3633-00	7.8975-01
7.0649-01	4.3508-02	3.3742-02	2.1626-01	1.4902-00	6.8242-01
7.5585-01	4.1026-02	3.3213-02	1.6991-01	1.5859-00	5.8244-01
7.9808-01	3.7894-02	3.1507-02	1.3256-01	1.6655-00	4.8771-01
8.3280-01	3.4384-02	2.9257-02	1.0356-01	1.7292-00	4.0456-01
8.6177-01	3.0804-02	2.6654-02	8.0768-02	1.7810-00	3.3257-01
1 3					
4.5339-01	1.5364-02	1.5364-02	5.1589-01	9.3750-01	9.6537-01
4.5769-01	2.6685-02	2.6685-02	4.8894-01	9.6875-01	9.4565-01
5.7828-01	3.1091-02	3.1091-02	3.5953-01	1.2187-00	8.8997-01
6.4108-01	3.4702-02	3.4702-02	2.8952-01	1.3516-00	8.0700-01
7.0503-01	3.5209-02	3.5209-02	2.2456-01	1.4805-00	6.9873-01
7.5446-01	3.4602-02	3.4602-02	1.7634-01	1.5781-00	5.9657-01
7.9678-01	3.2813-02	3.2813-02	1.3760-01	1.6592-00	4.9986-01
8.3161-01	3.0454-02	3.0454-02	1.0749-01	1.7241-00	4.1474-01
8.6069-01	2.7737-02	2.7737-02	8.3836-02	1.7769-00	3.4102-01
1 4					
4.2455-01	7.5450-02	1.2950-02	4.8705-01	9.3750-01	9.0769-01
4.4384-01	5.6162-02	2.4912-02	4.7509-01	9.6875-01	9.1795-01
5.6497-01	6.0029-02	2.8779-02	3.4622-01	1.2187-00	8.6334-01
6.3149-01	5.6014-02	3.2577-02	2.7992-01	1.3516-00	7.8781-01
6.9734-01	5.2696-02	3.3125-02	2.1688-01	1.4805-00	6.8337-01
7.4856-01	4.8319-02	3.2694-02	1.7043-01	1.5781-00	5.8476-01
7.9217-01	4.3770-02	3.1075-02	1.3299-01	1.6592-00	4.9064-01
8.2803-01	3.9157-02	2.8903-02	1.0391-01	1.7241-00	4.0759-01
8.5791-01	3.4668-02	2.6367-02	8.1055-02	1.7769-00	3.3546-01
1 5					
4.2157-01	1.5928-02	1.5928-02	5.4657-01	8.7500-01	9.5252-01
4.4181-01	2.6941-02	2.6941-02	5.0431-01	9.3750-01	9.4221-01
5.6199-01	3.1764-02	3.1764-02	3.7449-01	1.1875-00	9.0132-01
6.2866-01	3.5399-02	3.5399-02	3.0054-01	1.3281-00	8.2154-01
6.9444-01	3.6033-02	3.6033-02	2.3350-01	1.4609-00	7.1547-01
7.4580-01	3.5449-02	3.5449-02	1.8330-01	1.5625-00	6.1270-01
7.8957-01	3.3669-02	3.3669-02	1.4309-01	1.6465-00	5.1472-01
8.2565-01	3.1281-02	3.1281-02	1.1179-01	1.7139-00	4.2783-01
8.5576-01	2.8519-02	2.8519-02	8.7204-02	1.7686-00	3.5229-01
1 6					
3.9422-01	1.0577-01	1.2025-02	4.8797-01	9.0625-01	8.7341-01
4.2897-01	7.1033-02	2.4158-02	4.7584-01	9.5312-01	9.0261-01
5.5015-01	7.4847-02	2.7972-02	3.4703-01	1.2031-00	8.5592-01
6.2047-01	6.7031-02	3.1875-02	2.8063-01	1.3398-00	7.8560-01
6.8820-01	6.1804-02	3.2507-02	2.1749-01	1.4707-00	6.8413-01
7.4126-01	5.5612-02	3.2174-02	1.7095-01	1.5703-00	5.8696-01
7.8625-01	4.9686-02	3.0643-02	1.3342-01	1.6528-00	4.9348-01
8.2326-01	4.3931-02	2.8550-02	1.0426-01	1.7190-00	4.1057-01
8.5405-01	3.8532-02	2.6081-02	8.1341-02	1.7727-00	3.3831-01
1 7					
3.8976-01	1.6492-02	1.6492-02	5.7726-01	8.1250-01	9.3186-01
4.2594-01	2.7187-02	2.7187-02	5.1969-01	9.0625-01	9.3684-01
5.4571-01	3.2413-02	3.2413-02	3.8946-01	1.1562-00	9.1076-01
6.1627-01	3.6072-02	3.6072-02	3.1158-01	1.3047-00	8.3502-01
6.8387-01	3.6833-02	3.6833-02	2.4246-01	1.4414-00	7.3150-01
7.3716-01	3.6274-02	3.6274-02	1.9029-01	1.5469-00	6.2838-01
7.8239-01	3.4505-02	3.4505-02	1.4860-01	1.6338-00	5.2930-01
8.1971-01	3.2092-02	3.2092-02	1.1610-01	1.7036-00	4.4074-01
8.5084-01	2.9288-02	2.9288-02	9.0585-02	1.7603-00	3.6344-01
1 8					
3.6390-01	1.3610-01	1.1100-02	4.8890-01	8.7500-01	8.3717-01
4.1410-01	8.5905-02	2.3405-02	4.7660-01	9.3750-01	8.8678-01
5.3534-01	8.9665-02	2.7165-02	3.4784-01	1.1875-00	8.4801-01
6.0945-01	7.8048-02	3.1173-02	2.8133-01	1.3281-00	7.8311-01
6.7905-01	7.0952-02	3.1890-02	2.1811-01	1.4609-00	6.8469-01
7.3397-01	6.2905-02	3.1655-02	1.7147-01	1.5625-00	5.8903-01
7.8034-01	5.5601-02	3.0211-02	1.3385-01	1.6465-00	4.9625-01
8.1848-01	4.8704-02	2.8196-02	1.0462-01	1.7139-00	4.1349-01
8.5018-01	4.2396-02	2.5794-02	8.1628-02	1.7686-00	3.4113-01

1 9					
3.5794-01	1.7056-02	1.7056-02	6.0794-01	7.5000-01	9.0339-01
4.1008-01	2.7424-02	2.7424-02	5.3508-01	8.7500-01	9.2953-01
5.2946-01	3.3038-02	3.3038-02	4.0446-01	1.1250-00	9.1830-01
6.0390-01	3.6722-02	3.6722-02	3.2265-01	1.2812-00	8.4745-01
6.7333-01	3.7608-02	3.7608-02	2.5145-01	1.4219-00	7.4681-01
7.2855-01	3.7078-02	3.7078-02	1.9730-01	1.5312-00	6.4362-01
7.7523-01	3.5322-02	3.5322-02	1.5413-01	1.6211-00	5.4360-01
8.1379-01	3.2887-02	3.2887-02	1.2043-01	1.6934-00	4.5348-01
8.4593-01	3.0043-02	3.0043-02	9.3981-02	1.7520-00	3.7448-01
2 2					
4.2455-01	7.5450-02	1.2950-02	4.8705-01	9.3750-01	9.0769-01
4.4384-01	5.6162-02	2.4912-02	4.7509-01	9.6875-01	9.1795-01
5.6497-01	6.0029-02	2.8779-02	3.4622-01	1.2187-00	8.6334-01
6.3149-01	5.6014-02	3.2577-02	2.7992-01	1.3516-00	7.8781-01
6.9734-01	5.2656-02	3.3125-02	2.1688-01	1.4805-00	6.8337-01
7.4856-01	4.8319-02	3.2694-02	1.7043-01	1.5781-00	5.8476-01
7.9217-01	4.3710-02	3.1075-02	1.3299-01	1.6592-00	4.9064-01
8.2803-01	3.9157-02	2.8903-02	1.0391-01	1.7241-00	4.0759-01
8.5791-01	3.4668-02	2.6367-02	8.1055-02	1.7769-00	3.3546-01
2 3					
4.2310-01	4.5650-02	1.4400-02	5.1685-01	9.0625-01	9.3116-01
4.4284-01	4.1533-02	2.5908-02	4.8972-01	9.5312-01	9.3036-01
5.6350-01	4.5819-02	3.0254-02	3.6037-01	1.2031-00	8.8261-01
6.3009-01	4.5694-02	3.3975-02	2.9024-01	1.3398-00	8.0484-01
6.9590-01	4.4354-02	3.4568-02	2.2520-01	1.4707-00	6.9954-01
7.4719-01	4.1876-02	3.4063-02	1.7687-01	1.5703-00	5.9880-01
7.9088-01	3.8713-02	3.2365-02	1.3804-01	1.6528-00	5.0273-01
8.2685-01	3.5215-02	3.0088-02	1.0785-01	1.7190-00	4.1775-01
8.5684-01	3.1597-02	2.7439-02	8.4133-02	1.7727-00	3.4390-01
2 4					
3.9422-01	1.0577-01	1.2025-02	4.8797-01	9.0625-01	8.7341-01
4.2897-01	7.1033-02	2.4158-02	4.7584-01	9.5312-01	9.0261-01
5.5015-01	7.4847-02	2.7972-02	3.4703-01	1.2031-00	8.5592-01
6.2047-01	6.7031-02	3.1875-02	2.8063-01	1.3398-00	7.8560-01
6.8820-01	6.1804-02	3.2507-02	2.1749-01	1.4707-00	6.8413-01
7.4126-01	5.5612-02	3.2174-02	1.7095-01	1.5703-00	5.8696-01
7.8625-01	4.9686-02	3.0643-02	1.3342-01	1.6528-00	4.9348-01
8.2326-01	4.3931-02	2.8550-02	1.0426-01	1.7190-00	4.1057-01
8.5405-01	3.8532-02	2.6081-02	8.1341-02	1.7727-00	3.3831-01
2 5					
3.9133-01	4.6175-02	1.4925-02	5.4758-01	8.4375-01	9.1449-01
4.2698-01	4.1755-02	2.6140-02	5.0511-01	9.2187-01	9.2599-01
5.4723-01	4.6522-02	3.0897-02	3.7535-01	1.1719-00	8.9304-01
6.1770-01	4.6366-02	3.4647-02	3.0129-01	1.3164-00	8.1887-01
6.8533-01	4.5136-02	3.5370-02	2.3416-01	1.4512-00	7.1594-01
7.3855-01	4.2704-02	3.4892-02	1.8386-01	1.5547-00	6.1473-01
7.8369-01	3.9523-02	3.3205-02	1.4355-01	1.6401-00	5.1747-01
8.2090-01	3.6028-02	3.0902-02	1.1216-01	1.7087-00	4.3076-01
8.5192-01	3.2362-02	2.8211-02	8.7512-02	1.7644-00	3.5511-01
2 6					
3.6390-01	1.3610-01	1.1100-02	4.8890-01	8.7500-01	8.3717-01
4.1410-01	8.5905-02	2.3425-02	4.7660-01	9.3750-01	8.8678-01
5.3534-01	8.9665-02	2.7165-02	3.4784-01	1.1875-00	8.4801-01
6.0945-01	7.8048-02	3.1173-02	2.8133-01	1.3281-00	7.8311-01
6.7905-01	7.0952-02	3.1890-02	2.1811-01	1.4609-00	6.8469-01
7.3397-01	6.2905-02	3.1655-02	1.7147-01	1.5625-00	5.8903-01
7.8034-01	5.5601-02	3.0211-02	1.3385-01	1.6465-00	4.9625-01
8.1848-01	4.8704-02	2.8196-02	1.0462-01	1.7139-00	4.1349-01
8.5018-01	4.2396-02	2.5794-02	8.1626-02	1.7686-00	3.4113-01
2 7					
3.5955-01	4.6700-02	1.5450-02	5.7830-01	7.8125-01	8.9000-01
4.1114-01	4.1988-02	2.6363-02	5.2051-01	8.9062-01	9.1969-01
5.3098-01	4.7140-02	3.1515-02	3.9036-01	1.1406-00	9.0157-01
6.0533-01	4.7014-02	3.5296-02	3.1236-01	1.2930-00	8.3186-01
6.7479-01	4.5913-02	3.6147-02	2.4315-01	1.4316-00	7.3163-01
7.2993-01	4.3510-02	3.5698-02	1.9086-01	1.5391-00	6.3020-01
7.7652-01	4.0373-02	3.4025-02	1.4908-01	1.6274-00	5.3192-01
8.1498-01	3.6827-02	3.1700-02	1.1649-01	1.6985-00	4.4359-01
8.4701-01	3.3120-02	2.8969-02	9.0904-02	1.7561-00	3.6622-01
2 8					
3.3357-01	1.6642-01	1.0175-02	4.8982-01	8.4375-01	7.9899-01
3.9922-01	1.0076-01	2.2651-02	4.7735-01	9.2187-01	8.7047-01
5.2052-01	1.0446-01	2.6358-02	3.4864-01	1.1719-00	8.3962-01
5.9843-01	8.9085-02	3.0471-02	2.8203-01	1.3164-00	7.8035-01
6.6990-01	8.0100-02	3.1272-02	2.1873-01	1.4512-00	6.8507-01
7.2668-01	7.0198-02	3.1135-02	1.7199-01	1.5547-00	5.9099-01
7.7442-01	6.1517-02	2.9779-02	1.3428-01	1.6401-00	4.9893-01
8.1371-01	5.3478-02	2.7843-02	1.0497-01	1.7087-00	4.1637-01
8.4632-01	4.6260-02	2.5508-02	8.1914-02	1.7644-00	3.4392-01
2 9					
3.2778-01	4.7225-02	1.5975-02	6.0903-01	7.1875-01	8.5770-01
3.9530-01	4.2291-02	2.6576-02	5.3592-01	8.5937-01	9.1145-01
5.1476-01	4.7735-02	3.2110-02	4.0539-01	1.1094-00	9.0819-01
5.9299-01	4.7639-02	3.5920-02	3.2345-01	1.2695-00	8.4379-01
6.6427-01	4.6665-02	3.6900-02	2.5216-01	1.4121-00	7.4660-01
7.2133-01	4.4295-02	3.6482-02	1.9789-01	1.5234-00	6.4524-01
7.6937-01	4.1174-02	3.4826-02	1.5463-01	1.6147-00	5.4609-01
8.0907-01	3.7608-02	3.2481-02	1.2084-01	1.6882-00	4.5625-01
8.4211-01	3.3864-02	2.9714-02	9.4310-02	1.7478-00	3.7721-01
3 3					
4.2165-01	1.5850-02	1.5850-02	5.4665-01	8.7500-01	9.5268-01
4.4185-01	2.6903-02	2.6903-02	5.0435-01	9.3750-01	9.4229-01
5.6202-01	3.1728-02	3.1728-02	3.7452-01	1.1875-00	9.0139-01
6.2869-01	3.5373-02	3.5373-02	3.0056-01	1.3281-00	8.2159-01
6.9446-01	3.6012-02	3.6012-02	2.3352-01	1.4609-00	7.1551-01
7.4582-01	3.5433-02	3.5433-02	1.8332-01	1.5625-00	6.1273-01
7.8959-01	3.3656-02	3.3656-02	1.4310-01	1.6465-00	5.1475-01
8.2566-01	3.1272-02	3.1272-02	1.1179-01	1.7139-00	4.2785-01
8.5577-01	2.8511-02	2.8511-02	8.7211-02	1.7686-00	3.5230-01

3 4							
3.9281-01	7.5936-02	1.3436-02	5.1781-01	8.7500-01	8.9500-01		
4.2799-01	5.6381-02	2.5130-02	4.9049-01	9.3750-01	9.1458-01		
5.4871-01	6.0656-02	2.9416-02	3.6121-01	1.1875-00	8.7476-01		
6.1910-01	5.6686-02	3.3248-02	2.9097-01	1.3281-00	8.0240-01		
6.8677-01	5.3440-02	3.3928-02	2.2584-01	1.4609-00	7.0015-01		
7.3991-01	4.9150-02	3.3515-02	1.7741-01	1.5625-00	6.0092-01		
7.8498-01	4.4613-02	3.1418-02	1.3849-01	1.6465-00	5.0553-01		
8.2209-01	3.9975-02	2.9721-02	1.0822-01	1.7139-00	4.2070-01		
8.5298-01	3.5643-02	2.7142-02	8.4430-02	1.7686-00	3.4674-01		
3 5							
3.8991-01	1.6356-02	1.6336-02	5.7741-01	8.1250-01	9.3217-01		
4.2601-01	2.7112-02	2.7112-02	5.1976-01	9.0625-01	9.3699-01		
5.4578-01	3.2341-02	3.2341-02	3.8953-01	1.1562-00	9.1090-01		
6.1632-01	3.6021-02	3.6021-02	3.1164-01	1.3047-00	8.3512-01		
6.8391-01	3.6791-02	3.6791-02	2.4251-01	1.4414-00	7.3158-01		
7.3220-01	3.6242-02	3.6242-02	1.9032-01	1.5469-00	6.2844-01		
7.8241-01	3.4480-02	3.4480-02	1.4363-01	1.6338-00	5.2935-01		
8.1973-01	3.2073-02	3.2073-02	1.1612-01	1.7036-00	4.4078-01		
8.5085-01	2.9273-02	2.9273-02	9.0600-02	1.7603-00	3.6347-01		
3 6							
3.6253-01	1.0622-01	1.2472-02	5.1878-01	8.4375-01	8.5689-01		
4.1315-01	7.1228-02	2.4353-02	4.9127-01	9.2187-01	8.9832-01		
5.3392-01	7.5454-02	2.8579-02	3.6205-01	1.1719-00	8.6643-01		
6.0810-01	6.7678-02	3.2521-02	2.9170-01	1.3164-00	7.9969-01		
6.7765-01	6.2585-02	3.3238-02	2.2648-01	1.4512-00	7.0057-01		
7.3264-01	5.6424-02	3.2986-02	1.7735-01	1.5547-00	6.0291-01		
7.7908-01	5.0513-02	3.1470-02	1.3894-01	1.6401-00	5.0824-01		
8.1733-01	4.4735-02	2.9355-02	1.0858-01	1.7087-00	4.2360-01		
8.4913-01	3.9296-02	2.6845-02	8.4727-02	1.7644-00	3.4954-01		
3 7							
3.5818-01	1.6822-02	1.6822-02	6.0818-01	7.5000-01	9.0386-01		
4.1019-01	2.7312-02	2.7312-02	5.3519-01	8.7500-01	9.2975-01		
5.2957-01	3.2930-02	3.2930-02	4.0457-01	1.1250-00	9.1852-01		
6.0398-01	3.6644-02	3.6644-02	3.2273-01	1.2812-00	8.4761-01		
6.7339-01	3.7545-02	3.7545-02	2.5152-01	1.4219-00	7.4693-01		
7.2860-01	3.7030-02	3.7030-02	1.9735-01	1.5312-00	6.4371-01		
7.7526-01	3.5284-02	3.5284-02	1.5417-01	1.6211-00	5.4367-01		
8.1382-01	3.2858-02	3.2858-02	1.2046-01	1.6934-00	4.5354-01		
8.4596-01	3.0020-02	3.0020-02	9.4003-02	1.7520-00	3.7453-01		
3 8							
3.3224-01	1.3651-01	1.1508-02	5.1974-01	3.1250-01	8.1683-01		
3.9830-01	1.6076-02	2.3576-02	4.9205-01	9.0625-01	8.8156-01		
5.1913-01	9.0242-02	2.7742-02	3.6288-01	1.1562-00	8.5760-01		
5.9711-01	7.8670-02	3.1795-02	2.9242-01	1.3047-00	7.9670-01		
6.6852-01	7.1711-02	3.2648-02	2.2712-01	1.4414-00	7.0080-01		
7.2535-01	6.3698-02	3.2448-02	1.7849-01	1.5469-00	6.0478-01		
7.7318-01	5.6412-02	3.1022-02	1.3939-01	1.6338-00	5.1088-01		
8.1256-01	4.9498-02	2.8988-02	1.0895-01	1.7036-00	4.2644-01		
8.4828-01	4.5149-02	2.6548-02	8.5024-02	1.7603-00	3.5232-01		
3 9							
3.2644-01	1.7307-02	1.7307-02	6.3894-01	6.8750-01	8.6773-01		
3.9437-01	2.7501-02	2.7501-02	5.5062-01	8.4375-01	9.2058-01		
5.1338-01	3.3494-02	3.3494-02	4.1963-01	1.0937-00	9.2422-01		
5.9166-01	3.7243-02	3.7243-02	3.3385-01	1.2578-00	8.5905-01		
6.6290-01	3.8275-02	3.8275-02	2.6055-01	1.4023-00	7.6157-01		
7.2002-01	3.7795-02	3.7795-02	2.0439-01	1.5156-00	6.3854-01		
7.6813-01	3.6069-02	3.6069-02	1.5973-01	1.6084-00	5.5771-01		
8.0793-01	3.3626-02	3.3626-02	1.2482-01	1.6831-00	4.6611-01		
8.4107-01	3.0754-02	3.0754-02	9.7420-02	1.7437-00	3.8547-01		
4 4							
3.6390-01	1.3610-01	1.1100-02	4.8890-01	8.7500-01	8.3717-01		
4.1410-01	8.8905-02	2.3405-02	4.7660-01	9.3750-01	8.8678-01		
5.3534-01	8.9665-02	2.7165-02	3.4784-01	1.1875-00	8.4801-01		
6.0945-01	7.8048-02	3.1173-02	2.8133-01	1.3281-00	7.8311-01		
6.7905-01	7.0930-02	3.1890-02	2.1811-01	1.4609-00	6.8469-01		
7.3397-01	6.2905-02	3.1655-02	1.7147-01	1.5625-00	5.8903-01		
7.8034-01	5.5601-02	3.0211-02	1.3385-01	1.6465-00	4.9625-01		
8.1848-01	4.8704-02	2.8196-02	1.0462-01	1.7139-00	4.1349-01		
8.5018-01	4.2396-02	2.5794-02	8.1628-02	1.7686-00	3.4113-01		
4 5							
3.6108-01	7.6422-02	1.3922-02	5.4858-01	8.1250-01	8.7450-01		
4.1216-01	5.6589-02	2.5339-02	5.0591-01	9.0625-01	9.0928-01		
5.3247-01	6.1279-02	3.0029-02	3.7622-01	1.1562-00	8.8428-01		
6.0673-01	5.7333-02	3.3896-02	3.0204-01	1.3047-00	8.1594-01		
6.7623-01	5.4239-02	3.4707-02	2.3482-01	1.4414-00	7.1621-01		
7.3129-01	4.9959-02	3.4334-02	1.8442-01	1.5469-00	6.1663-01		
7.7781-01	4.5436-02	3.2741-02	1.4402-01	1.6338-00	5.2013-01		
8.1616-01	4.0776-02	3.0522-02	1.1254-01	1.7036-00	4.3363-01		
8.4807-01	3.6204-02	2.7903-02	8.7819-02	1.7603-00	3.5791-01		
4 6							
3.3357-01	1.6642-01	1.0175-02	4.8982-01	8.4375-01	7.9899-01		
3.9922-01	1.0078-01	2.2651-02	4.7735-01	9.2187-01	8.7047-01		
5.2052-01	1.0448-01	2.0358-02	3.4864-01	1.1719-00	8.3962-01		
5.9843-01	8.9065-02	3.0471-02	2.8203-01	1.3164-00	7.8035-01		
6.6990-01	8.0100-02	3.1272-02	2.1973-01	1.4512-00	6.8507-01		
7.2668-01	7.0198-02	3.1135-02	1.7199-01	1.5547-00	5.9099-01		
7.7442-01	6.1517-02	2.9779-02	1.3428-01	1.6401-00	4.9893-01		
8.1371-01	5.3478-02	2.7843-02	1.0497-01	1.7087-00	4.1637-01		
8.4632-01	4.6280-02	2.5508-02	8.1914-02	1.7644-00	3.4392-01		
4 7							
3.2934-01	7.6908-02	1.4408-02	5.7934-01	7.5000-01	8.4618-01		
3.9634-01	5.6788-02	2.5538-02	5.2134-01	8.7500-01	9.0205-01		
5.1626-01	6.1868-02	3.0618-02	3.9126-01	1.1250-00	8.9189-01		
5.9439-01	5.7956-02	3.4519-02	3.1314-01	1.2812-00	8.2842-01		
6.6571-01	5.4993-02	3.5462-02	2.4384-01	1.4219-00	7.3157-01		
7.2269-01	5.0747-02	3.5122-02	1.9144-01	1.5312-00	6.3191-01		
7.7065-01	4.6241-02	3.3545-02	1.4956-01	1.6211-00	5.3446-01		
8.1025-01	4.1561-02	3.1307-02	1.1689-01	1.6934-00	4.4638-01		
8.4318-01	3.6952-02	2.8651-02	9.1222-02	1.7520-00	3.6896-01		

4 8							
3.0325-01	1.9675-01	9.2500-03	4.9075-01	8.1250-01	7.5884-01		
3.8435-01	1.1565-01	2.1898-02	4.7810-01	9.0625-01	8.5367-01		
5.0570-01	1.1930-01	2.5550-02	3.4945-01	1.1562-00	8.3074-01		
5.8742-01	1.0008-01	2.9770-02	2.8273-01	1.3047-00	7.7731-01		
6.6075-01	8.9249-02	3.0655-02	2.1935-01	1.4414-00	6.8526-01		
7.1938-01	7.7491-02	3.0616-02	1.7251-01	1.5469-00	5.9282-01		
7.6890-01	6.7433-02	2.9347-02	1.3472-01	1.6338-00	5.0153-01		
8.0894-01	5.8251-02	2.7489-02	1.0532-01	1.7036-00	4.1919-01		
8.4245-01	5.0123-02	2.5221-02	8.2201-02	1.7603-00	3.4667-01		
4 9							
2.9761-01	7.7393-02	1.4893-02	6.1011-01	6.6750-01	8.1006-01		
3.8052-01	5.6978-02	2.5728-02	5.3677-01	8.4375-01	8.9288-01		
5.0007-01	6.2433-02	3.1183-02	4.0632-01	1.0937-00	8.9760-01		
5.8207-01	5.8556-02	3.5118-02	3.2426-01	1.2573-00	8.3986-01		
6.5521-01	5.5723-02	3.6192-02	2.5287-01	1.4023-00	7.4620-01		
7.1411-01	5.1512-02	3.5887-02	1.9849-01	1.5156-00	6.4673-01		
7.6352-01	4.7026-02	3.4330-02	1.5512-01	1.6084-00	5.4850-01		
8.0435-01	4.2330-02	3.2076-02	1.2124-01	1.6831-00	4.5896-01		
8.3829-01	3.7686-02	2.9385-02	9.4639-02	1.7437-00	3.7991-01		
5 5							
3.5826-01	1.6743-02	1.6743-02	6.0826-01	7.5000-01	9.0401-01		
4.1023-01	2.7274-02	2.7274-02	5.3523-01	8.7500-01	9.2983-01		
5.2961-01	3.2894-02	3.2894-02	4.0461-01	1.1250-00	9.1859-01		
6.0401-01	3.6618-02	3.6618-02	3.2276-01	1.2812-00	8.4766-01		
6.7341-01	3.7525-02	3.7525-02	2.5154-01	1.4219-00	7.4697-01		
7.2661-01	3.7014-02	3.7014-02	1.9736-01	1.5312-00	6.4375-01		
7.7528-01	3.5272-02	3.5272-02	1.5418-01	1.6211-00	5.4370-01		
8.1383-01	3.2848-02	3.2848-02	1.2047-01	1.6934-00	4.5356-01		
8.4596-01	3.0013-02	3.0013-02	9.4011-02	1.7520-00	3.7454-01		
5 6							
3.3083-01	1.0667-01	1.2919-02	5.4958-01	7.8125-01	8.3256-01		
3.9734-01	7.1413-02	2.4538-02	5.0671-01	8.9062-01	8.9209-01		
5.1771-01	7.6037-02	2.9162-02	3.7709-01	1.1406-00	8.7503-01		
5.9576-01	6.8300-02	3.3144-02	3.0279-01	1.2930-00	8.1273-01		
6.6713-01	6.3341-02	3.4045-02	2.3549-01	1.4316-00	7.1630-01		
7.2404-01	5.7214-02	3.3777-02	1.8497-01	1.5391-00	6.1842-01		
7.7192-01	5.1320-02	3.2277-02	1.4448-01	1.6274-00	5.2272-01		
8.1141-01	4.5524-02	3.0143-02	1.1292-01	1.6985-00	4.3645-01		
8.4423-01	4.0047-02	2.7596-02	8.8127-02	1.7561-00	3.6066-01		
5 7							
3.2660-01	1.7151-02	1.7151-02	6.3910-01	6.8750-01	6.6804-01		
3.9445-01	2.7426-02	2.7426-02	5.5070-01	8.4375-01	9.2073-01		
5.1345-01	3.3422-02	3.3422-02	4.1970-01	1.0937-00	9.2437-01		
5.9171-01	3.7191-02	3.7191-02	3.3380-01	1.2578-00	8.5915-01		
6.6294-01	3.8234-02	3.8234-02	2.6059-01	1.4023-00	7.6165-01		
7.2005-01	3.7763-02	3.7763-02	2.0442-01	1.5156-00	6.5860-01		
7.6816-01	3.6044-02	3.6044-02	1.5976-01	1.6084-00	5.5776-01		
8.0795-01	3.3607-02	3.3607-02	1.2484-01	1.6831-00	4.6615-01		
8.4109-01	3.0739-02	3.0739-02	9.7435-02	1.7437-00	3.8550-01		
5 8							
3.0058-01	1.3692-01	1.1915-02	5.5058-01	7.5000-01	7.8867-01		
3.8251-01	8.6238-02	2.3738-02	5.0751-01	8.7500-01	8.7440-01		
5.0296-01	2.0795-02	2.8245-02	3.7796-01	1.1250-00	8.6529-01		
5.8480-01	7.9267-02	3.2392-02	3.0355-01	1.2812-00	8.0924-01		
6.5802-01	7.2444-02	3.3382-02	2.3615-01	1.4219-00	7.1620-01		
7.1678-01	6.4469-02	3.3219-02	1.8553-01	1.5312-00	6.2008-01		
7.6604-01	5.7204-02	3.1814-02	1.4494-01	1.6211-00	5.2522-01		
8.0666-01	5.0271-02	2.9763-02	1.1330-01	1.6934-00	4.3922-01		
8.4039-01	4.3889-02	2.7288-02	8.8435-02	1.7520-00	3.6339-01		
5 9							
2.9494-01	1.7559-02	1.7559-02	6.6994-01	6.2500-01	8.2426-01		
3.7868-01	2.7568-02	2.7568-02	5.6618-01	8.1250-01	9.0971-01		
4.9732-01	3.3927-02	3.3927-02	4.3482-01	1.0625-00	9.2824-01		
5.7945-01	3.7741-02	3.7741-02	3.4507-01	1.2344-00	8.6959-01		
6.5249-01	3.8919-02	3.8919-02	2.6968-01	1.3828-00	7.7562-01		
7.1151-01	3.8490-02	3.8490-02	2.1151-01	1.5000-00	6.7302-01		
7.6105-01	3.6797-02	3.6797-02	1.6535-01	1.5957-00	5.7154-01		
8.0208-01	3.4349-02	3.4349-02	1.2922-01	1.6729-00	4.7857-01		
8.3622-01	3.1452-02	3.1452-02	1.0087-01	1.7354-00	3.9635-01		
6 6							
3.0325-01	1.9675-01	9.2500-03	4.9075-01	8.1250-01	7.5884-01		
3.8435-01	1.1565-01	2.1898-02	4.7810-01	9.0625-01	8.5367-01		
5.0570-01	1.1930-01	2.5550-02	3.4945-01	1.1562-00	8.3074-01		
5.8742-01	1.0008-01	2.9770-02	2.8273-01	1.3047-00	7.7731-01		
6.6075-01	8.9249-02	3.0655-02	2.1935-01	1.4414-00	6.8526-01		
7.1938-01	7.7491-02	3.0616-02	1.7251-01	1.5469-00	5.9282-01		
7.6890-01	6.7433-02	2.9347-02	1.3472-01	1.6338-00	5.0153-01		
8.0894-01	5.8251-02	2.7489-02	1.0532-01	1.7036-00	4.1919-01		
8.4245-01	5.0123-02	2.5221-02	8.2201-02	1.7603-00	3.4667-01		
6 7							
2.9913-01	1.0712-01	1.3365-02	5.8038-01	7.1875-01	8.0042-01		
3.8154-01	7.5899-02	2.4714-02	5.2216-01	8.5937-01	8.8392-01		
5.0153-01	7.6599-02	2.9721-02	3.9215-01	1.1094-00	8.8172-01		
5.8345-01	6.8898-02	3.3742-02	3.1391-01	1.2695-00	8.2471-01		
6.5563-01	6.4073-02	3.4776-02	2.4452-01	1.4121-00	7.3132-01		
7.1545-01	5.7983-02	3.4545-02	1.9202-01	1.5234-00	6.3349-01		
7.6479-01	5.2109-02	3.3066-02	1.5004-01	1.6147-00	5.3691-01		
8.0551-01	4.6295-02	3.0915-02	1.1728-01	1.6882-00	4.4913-01		
8.3934-01	4.0784-02	2.8333-02	9.1540-02	1.7478-00	3.7167-01		
6 8							
2.7292-01	2.2707-01	3.3250-03	4.9167-01	7.8125-01	7.1675-01		
3.6948-01	1.3052-01	2.1144-02	4.7886-01	8.9062-01	8.3637-01		
4.9088-01	1.3412-01	2.4743-02	3.5026-01	1.1406-00	8.2136-01		
5.7640-01	1.1110-01	2.9058-02	2.8343-01	1.2930-00	7.7400-01		
6.5160-01	9.8397-02	3.0037-02	2.1996-01	1.4316-00	6.8525-01		
7.1209-01	8.4784-02	3.0096-02	1.7303-01	1.5391-00	5.9453-01		
7.6259-01	7.3348-02	2.8915-02	1.3515-01	1.6274-00	5.0405-01		
8.0416-01	6.3025-02	2.7136-02	1.0568-01	1.6985-00	4.2196-01		
8.3859-01	5.3987-02	2.4934-02	8.2487-02	1.7561-00	3.4939-01		

6 9					
2.6744-01	1.0756-01	1.3812-02	6.1119-01	6.5625-01	7.6046-01
3.6515-01	7.1755-02	2.4840-02	5.3762-01	8.2812-01	8.7382-01
4.8537-01	7.7130-02	3.0255-02	4.0724-01	1.0781-00	8.8651-01
5.7115-01	6.9473-02	3.4316-02	3.2506-01	1.2461-00	8.3565-01
6.4616-01	6.4781-02	3.5484-02	2.5358-01	1.3926-00	7.4562-01
7.0690-01	5.8729-02	3.5291-02	1.9908-01	1.5078-00	6.4811-01
7.5767-01	5.2678-02	3.3835-02	1.5562-01	1.6021-00	5.5082-01
7.9963-01	4.7051-02	3.1670-02	1.2165-01	1.6780-00	4.6162-01
8.3447-01	4.1507-02	2.9056-02	9.4968-02	1.7395-00	3.8257-01
7 7					
2.9502-01	1.7481-02	1.7481-02	6.7002-01	6.2500-01	8.2441-01
3.7872-01	2.7530-02	2.7530-02	5.6622-01	8.1250-01	9.0978-01
4.9736-01	3.3891-02	3.3891-02	4.3486-01	1.0625-00	9.2831-01
5.7947-01	3.7715-02	3.7715-02	3.4510-01	1.2344-00	8.6964-01
6.5251-01	3.8898-02	3.8898-02	2.6970-01	1.3828-00	7.7566-01
7.1153-01	3.8474-02	3.8474-02	2.1153-01	1.5000-00	6.7305-01
7.6107-01	3.6705-02	3.6705-02	1.6536-01	1.5957-00	5.7157-01
8.0209-01	3.4340-02	3.4340-02	1.2923-01	1.6729-00	4.7859-01
8.3623-01	3.1444-02	3.1444-02	1.0088-01	1.7354-00	3.9637-01
7 8					
2.6893-01	1.3732-01	1.2323-02	5.8143-01	6.8750-01	7.5270-01
3.6674-01	8.6389-02	2.3889-02	5.2299-01	8.4375-01	8.6531-01
4.8680-01	9.1323-02	2.8823-02	3.9305-01	1.0937-00	8.7106-01
5.7250-01	7.9840-02	3.2965-02	3.1469-01	1.2578-00	8.2073-01
6.4755-01	7.3153-02	3.4091-02	2.4521-01	1.4023-00	7.3088-01
7.0822-01	6.5219-02	3.3969-02	1.9259-01	1.5156-00	6.3494-01
7.5892-01	5.7977-02	3.2586-02	1.5052-01	1.6084-00	5.3929-01
8.0078-01	5.1030-02	3.0522-02	1.1767-01	1.6831-00	4.5181-01
8.3551-01	4.4616-02	2.8014-02	9.1859-02	1.7437-00	3.7435-01
7 9					
2.6344-01	1.7810-02	1.7810-02	7.0094-01	5.6250-01	7.7297-01
3.6300-01	2.7625-02	2.7625-02	6.8175-01	7.8125-01	8.9690-01
4.8129-01	3.4336-02	3.4336-02	4.5004-01	1.0312-00	9.3035-01
5.6725-01	3.8214-02	3.8214-02	3.5632-01	1.2109-00	8.7908-01
6.4210-01	3.9537-02	3.9537-02	2.7882-01	1.3633-00	7.8895-01
7.0302-01	3.9164-02	3.9164-02	2.1865-01	1.4844-00	6.8705-01
7.5400-01	3.7506-02	3.7506-02	1.7099-01	1.5830-00	5.8509-01
7.9624-01	3.5056-02	3.5056-02	1.3364-01	1.6626-00	4.9085-01
8.3139-01	3.2136-02	3.2136-02	1.0434-01	1.7271-00	4.0713-01
8 8					
2.4260-01	2.5740-01	7.4000-03	4.9260-01	7.5000-01	6.7270-01
3.5461-01	1.4539-01	2.0390-02	4.7961-01	8.7500-01	8.1859-01
4.7606-01	1.4894-01	2.3936-02	3.5106-01	1.1250-00	8.1150-01
5.6538-01	1.2212-01	2.8366-02	2.8413-01	1.2812-00	7.7042-01
6.4246-01	1.0794-01	2.9420-02	2.2058-01	1.4219-00	6.8506-01
7.0480-01	9.2077-02	2.9577-02	1.7355-01	1.5312-00	5.9612-01
7.5667-01	7.9264-02	2.8483-02	1.3558-01	1.6211-00	5.0650-01
7.9939-01	6.7798-02	2.6782-02	1.0603-01	1.6934-00	4.2467-01
8.3473-01	5.7851-02	2.4648-02	8.2774-02	1.7520-00	3.5207-01
8 9					
2.3727-01	1.3773-01	1.2731-02	6.1227-01	6.2500-01	7.0891-01
3.5097-01	8.6532-02	2.4032-02	5.3847-01	8.1250-01	8.5428-01
4.7067-01	9.1828-02	2.9328-02	4.0917-01	1.0625-00	8.7494-01
5.6024-01	8.0390-02	3.3515-02	3.2586-01	1.2344-00	8.3116-01
6.3710-01	7.3838-02	3.4776-02	2.5429-01	1.3828-00	7.4484-01
6.9968-01	6.6946-02	3.4696-02	1.9968-01	1.5000-00	6.4936-01
7.5182-01	5.8730-02	3.3339-02	1.5611-01	1.5957-00	5.5307-01
7.9491-01	5.1772-02	3.1265-02	1.2206-01	1.6729-00	4.6423-01
8.3065-01	4.5329-02	2.8727-02	9.5296-02	1.7354-00	3.8520-01
9 9					
2.3194-01	1.8062-02	1.8062-02	7.3194-01	5.0000-01	7.1388-01
3.4733-01	2.7673-02	2.7673-02	5.9733-01	7.5000-01	8.8215-01
4.6528-01	3.4720-02	3.4720-02	4.6528-01	1.0000-00	9.3056-01
5.5509-01	3.8663-02	3.8663-02	3.6759-01	1.1875-00	8.8752-01
6.3174-01	4.0131-02	4.0131-02	2.8799-01	1.3437-00	8.0157-01
6.9456-01	3.9815-02	3.9815-02	2.2581-01	1.4687-00	7.0064-01
7.4696-01	3.8196-02	3.8196-02	1.7665-01	1.5703-00	5.9835-01
7.9043-01	3.5747-02	3.5747-02	1.3808-01	1.6523-00	5.0295-01
8.2657-01	3.2806-02	3.2806-02	1.0782-01	1.7187-00	4.1779-01

of nine lines each. At the top of each block, numbers such as 1 3, 2 5, etc. appear. The pair of numbers 2 3 imply that from Table 1 initial conditions 2 and 3 are chosen for F_M and F_F . The first line below this pair of numbers contain $F_2(11)$, $F_2(10)$, $F_2(01)$, $F_2(00)$, M , and V . The second line the corresponding values for F_3 . Etc. Numbers, such as 4.8040-01, 1.9600-02, etc. should be read as 4.8040×10^{-1} , 1.9600×10^{-2} , etc.

Discussion

HALDANE (1949) obtained an expression for $F_2(11)$ under the condition that

$$F_F = F_M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \quad (18)$$

His result was

$$F = (1/16)(4c'^4 + 4c'^2c^2 + 2c^2), \quad (19)$$

where F is a generalization of inbreeding

coefficient (*i. e.*, our $F_2(11)$), c is the recombination probability and $c' = 1 - c$. If the corresponding result in the preceding section is written in terms of c and c' .

$F_2(11) = (1/16)(4c'^4 + 4c'^2c^2 + c)$, (20)
under condition (18).

In Figs. 2 & 3, some results of the numerical computations are plotted. In the figures, fairly large differences are seen in $F_2(11)$ and $F_2(10)$ with varying recombination probabilities, where numbers such as 1, 4, 5, 8, and 9 denote initial conditions, *i. e.*, when number "i" is written, initial condition numbered by "i" in Table 1 is used for F_F and F_M . With increasing recombination probabilities, $F_2(11)$ decreases in general while $F_2(10)$ increases. This has been also expected from considerations on single locus case. From further computations, it is also found that mutual differences in $F_{10}(11)$, for example, can be still as big as approximately ten percent, for different values of recombination probabilities. This implies existence of recombination effect of a fair amount even after several generations.

Summary

Generalized forms of inbreeding coefficient and coefficient of parentage are shown for a case of full-sibbing, where two linked loci are involved. In order to simplify the model for calculation, two individuals at the beginning of the pedigree are assumed to be not

related by descent. Both of initial parents are assumed to be inbred to various degrees. These degrees of homozygosity by descent are used as initial conditions. As well as the variety of these initial conditions. Seven recombination probabilities are used in numerical computations. Degrees of homozygosity by descent are calculated of offsprings at the second, third, ..., tenth generation. Weighted mean and variance of these are also calculated. Numerical results show existence of fairly large recombination effect.

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同胞交配の近交係数におよぼされる交叉の影響

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同胞交配を仮定し、近交係数における交叉の影響を数値的に調べた。例として、2遺伝子座の場合をとり、近交係数の数式表現を示し、それに各種の初期条件を与え

て数世代の近交係数の変化を計算した。組み換えの確率として、7種類の確率を用いた。多くの場合に比較的大きい交叉の影響が認められた。