Statistical Property of the Selection by the Plant Type Index Given by Quotient of Two Traits

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Synopsis. When plant type indices are given by the quotients of two quantitative traits following normal law, the statistical means are studied to evaluate, from statistics of component traits, (1) heritability value of the indices, (2) the regression function of the quotient on its component traits and (3) the correlation coefficient between components and the quotient. And the usefulness of such indices are suggested as the result of the application of these means to the experimental data in rice breeding.

Introduction

The plant types as the breeding objectives have recently drawn the attentions of breeders. JENNING, R. (1964) emphasized the breeding of a type of a rice plant capable of more effective performance under tropical conditions, summarizing the works explaining the association of yielding ability with plant type. MORISHIMA, H. *et al.* (1967) have reported a trial to deal with plant types by multivariate analysis, referring the plant type scores which may serve as criteria for selection. On the other hand, we have many instances where plant types are expressed by the ratio of a portion to another. KATAYAMA, Y. and T. NAGATOMO (1963) classified various types of peanuts by plant type index given by the ratio of the length of the longest branch to that of the main stem. SYAKUDO, K. and S. KAWABATA (1965) have reported the genetical behaviour of this index together with those of other main characters in F_2 population. GOTOH, K. (1963) expressed plant type of soybeans by the ratio of number of pods on main stem to number of pods per plant, and conducted genetical analysis on this trait as well as on other agronomic characters. Sometimes the shape of a plant portion is also figured by the ratio. SAMARAI, S.M. *et al.* (1969) studied the genetical behaviour of grain shape in rice, representing it by the ratio of length to width.

Particular interest will be drawn to such indices as involving yielding portion, for example, the ratio of panicle weight per plant to total weight or vegetative portion of a plant.

These indices, as indicated by some workers (MURAYAMA, N., 1966, SAMARAI, S.M. 1969), imply the productivity of plants.

From the standpoint of selection works, the prediction of the impact of selection by such indices on yield must be worthy of theoretical considerations.

In the present paper the authors wish to introduce some statistical means to deal with plant type index, and thus, find out some peculiar features of the selection by plant type index.

So far as statistical theories concern, the property of the quotient of two normal variates have been studied around 1930 in relation to anthropometric researches.

A. S. MERRILL (1928) has calculated the first five moments of the distribution of the quotient around its mean, when the components follow the normal law. And he also showed the skewness and the kurtosis of this distribution, indicating that the degree of deviation of this distribution from normality varies depending upon the coefficient of variation and correlation coefficients. R. C. GEARY (1930) has proved that if the ratio μ_x/σ_x is larger than 3, then approximately,

$$t = \frac{v\mu_y - \mu_x}{\sqrt{\sigma_y^2 v^2 - 2\rho\sigma_x\sigma_y v + \sigma_x^2}} \quad (1)$$

distributes normally with unit standard deviation, where v is the quotient of two normal variates x and y , with mean μ_x and μ_y and standard deviation σ_x and σ_y respectively, and ρ is correlation coefficient between them. E. C. FIELLER (1934) also studied this problem in somewhat general manner, and reached the same conclusions.

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The approximate evaluation of heritability of indices by the data of components

Under the assumption that both the components x and y follow the normal law with correlation coefficient ρ between them and standard deviation σ_x and σ_y , respectively, F. S. MERRILL (1928) has shown that the second moment μ_2 of the quotient of x and y around its mean is given by

$$\begin{aligned} \mu_2 = (\mu_x/\mu_y)^2 \{ & V_1^2 - 2\rho V_1 V_2 + V_2^2 + 8V_1^4 \\ & - 16\rho V_1^3 V_2 + 3V_1^2 V_2^2 + 5\rho^2 V_1^2 V_2^2 + 69V_1^6 \\ & - 138\rho V_1^5 V_2 + 15V_1^4 V_2^2 + 54\rho^2 V_1^4 V_2^2 \\ & + 696V_1^8 - 1392\rho V_1^7 V_2 + 105V_1^6 V_2^2 \\ & + 591\rho^2 V_1^6 V_2^2 \} \end{aligned} \quad (2)$$

were $V_1 = \sigma_y/\mu_y$ and $V_2 = \sigma_x/\mu_x$.

When V_1 and V_2 is small enough to allow the reduction of above formula, the variance of the quotient σ_v^2 becomes

$$\sigma_v^2 = (\mu_x/\mu_y)^2 (V_1^2 - 2V_1 V_2 + V_2^2) \quad (3)$$

Actually, the utilization of the formula (3) must be limited in the instances where coefficient of variation do not exceed 1/3.

Before definition of heritability of the quotient of two traits, let us assume following model. For a given quantitative trait X , frequency distributions of genetic value x_g and environmental value x_e are defined respectively as normal distribution as follows:

$$\begin{aligned} & N(\mu_x, \sigma_{xg}^2) \\ & N(0, \sigma_{xe}^2) \end{aligned} \quad (4)$$

In addition x_g distributes independently of x_e , and phenotypic value x_p is given by

$$x_p = x_g + x_e \quad (5)$$

Then the distribution of x_p can be derived as the marginal distribution from the joint distribution of x_p and x_g , which is directly given through the transformation of variable by (5) in the joint distribution of x_g and x_e , thus, the distribution of x_p becomes

$$N(\mu_x, \sigma_p^2) \quad (6)$$

where

$$\sigma_p^2 = \sigma_g^2 + \sigma_e^2$$

Applying above assumptions also to another quantitative trait Y in similar notation, let us define the phenotypic value of the quotient of X and Y by

$$v_p = x_p/y_p \quad (7)$$

As shown above, the variance of v_p , σ_{vp}^2 can

be given approximately by

$$\begin{aligned} \sigma_{vp}^2 = & (\mu_x/\mu_y)^2 (\sigma_{xp}^2/\mu_x^2 \\ & - 2\rho_p \sigma_{xp} \sigma_{yp}/\mu_x \mu_y + \sigma_{yp}^2/\mu_y^2) \end{aligned} \quad (8)$$

where ρ_p is correlation coefficient between x_p and y_p .

Under the following additive assumption,

$$\rho_p \sigma_{xp} \sigma_{yp} = \rho_g \sigma_{xg} \sigma_{yg} + \rho_e \sigma_{xe} \sigma_{ye} \quad (9)$$

$$\begin{aligned} \sigma_{vp}^2 = & (\mu_x/\mu_y)^2 (\sigma_{xg}^2/\mu_x^2 - 2\rho_g \sigma_{xg} \sigma_{yg}/\mu_x \mu_y \\ & + \sigma_{yg}^2/\mu_y^2 + \sigma_{xe}^2/\mu_x^2 - 2\rho_e \sigma_{xe} \sigma_{ye}/\mu_x \mu_y \\ & + \sigma_{ye}^2/\mu_y^2) \\ = & \sigma_{vg}^2 + \sigma_{ve}^2 \end{aligned} \quad (10)$$

where

$$\left. \begin{aligned} \sigma_{vg}^2 = & (\mu_x/\mu_y)^2 (\sigma_{xg}^2/\mu_x^2 \\ & - 2\rho_g \sigma_{xg} \sigma_{yg}/\mu_x \mu_y + \sigma_{yg}^2/\mu_y^2) \\ \sigma_{ve}^2 = & (\mu_x/\mu_y)^2 (\sigma_{xe}^2/\mu_x^2 \\ & - 2\rho_e \sigma_{xe} \sigma_{ye}/\mu_x \mu_y + \sigma_{ye}^2/\mu_y^2) \end{aligned} \right\} \quad (11)$$

and ρ_g and ρ_e is genetic and environmental correlation coefficient, respectively.

In above formula for σ_{vp}^2 , σ_{vg}^2 is able to be interpreted as the approximate expression of the variance of $v_g (= x_g/y_g)$, while σ_{ve}^2 does not exactly correspond to the variance of x_e/y_e , since environmental values have not positive means by definitions. However, it is noteworthy that σ_{vp}^2 is shown to be divided into two parts, i.e. one affected by genetic variance-covariance, and the other varying with environmental variance-covariance.

Therefore, we are able to compare σ_{vp}^2 , at least, with σ_{vg}^2 and afterwards we may define the heritability of the quotient by

$$h_v^2 = \sigma_{vg}^2/\sigma_{vp}^2 \quad (12)$$

The regression functions of a quotient on its components

i) Regression function of a quotient on the denominator, $v(y)$

Under the assumption that x and y follow the binormal distribution, defined by

$$\begin{aligned} P(x, y) = & \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left[-\frac{1}{2(1-\rho^2)} \right. \\ & \times \left\{ \left(\frac{x-\mu_x}{\sigma_x} \right)^2 - 2\rho \left(\frac{x-\mu_x}{\sigma_x} \right) \right. \\ & \times \left. \left(\frac{y-\mu_y}{\sigma_y} \right) + \left. \left(\frac{y-\mu_y}{\sigma_y} \right)^2 \right\} \right] \end{aligned} \quad (13)$$

by the following transformations of variable,

$$y = y, \quad v = x/y, \quad \left| \frac{\partial(y, v)}{\partial(x, y)} \right| = \left| -\frac{1}{y} \right|$$

the joint distribution function of y and $v, q(y, v)$ becomes

$$q(y, v) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left[-\frac{1}{2(1-\rho^2)} \right. \\ \times \left\{ \left(\frac{yv - \mu_x}{\sigma_x} \right)^2 - 2\rho \left(\frac{yv - \mu_x}{\sigma_y} \right) \right. \\ \left. \times \left(\frac{y - \mu_y}{\sigma_y} \right) + \left(\frac{y - \mu_y}{\sigma_y} \right)^2 \right\} \left. \right] \quad (14)$$

Rewriting $q(y, v)$ we get

$$q(y, v) = \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2} \\ \times \frac{y}{\sqrt{2\pi}\sigma_x\sqrt{1-\rho^2}} e^{-\frac{1}{2}\left(\frac{v - \frac{\sigma_y\mu_x + \rho\sigma_x(y-\mu_y)}{\sigma_y\sigma_x}}{\sqrt{1-\rho^2}\sigma_x/y}\right)^2} \quad (15)$$

Therefore, the conditional probability density function of v for given y is normal and regression function $v(y)$ will be given by

$$v(y) = \{\sigma_y\mu_x + \rho\sigma_x(y - \mu_y)\} / \sigma_y y \quad (16)$$

ii) **Regression of a quotient on the numerator,**
 $v(x)$

The regression function of v on x is the expected value of v , corresponding to a given fixed value of x .

When x is fixed the expectation of $v(=x/y)$ is obtained by means of conditional probability density function for y given x , i.e.

$$P(y/x) = \frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_y} \exp \left[-\frac{1}{2(1-\rho^2)\sigma_y^2} \right. \\ \left. \times \left\{ (y - \mu_y) - \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) \right\}^2 \right] \quad (17)$$

Integration of the product of x/y and $P(y/x)$ in respect to y will give the mean of v for a given value of x .

Hence

$$v(x) = x \int_{-\infty}^{\infty} \frac{1}{y} \frac{1}{\sqrt{2\pi(1-\rho^2)}\sigma_y} \\ \times \exp \left[-\frac{1}{2(1-\rho^2)\sigma_y^2} \left\{ (y - \mu_y) - \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) \right\}^2 \right] dy \quad (18)$$

Transforming variable by following equations,

$$\left\{ \begin{aligned} \left((y - \mu_y) - \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) \right) / \sqrt{1-\rho^2}\sigma_y &= t \\ dy &= \sqrt{1-\rho^2}\sigma_y dt \end{aligned} \right\} \quad (19)$$

the above integral is expressed by

$$v(x) = x \int_{-\infty}^{\infty} \frac{1}{\sqrt{1-\rho^2}\sigma_y t + \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \\ \times \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \frac{x}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y}$$

$$\times \int_{-\infty}^{\infty} \left\{ 1 + \frac{\sqrt{1-\rho^2}\sigma_y}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \cdot t \right\}^{-1} \\ \times \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \quad (20)$$

Considering following relation,

$$\left\{ 1 + \frac{\sqrt{1-\rho^2}\sigma_y t}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \right\}^{-1} \\ = \left[1 - \frac{\sqrt{1-\rho^2}\sigma_y}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} t \right. \\ \left. + \left\{ \frac{\sqrt{1-\rho^2}\sigma_y}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \right\}^2 t^2 - \dots \right]$$

within the range of t restricted by following inequality,

$$\left| \frac{\sqrt{1-\rho^2}\sigma_y}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \cdot t \right| \equiv \left| \frac{t}{c(x)} \right| < 1 \quad (21)$$

$$v(x) = \frac{x}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \\ \times \int_{-c(x)}^{c(x)} \left\{ 1 - \frac{\sqrt{1-\rho^2}\sigma_y}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \cdot t + \dots \right\} \\ \times \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt + F(x)$$

where $F(x)$ is the value of integration corresponding to the t value outside of the range (21).

If we assume $|c(x)| > 3$, namely, the condition that

$$\sqrt{1-\rho^2}\sigma_y / \left\{ \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y \right\} < \frac{1}{3} \quad (22)$$

is satisfied, then the range $-3 < t < 3$ will cover actually almost all frequency of t , and $F(x)$ will be negligible, and all terms of order above the second in series of t are able to be neglected.

Consequently, based upon wellknown property of probability integral of normal variate, $v(x)$ reduces approximately to

$$v(x) = \frac{x}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \\ \equiv \frac{1}{\rho \frac{\sigma_y}{\sigma_x}} - \frac{\mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x}{\rho \frac{\sigma_y}{\sigma_x} \left\{ \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y \right\}} \quad (23)$$

Furthermore, if $\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) / \mu_y$ is small enough, for example, less than 1/3, $v(x)$ may be approximately rewritten as follows,

$$\begin{aligned} v(x) &= \frac{1}{\left(\rho \frac{\sigma_y}{\sigma_x}\right)} - \frac{1}{\rho \frac{\sigma_y}{\sigma_x} \mu_y} \left(\mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x \right) \\ &\quad \times \left\{ 1 + \frac{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x)}{\mu_y} \right\}^{-1} \\ &= \frac{1}{\left(\rho \frac{\sigma_y}{\sigma_x}\right)} - \frac{1}{\rho \frac{\sigma_y}{\sigma_x} \mu_y} \left(\mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x \right) \\ &\quad \times \left\{ 1 - \frac{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x)}{\mu_y} + \left(\frac{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x)}{\mu_y} \right)^2 \dots \right\} \\ &\doteq \frac{1}{\left(\rho \frac{\sigma_y}{\sigma_x}\right)} - \frac{1}{\rho \frac{\sigma_y}{\sigma_x} \mu_y} \left(\mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x \right) \\ &\quad \times \left\{ 1 - \frac{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x)}{\mu_y} \right\} \\ &= \left\{ \left(\mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x \right) x + \rho \frac{\sigma_y}{\sigma_x} \mu_x^2 \right\} / \mu_y^2 \end{aligned} \quad (24)$$

Before the application of the regression function $v(x)$, we have to take the validity of the assumptions into account. First of all, the assumption $|c(x)| > 3$ must be examined in the range of x given by inequality,

$$|x - \mu_x| < 3\sigma_x \quad (25)$$

since this range may cover almost whole frequency of x in actual evaluation. Therefore, we have to check the following condition,

$$\begin{aligned} \left| \frac{1}{c(x)} \right| &= \left| \frac{\sqrt{1 - \rho^2} \sigma_y}{\rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) + \mu_y} \right| \\ &< \left| \frac{\sqrt{1 - \rho^2} \sigma_y}{\mu_y \pm 3\rho\sigma_y} \right| < \frac{1}{3} \end{aligned} \quad (26)$$

Next, when $\left| \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x) / \mu_y \right|$ is small enough, for instance, 1/3 in the range of x restricted by (25), namely,

$$|3\rho\sigma_y|/\mu_y < \frac{1}{3} \quad (27)$$

the linear approximation to regression function (23) is given by (24).

Correlation coefficients between a quotient and its numerator.

Since the linear regression function of a quotient on its numerator is given by the

equation (24), though approximately, the covariance between a quotient and its numerator will be given by

$$\frac{COV(x, v)}{\sigma_x^2} = \left(\mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x \right) / \mu_y^2 \quad (28)$$

So that the correlation coefficient between a quotient and its numerator, $R_{x \cdot v}$ can be approximated by

$$\begin{aligned} R_{x \cdot v} &= \frac{COV(x, v)}{\sigma_x^2} \cdot \frac{\sigma_x}{\sigma_v} \\ &= 1/\mu_y^2 \cdot \left(\mu_y - \rho \frac{\sigma_y}{\sigma_x} \mu_x \right) \\ &\quad \times \frac{\sigma_x}{\mu_x/\mu_y \cdot \sqrt{V_x^2 - 2\rho V_x V_y + V_y^2}} \\ &= \frac{V_x - \rho V_y}{\sqrt{V_x^2 - 2\rho V_x V_y + V_y^2}} \\ &= \frac{(V_x/V_y - \rho)}{\sqrt{(V_x/V_y - \rho)^2 + 1 - \rho^2}} \end{aligned} \quad (29)$$

Thus, by formula (29), if the quotients V_x/V_y of the coefficient of variation and correlation coefficient ρ between the numerator and the denominator are given, we are able to get rough estimate of the correlation coefficient between a quotient and its numerator.

Experimental Results

i) The heritability of the quotient of two traits

The experimental data cited here were obtained as the results of the triply replicated growing of 50 F_3 progeny lines from the hybridization of rice varieties, Kinmaze and Chiyohikari, which are contrasting with each other in their plant types. After the harvest, we measured some agronomic characters on each line composed of three randomized plots having five plants, and conducted variance and covariance analysis*, through which we got evaluation of genetic and phenotypic variance, as well as genotypic and phenotypic correlation coefficients. The results of evaluation were shown in Table 1~2.

By the application of formula (12) to these data, we have the heritability values of indices, as shown in Table 3, where the result of 'the direct calculation' of heritability values

* Calculations were carried out in the Computing Center for Research in Agriculture, Forestry and Fishery by the program; Kumagai, K.: Analysis of variance, calculation of heritability values and correlation coefficients based on the data of the experiment with randomized block.

Table 1. Means, variance components and some parameters derived from them for 5 traits of F_3 lines

Characters Parameters	Total weight	Panicle weight	Culm length	Panicle length	No. of tillers
Phenotypic variance	91.417	24.200	28.553	2.091	4.622
Genotypic variance	43.147	12.079	19.929	1.544	2.213
Heritability percent	47.198	49.916	69.797	73.836	47.898
Means (M)	67.641 gr	32.583 gr	85.911 cm	20.663 cm	13.469
$V_p = \sigma_p/M$	0.1414	0.1510	0.0622	0.0688	0.1596
$V_g = \sigma_g/M$	0.0971	0.1067	0.0520	0.0601	0.1104

Table 2. Genotypic and Phenotypic correlations between 5 traits of F_3 lines

Characters	Total weight	Panicle weight	Culm length	Panicle length	No. of tillers
Total weight	—	0.6003**	0.6062**	0.0856	0.5475**
Panicle weight	0.3004	—	0.3849**	0.6525**	0.3258*
Culm length	0.7530	0.4329	—	0.1499	0.1430
Panicle length	0.0157	0.8851	0.2290	—	-0.2381
No. of tillers	0.3940	-0.0252	0.0612	-0.3402	—

*,** Significant at the 5% and 1% level respectively. Above the diagonal : Phenotypic. Below the diagonal: Genotypic.

Table 3. Heritability percent of indices by formula (12)

Characters	Total weight	Panicle weight	Culm length	Panicle length	No. of tillers
Total weight	47.198	84.974 (85.632)	34.27 (35.01)	55.77 (54.50)	63.47 (63.96)
Panicle weight		49.916	47.712 (47.986)	26.109 (26.980)	74.118 (73.715)
Culm length			69.797	66.67 (64.74)	53.55 (52.46)
Panicle length				73.836	57.35 (54.91)
No. of tillers					47.898

Heritability of component traits is shown on diagonal.

Heritability derived from the analysis of variance on the ratios in each plot is described in parenthesis.

were also described. By the direct calculation, we mean the calculation of the quotient in each plot and the subsequent analysis of variance for them. Thus we are able to examine the validity of formula (12) in estimating the heritability values of quotients. As the results, fairly good accordance is found between the values by formula (12) and those obtained by the analysis of variances on the quotient in each plot.

And the effectiveness of the approximate working method introduced here also implies the factors affecting the heritability of plant type index as expressed by quotient. Some quotients are found to assume heritability

value higher than their components. For instance, the heritability of the quotient of panicle weight and total weight is about 0.85, whereas, the heritability of panicle weight and total weight is 0.47 and 0.50, respectively. This fact suggests that plant type may be an efficient index in selection in some cases.

ii) The regression of a quotient on its numerator

Here we would like to illustrate the regression of v on x , because practical implication of this regression may be evident.

The results of evaluation of regression function is shown in Fig. 1 with the distribution of actual values in the case of ratio of panicle

Fig. 1. Distribution of actual values and the regression function of the ratio of panicle weight to total weight on panicle weight by formula (23).

Fig. 2. Illustration of equation (29).

Table 4. Correlation coefficient between ratio and numerator by formula (29) (phenotypic value)

Numerator \ Ratio	Panicle weight to total weight	Panicle weight to culm length	Panicle weight to No. of tillers	Panicle weight to No. of tillers
Panicle weight	0.505 (0.486)	0.863 (0.906)	0.619 (0.549)	0.898 (0.889)

The value calculated from original data is described in parenthesis.

weight to total weight.

From Figure 1, considerable accordance is confirmed between observed data and regression function derived in previous section.

iii) Correlation coefficient between a quotient and its numerator

As shown in previous section, correlation coefficient between a quotient and its numerator is given as a function of the quotient of the coefficients of variation of two traits and the correlation coefficient between them, so that we are able to illustrate this function in general form as shown in Figure 2. And the estimation of correlation coefficient between index and component, by formula (29), is described together with the result of the direct calculation in Table 4.

The values of correlation coefficients derived by formula (29) are shown to be approximate to the values obtained by direct calculation. However the deviation of the values by formula (29) from those by direct evaluation seems not to be negligible. It must be brought about through several steps of approximation. Nevertheless the formula (29) may be useful as a mean for rough estimation.

Discussion

As has been illustrated in previous section, we could evaluate approximately the statistical parameters of any given indices, provided that the primary analyses are conducted on the component characters. Besides the approximate estimations, we are able to point out the factor affecting the parameters of indices. In connection with that, it will be worth mentioning that heritability value of an index varies with genotypic and phenotypic correlation coefficient as shown by formula (8), (11) and (12). Because of the parallel effect of soil fertility on two portions of a whole plant, the

phenotypic correlation coefficient tends to be relatively high in comparison with genotypic correlation coefficient which will be determined by genotypic diversity in a given population. This gives the heritability of indices the possibility to assume values higher than the heritability of the component traits. The results of the estimation of heritability of indices reported by several workers (GOTOH, K., 1963, SYAKUDO, K. and S. KAWABATA, 1965, SAMARAI, S. M. *et al.*, 1969) seem to support above generalization.

From the standpoint of selection practice, plant type indices may serve as effective criteria in indirect selection, when their heritabilities are high.

As is well known, we are able to predict the efficiency of indirect selection in comparison with direct selection, according to following inequality (FALCONER, D. S., 1960),

$$h_x < r_g \cdot h_y$$

where, h_x is the square root of heritability of aimed trait, and h_y is that of indirect trait and r_g is genetic correlation coefficient between them.

Since parameters necessary in above inequality are able to be given based on primary data, the utility of a given index as a indirect trait will be estimated.

However, with the aim of getting simple results, approximation were boldly employed in present paper, this gives limitations for accurate treatment of the problems. Moreover, when the distribution of a quotient deviates largely from normality, the condition assumed for above inequality is not satisfied. Therefore, more exact way to deal with those problems remains for further studies. And a probable approach towards those will be made through the introduction of tetra-normal distribution from which the joint frequency function of phenotypic and genotypic value of a given quotient may be derived by transformation of variables.

In practical selection, the general tendency that plant type indices have the possibility to assume relatively high heritability values should be taken into account. Especially under the restriction of field size, we are allowed to grow large number of lines for the selection of the traits having high heritability values,

because of little need of replication to reduce environmental fluctuations as has been pointed out by our previous paper (H. IKEHASHI *et al.* 1969). In the other words, the selection on plant type index which may assume high heritability must be important at the initial stage of selection, where material lines are in general grown without replication. And the impact of such selection on the components traits will be predictable by statistical means introduced here.

Summary

The plant type index given by a ratio of a trait to another is often employed as criterion in selection, though little attention has been drawn to its statistical property. In order to promote the application of such indices, the authors introduced some statistical means, through which heritability of indices, regression function of a ratio on its components and the correlation coefficient between component and ratio are approximately evaluated based on the statistics of component traits. As the result, the effect of selection by any given index is shown to be predictable depending on the statistics of component traits. And the particular case where an index may assume heritability higher than that of components traits is suggested.

Applications of these theoretical results to F_3 lines of rice plants confirmed the validity of the introduced formula. However, correlation coefficient between components and ratio approximated by statistics of component traits gave relatively large deviation from the value of correlation coefficient calculated directly based on quotient in each plot. Nevertheless these formulae are considered to serve as easy means to estimate the impact of selection by plant type indices.

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2 形質の比で与えられる草型指数による選抜の統計的性質

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従来、計量育種の研究対象は主に形質の絶対量であつたが、育種家は、草型指数などとして、形質の相対比をしばしば重視している。この間隙をうめる見地から、形質比の統計的性質を調べ、実際への適用を試みた。

2次元正規分布の2変数の比の統計的性質の検討から、構成2形質の統計量により、それらの比の統計量を推定する式を求めた。

これらの式を水稻の F_3 系統の成績に適用し以下の結果を得た。①形質比の遺伝力は、構成形質の統計量から

推定でき、その値は2形質の表現型相関→大、遺伝相関→小のとき大きく、実際にかなり高くなり得る。②形質比とその分子形質の回帰式も、構成形質の統計量から、実際の値の分布と一致したものが得られた。③形質比と分子形質の相関係数も、近似度は劣るが、構成形質の統計量から推定できる。

形質比の遺伝力は構成形質より高くなり得ることから、これによる選抜が効果的な場合上の結果から、その効果の推定ができよう。