PROGRAMMING FOR DIGITAL COMPUTATION OF SUSPENSION BRIDGES UNDER VERTICAL, HORIZONTAL AND TORSIONAL LOADINGS

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1. Introduction

A treatment is introduced over "how-to" utilize the electronic digital computers for the calculation of suspension bridges under horizontal and torsional forces as well as vertical loadings.

Because of non-linearity of suspension bridges on the spatial deformation, statical values such as bending moments of stiffening girder are not easily determined that we may operate manual calculs.

Equilibrium equations for the structure are three differential equations on the deflection v , the horizontal displacement u , and the torsional deformations φ of the stiffening girder, and they are not independent each other, in other words, each equation shows non-linear differential one including every other variable such as u, v, φ and various static values. Some elastic equations are also to be considered in order to accomodate elastic conditions for cables.

Problems are to solve these equations and to obtain the numerical values for given suspension bridge. When we use the electronic digital computers, it is the most important task that how to compose the programme of calculation.

Since the most of the engineers who are concerned with the structural mechanic have few chances really to operate the digital computers by themselves, and operators or programmers of the computers also know few knowledge about the real meaning of the equations, so previous treatment of equations is necessary before they are undertaken by the programmers.

The previous treatment is, in short, to compose the calculating flow chart and to give the practical calculating formulas instead of analy-

tical display. When we are to utilize the digital computers, usual theoretical and analytical results and methods of calculs are not always suitable, we must therefore serch somewhat different sort of calculating methods.

The author introduces in the following articles how to compose the flow chart for the calculation of the vertical, horizontal and torsional behaviours of suspension bridge. In the second article, theoretical and analytical presentation is dealed. The third article shows the modification of such theoretical equations to practical formulas to fit in the practical solutions. The last article gives the conclusive results of the flow chart. This article be sent to any operators or programmers, we would obtain the values of given suspension bridge without any explanation.

2. Fundamental equations

Let a suspension bridge be considered as are shown in Figs. 1 and 2 with definitions of

Fig. 1

Fig. 2

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spatial co-ordinates, and of directions of deformations and forces. Two parallel cables are to carry total weight of structures by the infinitely close spaced curtain-like suspenders causing no stresses in the stiffening girder under the standard state, that is, there are no loadings and under usual temperature.

The shapes of the cables and the stiffening girder are to be defined as eqs. (1) and (2), wherein the cables take a symmetric curve of parabola and the stiffening girder is to have the initially longitudinal parabolic camber.

$$Y_c = \frac{4f}{l^2} z(l-z) \dots\dots\dots (1)$$

$$Y_J = h - \frac{4i}{l^2} z(l-z) \dots\dots\dots (2)$$

The deflection v and torsional angle φ of the stiffening girder cause the deflection v_1 , and v_2 of the left and right cables, respectively.

$$\left. \begin{aligned} v_1 &= v - b\varphi/2 \\ v_2 &= v + b\varphi/2 \end{aligned} \right\} \dots\dots\dots (3)$$

Each cable then varies its tension. Usually we consider the horizontal reaction of tensile forces that varies from H_w to $(H_w + H_1)$ and $(H_w + H_2)$ for each cable. By the close spaced suspenders, each cable gives the distributing load p_1 and p_2 to the stiffening girder.

$$\left. \begin{aligned} p_1 &= (H_w + H_1) \frac{d^2}{dz^2} (Y_c + v_1) \\ p_2 &= (H_w + H_2) \frac{d^2}{dz^2} (Y_c + v_2) \end{aligned} \right\} \dots\dots\dots (4)$$

As p_1 and p_2 work along the left and right side of the stiffening girder, they are composed so as to cause the vertical and horizontal forces along the sectional gravity center and, the twisting moment around the shear center or twisting center of the stiffening girder.

External forces or loadings against the stiffening girder are, the vertical loadings p_v like traffic weight, also the dead weight p_d of stiffening girder itself, the horizontal forces p_H like wind thrust, and the torque t_p which would occur when the traffic loads do not lie just on the center axis of bridge width or when wind thrust works so as to make a torque around the shear center.

The stiffening girder is then affected under the three component of forces as shown eqs. (5)

$$\left. \begin{aligned} \text{vertical} & \quad p_v + p_d - (p_1 + p_2) \\ \text{horizontal} & \quad p_H - \theta(p_1 + p_2) \\ \text{torque} & \quad t_p + \frac{b}{2}(p_1 - p_2) \end{aligned} \right\} \dots\dots\dots (5)$$

where θ is the slope angle of the suspenders, and it may be allowed to assume nearly equal to the ratio u/h . The dead weight p_d balances to the cable reaction as eq. (6)

$$p_d = -2H_w \frac{d^2 Y_c}{dz^2} = 2 \cdot \frac{8f}{l^2} \cdot H_w \dots\dots\dots (6)$$

Succeedingly, we consider the deformation and static values of the stiffening girder itself. Sectional bending moments around the two principal axis ξ, η and torsional moment around the longitudinal axis are;

$$\left. \begin{aligned} M_\xi &= -EJ_\xi \left[\frac{d^2 v}{dz^2} - \frac{d^2 u}{dz^2} \varphi \right] \\ M_\eta &= -EJ_\eta \left[\frac{d^2 u}{dz^2} + \frac{d^2 v}{dz^2} \varphi \right] \\ M_z &= -EC \frac{d^3 \varphi}{dz^3} + GK \frac{d\varphi}{dz} \end{aligned} \right\} \dots\dots\dots (7)$$

As for the reduced components inside the vertical and horizontal plain, we obtain from M_ξ and M_η ,

$$\left. \begin{aligned} M_x &= -EJ_\xi \frac{d^2 v}{dz^2} + (EJ_\xi - EJ_\eta) \varphi \frac{d^2 u}{dz^2} \\ M_y &= -EJ_\eta \frac{d^2 u}{dz^2} + (EJ_\xi - EJ_\eta) \varphi \frac{d^2 v}{dz^2} \end{aligned} \right\} \dots\dots\dots (8)$$

As the axis of the stiffening girder runs with the spatial curvature, bending moments M_x and M_y cause the additional torque around the axis;

$$t(z) = \frac{d^2 u}{dz^2} M_x - \left(\frac{d^2 Y_J}{dz^2} + \frac{d^2 v}{dz^2} \right) M_y \dots\dots\dots (9)$$

Differentiate the eqs. (8) twice by z , and the M_z of eq. (7) once, and we obtain three fundamental differential equations upon u , v and φ . Considering eqs. (5) and (9), and assuming the shear center of the section is not so deviated from the gravity center, we get;

$$\begin{aligned} EJ_\xi \frac{d^4 v}{dz^4} - (EJ_\xi - EJ_\eta) \frac{d^2}{dz^2} \left[\varphi \frac{d^2 u}{dz^2} \right] &= p_v, \\ + (2H_w + H_1 + H_2) \frac{d^2 v}{dz^2} + \frac{b}{2} (H_2 - H_1) \\ \times \frac{d^2 \varphi}{dz^2} - (H_1 + H_2) \frac{8f}{l^2} &\dots\dots\dots (10-1) \end{aligned}$$

$$EJ_\eta \frac{d^4 u}{dz^4} - (EJ_\xi - EJ_\eta) \frac{d^2}{dz^2} \left[\varphi \frac{d^2 v}{dz^2} \right] = p_H$$