GENERALIZED METHOD FOR DESIGNING RETRACTABLE FENDERS SYSTEM

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ABSTRACT

This paper describes a generalized method for designing the retractable fenders system. The presented hypothetical method of design is based on the mathematical analysis of the frictional resistance created during retraction, between the fender frame and both the ship hull and the sliding surface of the supporting brackets.

In the design procedure included herein, a particular attention is directed to the influence of the combined dynamic behaviour of the system, including the berthing ship, the fender and the marine structure.

INTRODUCTION

A fender is an important component of waterfront facilities, designed to absorb shock from the berthing ship impact, for the purpose of protecting both the ship and the marine structure. Retractable fender system has showed a satisfactory performance in this field1,2).

This type of fenders usually consists of a frame with front surface composed of vertical timber members fixed to horizontal wales. The frame is suspended by bars sliding on supporting brackets built in the berthing structure. The fender frame with its own weight, or with other auxiliary weight, moves, under the ship impact, inward and upward on the supporting brackets, thereby utilizing the weight of the fender to absorb the kinetic energy of the berthing ship. It has an economic feature, than other type of resilient fenders, in using the weight of the fender and the frictional resistance to retraction in order to absorb large quantity of kinetic energy3). Besides, in retractable fender, the rate of energy absorption increases with the retractable distance, making it suitable for berthing of large ships3).

THE EXISTING METHODS OF RETRACTABLE FENDER SYSTEM DESIGN

The outside force required to move the fender frame along the sliding surface of the supporting brackets is a direct function of:

i) The inclination of the sliding surface.

ii) The effective retractable weight.

iii) The coefficients of friction of the sliding surface.

Most designers select the sliding surface to have a constant inclination through the whole retraction1,2). In this case the deduced load-displacement relation will be of a constant load value, permitting the force of impact required to initiate the movement of the fender frame to be large. Thus, the berthing structure will be exposed to heavy loading from the beginning, which may create resonance response in the structure during retraction. This point has been regarded by Blancato who treated it in two ways: In the first he let the fender frame with its own weight only, moves after impact, then some weights which are raised at subsequent intervals will start to move one after another as the frame movement continues. In the second the sliding surface was designed to take the form of a broken-line shape with a small curved part at the beginning. These two solutions permit the acting outside load to be relatively small at the beginning of retraction, after then it increases in steps when the effective weight is increased, which is the case of Blancato first solution or when the inclination is changed which is the case of the second one. To the authors, the deduced stepped load-displacement relation will expose the berthing structure to sudden shocks at the points where the load is changed, which may increase the dynamic response of the berthing structure.

As regards the frictional resistance to retraction, some designers consider the frictional resistance to the motion of the effective weight on the sliding surface of the supporting brackets only3). They
overlook another frictional resistance created between the surfaces of contact of the ship hull and fender front surface. This latter force has a considerable effect, as will be seen later on increasing the outside load required to move the fender weight. Thus, neglecting of such force, which actually exists, is neither on the safe side in designing the berthing structure, nor on the economic side for fender design.

In the view of the preceding informations, the authors present a modified method for designing retractable fenders system. The modification involves two points:

i) Selection of the sliding surface to be curved in shape in the whole retraction. With this arrangement, the acting load on the structure will be increased gradually and smoothly during retraction.

ii) Consideration of the dynamic behaviour of the berthing system (ship, fender and structure) during berthing, in the design of the fender system (fender and structure).

PRINCIPLES OF RETRACTABLE FENDER SYSTEM

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Fig. 1 shows a typical retractable fender under the action of berthing ship. Fig. 2 represents the free diagram of the system at any point during retraction.

From Fig. 2 the horizontal load $P$ required to move the weight $W$ inward and upward along a plane inclined an angle $\theta$ should satisfy the following equation:

$$ P \cos \theta = (F_1 + W) \sin \theta + F_2 \quad \text{(1)} $$

from Fig. 1 the force $F_1$ equals to:

$$ F_1 = f \cdot P \quad \text{(2)} $$

in which $f$ denotes the coefficient of friction between the ship hull surface and the fender front surface. Also $F_2$ equals to:

$$ F_2 = (P \sin \theta + (F_1 + W) \cos \theta) \mu \quad \text{(3)} $$

where $\mu$ is the coefficient of friction between the weight suspending bars and the sliding surface of the supporting brackets.

Substituting (2) and (3) in (1) we obtain:

$$ P (\cos \theta - f \sin \theta - \mu \sin \theta - f \mu \cos \theta) = W (\sin \theta + \mu \cos \theta) $$

from which,

$$ P = \frac{\mu + \tan \theta}{1 - \mu (f + \mu) \tan \theta} W \quad \text{(4)} $$

For general utilizing of equation (4), let the sliding plane having a curved shape represented by the equation:

$$ y = G(x) \quad \text{(5)} $$

where $y$, $x$ are the horizontal and the vertical retraction of the weight $W$ at any time. From (5) we have:

$$ y' = G'(x) \quad \text{(6)} $$

represents the inclination of the sliding surface at a retractable distance $x$.

Substituting (6) into (4), the general form of the load $P$ will be obtained:

$$ P(x) = \frac{\mu + G'(x)}{1 - \mu (f + \mu) G'(x)} W \quad \text{(7)} $$

From (7) it can be seen that if the sliding surface takes the form of a curve represented by $G(X)$, the corresponding load-displacement relation will take the form of a curve too, thus permitting no sudden shocks during retraction. Smooth load-displacement curve can be easily controlled by good selection of the sliding surface equation $G(x)$.

CRITICAL VALUE OF THE LOAD $P$

From $P \sim W$ relation in (7), the load $P$ tends to infinity when the denominator of the right hand side of equation (7) tends to zero. Equating the denominator to zero we obtain:

$$ 1 - \mu (f + \mu) G'(x) = 0, 0 $$

from which,

$$ G'(x)_{cr} = \frac{1 - \mu f}{\mu + f} \quad \text{(8)} $$

$G'(x)_{cr}$ represents the critical inclination that should
be regarded in design.

**DEDUCTION OF THE SLIDING SURFACE SHAPE FOR GIVEN LOAD-DISPLACEMENT RELATION**

Sometimes, for convenience of the operation of berthing, fenders of specific load-displacement relations are recommended by some investigators. Such relations can be easily achieved in retractable fender system as follows:

From eq. (7) we can obtain,

\[
G'(x) = \frac{(1-\mu f)}{(\mu + f)} \frac{P(x)}{W} - \frac{\mu}{W} \quad \text{(9)}
\]

Integrating (9) with respect to \( x \), the sliding surface equation \( y = G(x) \) can be obtained which will equal to:

\[
y = G(x) = \int \left( \frac{(1-\mu f)}{(\mu + f)} \frac{P(x)}{W} - \frac{\mu}{W} \right) dx \quad \text{(10)}
\]

Integration of (10) can be carried out by numerical integration methods if it is difficult to be performed by ordinary integration.

Also the sliding surface shape can be deduced graphically in the following steps:

1. Plot the \( G(x) \sim P(x)/W \) relation given by equ. (9) for given values of \( \mu \) and \( f \). (for example as shown by Fig. 3 (a).

2. Divide the given \( P(x)/W \sim x \) curve into \( n \) equi-small-intervals.

3. At the mid-point of each interval the \( P(x)/W \) and \( G(x) \) values can be read from the curves stated in steps 1 & 2. Consider the deduced value of \( G'(x) \) is constant through the whole interval.

4. Starting from origin, taking \( y = 0 \) at \( x = 0 \), draw a line having an inclination equals to \( G'(x) \) corresponding to the first interval deduced as stated in step 3.

5. Repeating steps 3 & 4 for the second interval and so on till the "n"th interval. Finally the required sliding surface will be constructed by numerical computation.

**SELECTION OF THE MAX. AND MIN. INCLINATION OF THE SLIDING SURFACE**

The critical inclination \( G_{cr}(x) \) can be determined by substituting the given values of \( \mu \) and \( f \) into (h). The max. inclination should be selected of a lower value than \( G_{cr}'(x) \). That is due to:

i) Values of the coefficients of friction \( \mu \) and \( f \) are not absolutely constant during the whole life of the structure. They are affected by rusting, erosion, wetness and the amount of the oil covering the surfaces of contact. All these factors are not constant in the long run.

ii) For given values of \( \mu \), \( f \) and \( W \), the rate of change of the load \( P \) with respect to the inclination \( G'(x) \) increases sharply after a certain limit as seen in Fig. 3 which is plotted from equation (9).

A value between 0.5 and 0.6 of \( G'(x)_{cr} \) can be recommended for the max. inclination of the sliding surface at the end of retraction. (see Fig. 3 (b)).

As regards the min. Value of \( G'(x) \), it should satisfy two conditions;

i) Permitting the acting load required to initiate the movement of the fender weight to be not so large. This is accomplished by choosing the inclination at the beginning to be as small as possible.

![Fig. 3](image-url)
ii) When the ship separates from the fender at the end of berthing, the fender frame should descend freely under its own weight. For this purpose the following equation should hold:

\[ W \sin \theta > W \cos \theta \mu \]
\[ \tan \theta > \mu \]

or in general,

\[ G'(x) > \mu \]

A value equals to \((\mu + 0.5)\) can be recommended for the min. inclination of the sliding surface at the begining of retraction.

The inclination of the sliding surface in between the min. and the max. should be selected to permit increasing the acting load gradually and smoothly.

**DYNAMIC RESPONSE OF BERTHING STRUCTURE PROVIDED WITH RETRACTABLE FENDER**

In the analysis presented herein, the following assumptions are considered:

i) The kinetic energy of the berthing ship is dissipated by the fender weight retraction and the elastic deformation of the structure. All energy losses are neglected.

ii) The motion of the system after impact is a single-degree-of-freedom motion. The sway motion of the ship is only considered. Motion of three-degree-of-freedom, i.e. sway, swinging and rolling, has been presented by the authors elsewhere. The effect of both swing and rolling is relatively small compared to that of sway.

iii) Damping effect is neglected. This assumption is beering on the fact that designers are interested in the first peak of the motion, at which damping effect is not so great.

When a ship is berthing against a berthing structure provided with a retractable fender, the system will undergo the three following stages of motion:

**THE FIRST STAGE**

After the ship contacts the fender, there will be some elapsed time \((t_1)\) till the fender weight starts to retract. This due to that the outside acting load should reach a certain value \(P_1\) to initiate the fender motion, estimated by substituting \(G'(x) = G'(x)_{\text{min}}\) in equation (7).

\[ P_1 = \frac{\mu + G'(x)_{\text{min}}}{1 - \mu f - (\mu + f)G'(x)_{\text{min}}} W \]

From time of contact \((t_0)\) to the time \((t_1)\), the begining of retraction of the fender weight, all of the ship, fender and the berthing structure will vibrate together as one body. In this stage, the vibrated system undergoes a simple-degree-of-freedom motion. The following equation holds

\[ (M_1 + M_s + m)x_1 + k_1x_1 = 0 \]

where: \(M_1, M_s, m\), are the effective masses of the structure, ship and the fender respectively. \(k_1, x_1\) are the structure stiffness and deflection in direction of motion.

The initial velocity of the system \((\dot{x}_{10})\) is evaluated from consideration of kinetic energy conservation just before and after contact.

Assuming the velocity of the ship just before contact \(v_0\), acting in a plane normal to the face of berthing and passing through the center of gravity of the berthing structure.

The kinetic energy just before contact is given by

\[ \frac{1}{2} M_s v_0^2 \]

and the after contact is

\[ \frac{1}{2} (M_1 + M_s + m) x_{10}^2 \]

Equating the two energies, \(x_{10}\) will be given in the form:

\[ x_{10} = \frac{M_s}{M_1 + M_s + m} v_0 \]

However, the mass of fender and berthing structure are very small compared with the ship mass, that can be neglected giving \(x_{10}\) value as

\[ x_{10} = 0,0 \]

The solution of (12) with the above conditions is given in the form,

\[ x_1 = \frac{v_0}{w} \sin wt \]

where

\[ \omega = \frac{k_1}{M_1 + M_s + m} = \frac{k_1}{M_s} \]

At time \(t_1\) the structure is deflected a distance \(x_{11}\) given by;

\[ x_{11} = \frac{v_0}{w} \sin w t_{1} \]

besides, the structure reaction \(R_1\) will equal to the acting load \(P_1\) i.e.

\[ R_1 = k_1x_{11} = P_1 \]

Substituting (11) and (14) into (15) it yields to;

\[ k_1 \cdot \frac{v_0}{w} \sin wt_{1} = \frac{\mu + G'(x)_{\text{min}}W}{1 - \mu f - (\mu + f)G'(x)_{\text{min}}} \]

from which the elapsed time \(t_1\) will be

\[ t_1 = \frac{1}{w} \sin^{-1} \left( \frac{\mu + G'(x)_{\text{min}}W}{1 - \mu f - (\mu + f)G'(x)_{\text{min}}} \right) \]

**THE SECOND STAGE**

When the fender comes into action and starts to retract, the ship and fender will move together with velocity different from that of the structure. For
evaluation of the new velocities, the momentum and kinetic energy conservation just before contact and after the fender weight started to retract, are to be considered;

\[
\begin{align*}
\text{Momentum} & \quad M_2 \cdot v_2 = M_1 \cdot v_1 + (M_2 + m) v_2 \\
\text{K energy} & \quad \frac{1}{2} M_2 v_2^2 = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} (M_2 + m) v_2^2 + V_{st} \\
\end{align*}
\]

\(V_{st} = \text{strain energy stored by the structure up to time } t_1 = \frac{1}{2} k_1 x_{11}^2\)

Solution of (17) yields to

\[
\begin{align*}
v_1 = & \quad \frac{M_2}{M_1 + M_2} v_2 - \frac{k_1}{M_1 + M_2} (M_2 + M_1) x_11 \\
v_2 = & \quad \frac{M_2}{M_1 + M_2} v_2 - \frac{k_1}{M_1 + M_2} (M_2 + M_1) x_11 \\
\end{align*}
\]

where \(m\) is very small compared to \(M_2\) that can be neglected. In this stage, the following equation of motion holds ;

For structure

\[
M_1 \cdot \dot{x}_1 + k_1 x_1 - P(x) = 0
\]

For fender

\[
(M_2 + m) \cdot \dot{x}_2 + P(x) = 0
\]

\(P(x)\) is calculated from eqn. (7) by substituting for \(G'(x)\) with \(G'(x_2-x_1)\).

The initial conditions of motion are given as ;

\[
\begin{align*}
x_1 = 0 & \quad \dot{x}_1 = v_1 \\
x_2 = 0 & \quad \dot{x}_2 = v_2
\end{align*}
\]

Solution of eqn. (18) is carried out by numerical integration using digital computer.

**THE THIRD STAGE**

In case when the fender weight retracted to its max. displacement and the ship is not brought to rest, the fender will contact the structure and all the system will move together as one body. In this stage eqn. (12) will hold with the following initial conditions;

\[
\begin{align*}
x_1 = 0 & \quad \dot{x}_1 = v_1 \\
v_1 & \quad \text{is calculated by consideration of momentum conservation just before and after time } t_2, \text{ this leads to}:
\end{align*}
\]

\[
v_1 = \frac{M_1 \cdot \dot{x}_{12} + (M_2 + m) \cdot \dot{x}_{22}}{M_1 + M_2 + m}
\]

\(x_{12}, \dot{x}_{22}\) are the velocities of the structure and ship respectively at time \(t_2\), the time of max. retraction, estimated from eqn. (18).

**SHIP ENERGY DISSIPATION**

As stated before, the ship kinetic energy is assumed to be dissipated by fender retraction and structure elastic deformation.

Energy dissipated by fender =

\[
V_f = \int P(x) \, dx = \int [P(x_2-x_1)] \, d(x_2-x_1)
\]

Energy stored by structure =

\[
V_s = \int k_1 x_1 \, dx_1
\]

**APPLICATION**

A ship of 20,000 ton displacement weight is approaching to berth against a berthing structure provided with a retractable fender. Design of the structure and fender is required under the following data of berthing.

\[
\begin{align*}
\text{\(f_0\)} = 20 \text{ cm/sec} & \quad \text{\(M_1\}) = 0.3 \text{ ton-sec}^2/\text{cm} \\
\text{\(M_2\}) = 30.0 \text{ ton-sec}^2/\text{cm} & \quad \text{\(f_{\text{steel on steel}}\) = 0.3} \\
\text{\(f_{\text{steel on timber}}\) = 0.25} & \quad \text{\(\mu = 0.3\) (steel on steel)}
\end{align*}
\]

Assuming the sliding surface be curved in shape represented by

\[y = G(x) = Ax^B + Cx\]

from which

\[
y' = G'(x) = ABx^{B-1} + C
\]

Further assuming

\[
\text{\(y'_{\text{max}} = y'_{\text{min}}\)
\]

and

\[
\text{\(y'_{\text{max}} = X\)}
\]

where \(X\) denotes the max. retractable distance.

Substituting (22) into (21) the following is obtained ;

\[
G'(x) = (y'_{\text{max}} - y'_{\text{min}}) \left( \frac{x}{X} \right)^{B-1} + y'_{\text{min}}
\]

From (8) \(y_{\text{cr}}'\) equals ;

\[
y'_{\text{cr}} = \frac{1 - 0.30 + 0.25}{0.30 + 0.25} = 1.683
\]

From \(y_{\text{cr}}'\) value and from Fig. 3, \(y'_{\text{max}}\) can be taken as ;

\[
y'_{\text{max}} = 1.0
\]

Also, as stated before \(y'_{\text{min}}\) can be taken as ;

\[
y'_{\text{min}} = 0.35
\]

Substituting with these values into (23) it yields to ;

\[
y' = G'(x) = 0.65 \left( \frac{x}{X} \right)^{B-1} + 0.35
\]

Substituting with this value into (7) we obtain ;

\[
P(x) = \frac{1 + \left( \frac{x}{X} \right)^{B-1}}{1.127 - 0.846 \left( \frac{x}{X} \right)^{B-1}} W
\]

Trials has been made, using two structures having different stiffness namely, a rigid structure \((k_1 = 800 \text{ t/cm})\) and flexible one \((k_1 = 100 \text{ t/cm})\). Each of the two structures was provided with different types of retractable fender. The fender type can be easily
86

Komatsu, Salman:

designated by substituting in (24) different values for the parameters $X, B$ and $W$.

Solution has been carried out by digital computer. New Mark $\beta$ method, with $\beta=0.4$, was used for numerical integration of motion equations. The flow chart for computation is shown by Fig. 8. Table 1 includes the computed results, while Figs. 3, 4, 5, 6, 7 represent some relations graphically.

Fig. 3(a) represents the relation between the outside acting force $P(x)$ divided by the retractable weight $W$ and the sliding surface inclination $G'(x)$ for different combination of the coefficients $f$ and $\mu$. The curve is plotted by substituting $f$ and $\mu$ values in eq. "7". It can be seen that the derived curves possess an identical geometrical relation. If the $G'(x)$ values for each curve, is divided by its corresponding $G'(x)_{cr}$ value calculated from eq (8), the aforementioned relation can be put in dimensionless representation as shown by Fig. 3(b). From Fig. 3(b), it can be seen that the different combination of $f$ & $\mu$ almost leads to one curve, and that the rate of the change of $P(x)/W$ with respect to $G'(x)/G'(x)_{cr}$ increases sharply at $G'(x)$ value rea-
Generalized Method for Designing Retractable Fenders System

ches 60% of $G'(x)_{cr}$ value. This value of $G'(x)$ is recommended as max inclination of the sliding surface as stated before.

In Fig. 3, $P_i/W$ denotes the $P(x)/W$ value corresponding to $G'(x)=0$. The variation in $P_i/W$ values due to the different combination is so small that, the average value is considered in plotting Fig. 3.

Fig. 4 shows the shape of the sliding surfaces that used in the trials of the solved example.

These curves have the following common characteristics,

1) The max-retractable distance $X$ equals 20 cm.
2) $G'(x)_{min}$ and $G'(x)_{max}$ equal to 0.35 and 1.0 respectively.

Fig. 5 represents dynamic load and the energy dissipated capacity provided by four types of fender with the same retractable weight sliding on the four surfaces shown by Fig. 4.

Fig. 6 shows the relation between the dissipated energy by fender retraction and deflection of the structure with respect to the shape of sliding surface designated by the exponent $B$. It can be seen that at $B=1.2$, the strain energy stored by the structure is min.

Fig. 7 represents the typical dynamic response of a berthing ship and structure in the three stages of motion explained before, namely from time of contact “$t_0$” to $t_1$ when the fender comes into action and from $t_1$ to $t_2$ when the fender reaches its max travel and from $t_2$ to $t_3$ where the ship separates from the berthing structure.

COMMENTS ON THE RESULTS

From Table 1 the followings can be deduced:

1) For rigid structure, $(k_1=800 \text{ t/cm})$ the part of the ship kinetic energy that stored by the structure and that dissipated by fender, depends on the relative stiffness of the structure and fender, and consequently on their combined dynamic behaviour during berthing. This is clear from comparing cases 1 and 6 with 2 and 7 respectively. Fenders in cases 2 & 7 provide larger amount of energy dissipation than those in cases 1 & 6, while the energy stored by structure is on the contrary.

2) In case 3, the energy dissipated by fender and that stored by structure are almost the same as those included in case 6. Selecting one of the two cases for design will depend on the costs of the fender in each case (comprising fender frame, auxiliary weight and supporting brackets)

3) Also, the choice between case 3 or 6 (rigid structure.) and case 10 (flexible structure.) will depend on the function of the structure. If the berthing structure, carried heavy loads, there is no choice; deflection
must be limited and the structure should be rigid.
Moreover, the choice of any case for design, is
governed by the maximum acceptable hull load. For
ships from 15 000 to 20 000 tons, hull pressure of 35
psi is acceptable in general, with overloads of up
to 50 psi as an upperlimit. (7)

CONCLUSIONS

This paper has demonstrated the method of de-
sign a berthing structure provided with a retractable
fender system. Formulae have been derived by lin-
king the ship acting load with the fender effective
weight, sliding surface inclination and the coefficents
of friction of the sliding surfaces. It is also shown,
by the solved example, that the combined dynamic
behaviour of the system has a considerable effect on
design. By the design procedure included in the
solved example and the computer program given, it
is possible to design an optimum economic berthing
structure provided with the suitable fender.

NOTATIONS

\( k_s \) = Structure spring constant.
\( m \) = Mass of fender.
\( M_s \) = Effective mass of structure.
\( M_t \) = Virtual mass of ship in horizontal motion.
\( t_2 \) = Time of contact.
\( t_3 \) = Time of beginning of fender retraction.
\( t_\text{r} \) = Time of the end of fender retraction.
\( t_\text{s} \) = Time of separation of ship from the structure
(max. sway)
\( v_0 \) = Velocity of ship at time ‘to’ normal to the
face of the structure.
\( v_1 \) = Velocity of the structure, just after the fender
starting to retract.
\( v_2 \) = Velocity of the ship just after the fender start-
ing to retract.
\( v_3 \) = Velocity of ship and structure, just after the
fender reached its max. retractable distance.
\( x_1 \) = Deflection of the structure.
\( x_2 \) = Displacement of ship.
\( x \) = Fender retraction = \( x_2 - x_1 \).

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