EFFECTS OF DATA CONDITIONING ON MASS AND DRAG COEFFICIENTS

Robert T. HUDSPETH*, John W. LEONARD*, Minoru KUBOTA**, and Hisao KOTOGUCHI***

The effects of data conditioning on the mass and drag coefficients \(C_m\) and \(C_d\) are reviewed by two geometric and one numerical interpretations. Two geometric analyses of data conditioning proposed by Dean demonstrate that when the Dean eccentricity parameter \(E\) equals unity, the data are equally well-conditioned for determining \(C_m\) & \(C_d\). For simple harmonic data, the Dean eccentricity parameter may be shown to be proportional to the Keulegan-Carpenter parameter, \(K\), i.e., \(E = \frac{\sqrt{3} K}{2\pi^2}\). When \(E = 1.0\), then \(K \approx 11.40\) and the Dean error ellipse is a circle with zero eccentricity. The matrix condition number of the 2 \(\times\) 2 matrix used to determine \(C_m\) & \(C_d\) in a best least-squares sense becomes unity when \(K \approx 13.16\) and \(E \approx 1.15\). Two sets of experimental data are compared with the two geometric and one numerical analyses.

Keywords: mass coefficient, drag coefficient, Dean eccentricity parameter

1. INTRODUCTION

Much effort has been directed toward resolving the parametric dependency of the two empirical force coefficients, \(C_m\) and \(C_d\), that are used to estimate the wave-induced pressure loads on small members by the Morison equation (cf. Sarpkaya and Isaacson\(^1\) or Chakrabarti\(^2\)). The two most commonly used parameters are the Reynolds parameter, \(Re = U_m D / \nu\) and the Keulegan-Carpenter parameter, \(K = U_m T / D\). However, only Dean\(^3\) appears to have recognized the importance of the condition of the data when identifying these two empirical force coefficients in any parameter identification algorithm. Although the error ellipse concept was originally proposed by Dean to demonstrate geometrically the condition of data for identifying \(C_m\) and \(C_d\); his original development lacked the ability to demonstrate that the alignment of the axes of the error ellipse depends explicitly on either \(Re\) or \(K\). Because the data are relatively better-conditioned to identify the empirical force coefficient on the axis that is parallel to the semi-minor axis of the error ellipse, it is essential to be able to demonstrate that the alignment of the axes of the error ellipse depends explicitly on either \(Re\) or \(K\) or, preferably, both of these parameters.

It is possible to demonstrate that, for data with kinematics that are simple harmonic, the Dean eccentricity parameter, \(E\), is proportional to the Keulegan-Carpenter parameter, \(K\). Thus, the Dean eccentricity parameter, \(E\), provides an explicit measure of the parametric dependency of the alignment of the semi-minor axis of the error ellipse on the parameter \(K\). Specifically, when \(E = \frac{\sqrt{3} K}{2\pi^2} < 1.0\), then \(K < 11.40\) and the semi-minor axis of the error ellipse is parallel to the \(C_m\) axis. Conversely, when \(E > 1.0\), then \(K > 11.40\) and the semi-minor axis is parallel to the \(C_d\) axis. When \(E = 1.0\), then \(K = 11.40\) and the error ellipse is a circle with zero eccentricity. It is interesting to note that a value of \(K \approx 11.40\) is approximately the value of \(K\) at which the peak in \(C_d\) and the trough in \(C_m\) occur in the replotted Keulegan-Carpenter data (cf. Sarpkaya and Isaacson\(^1\) or Chakrabarti\(^2\)).

The Dean error ellipse methodology may be compared geometrically with an amplitude/phase analysis. In addition to demonstrating geometrically the importance of the condition of the data, the amplitude/phase error methodology also demonstrates the importance of errors in the amplitudes/phases of the kinematics. In contrast to the Dean error ellipses illustrated in Fig. 1, the amplitude/phase graphs demonstrate geometrically the parametric dependency of \(C_m\) and \(C_d\) on the parameter \(K\) (or \(E\)) by the magnitude of the slope of the contours of the dimensionless O'Brien force ratio, \(W = |f_m|/|f_m|\), passing through the origin for zero error in phase. The advantage of the amplitude/phase error methodology is that the separate plots required by the Dean error ellipse methodology for each fixed value of \(W = |f_m|/|f_m|\) may be replaced by a single plot with contours of fixed values of \(W\). Comparisons with synthetically phase-shifted laboratory data for \(E \geq 1.0\) (or \(K \geq 11.40\)) are excellent for phase shifts in the range of \(|\omega t| < \pi/8\).

A brief definition of the term data conditioning may be appropriate here. By data conditioning, we...
mean the ability of a least-squares algorithm to locate a global minimum on an error surface for given wave kinematic/force data (cf. Marquardt5). It is, of course, related to the numerical condition number of a least-squares error matrix (cf. Atkinson6). The error matrix number is computed by four standard measures for the Morison equation. The matrix condition number is identically equal to unity when $K=13.16$ and $E=1.15$.

Because the Morison equation represents the inertia of the fluid, it does not contain an explicit constitutive relationship for the viscous stress tensor. Therefore, it is not possible to demonstrate a similar explicit dependency of the alignment of the axes on the Reynolds parameter, $Re$.

2. DEAN ERROR ELLIPSE

The mean squared error, $\varepsilon^2$, between the “true” force per unit length (denoted by upper case unprimed letters), $F(wt)$, and the “computed” force per unit length (denoted by lower case primed letters), $f'(wt)$, may be estimated from

$$E^2 = \langle [F(wt) - f'(wt)]^2 \rangle$$

where the temporal averaging operator, $\langle \cdot \rangle$, is defined for simple harmonic data as $\langle \cdot \rangle = (2\pi)^{-1} \int_0^{2\pi} (\cdot) d(wt)$.

The “true” force is assumed to be represented exactly by the two-term Morison equation and is given by

$$F(wt) = F_m(wt) + F_d(wt) = K_m U(wt) + K_d U^2(wt)$$

and the “computed” force is given by

$$f'(wt) = f_m'(wt) + f_d'(wt) = K_m u(wt) + K_d u^2(wt)$$

where the “true” and “computed” generalized inertia and drag coefficients are, respectively

$$K_m = C_m \left[ \frac{\rho \pi D^2}{4} \right] ; \quad K_d = C_d \left[ \frac{\rho D}{2} \right]$$

$$K'_m = C'_m \left[ \frac{\rho \pi D^2}{4} \right] ; \quad K'_d = C'_d \left[ \frac{\rho D}{2} \right]$$

The “computed” inertia and drag coefficients are denoted by superscript primes (') in order to distinguish them from the “true” coefficients which are unprimed.

Substituting Eq. (2b) into Eq. (1) and expanding yields

$$\varepsilon^2 = \left[ \left( \frac{\rho D}{2} \right)^2 \langle u'^2 \rangle X^2 + \left( \frac{\rho \pi D^2}{4} \right)^2 \langle u'' \rangle Y^2 \right] + 2\left( \frac{\rho D}{2} \right) \left( \frac{\rho \pi D^2}{4} \right) \langle u' \rangle \langle u'' \rangle XY - 2\left( \frac{\rho D}{2} \right) \langle Fu \rangle \langle u'' \rangle X - 2\left( \frac{\rho \pi D^2}{4} \right) \langle Fu' \rangle Y + \langle F^2 \rangle$$

where $X = C_m$ and $Y = C_d$.

Equation (3) is the conic section equation for an ellipse (Dean and Dalrymple4 or Thomas7) whose origin has been translated and rotated; i.e.

$$(\alpha X)^2 + 2HXY + (\beta Y)^2 + 2GX + 2GY + C = 0$$

The coordinates of the translated and rotated origin [viz., $X_0 = \min(C_d)$ and $Y_0 = \min(C_m)$] may be computed from the data according to

$$X_0 = \min(C_d) = \frac{\langle u'' \rangle \langle Fu \rangle - \langle u' \rangle \langle Fu' \rangle}{(\rho \pi D^2/4) DET[X_0, Y_0]}$$

$$Y_0 = \min(C_m) = \frac{\langle u'' \rangle \langle Fu' \rangle - \langle u' \rangle \langle Fu \rangle}{(\rho \pi D^2/4) DET[X_0, Y_0]}$$

$$DET[X_0, Y_0] = \frac{3\omega^2 A^6}{16}$$

and the angle of rotation from

$$\cot 2\theta = \frac{\langle u'' \rangle - (\rho \pi D/2)^2 \langle u'' \rangle}{\pi D \langle u' \rangle \langle u'' \rangle}$$

If the “true” and “computed” kinematics are simple harmonic oscillations given by

$$U(wt) = A cos(wt) \quad \dot{U}(wt) = -A \omega \sin(wt) \quad \langle \dot{u} \rangle = \langle \dot{u}' \rangle$$

then the inner products $\langle \cdot \rangle$ required in Eq. (3) become

$$\langle u'' \rangle = \frac{3\alpha^4}{8} \quad \langle u'' \rangle = -\frac{\langle \omega \rangle^2}{2} \quad \langle u' \rangle \langle u'' \rangle = 0$$

and Eqs. (3 & 5) reduce to

$$\varepsilon^2 = \left( \frac{\rho D}{2} \right)^2 \langle u'^2 \rangle X^2 + \left( \frac{\rho \pi D^2}{4} \right)^2 \langle u'' \rangle Y^2$$

$$- 2\left( \frac{\rho D}{2} \right) \left( \frac{\rho \pi D^2}{4} \right) \langle u' \rangle \langle u'' \rangle XY$$

$$- 2\left( \frac{\rho \pi D^2}{4} \right) \langle Fu \rangle \langle u'' \rangle X - 2\left( \frac{\rho \pi D^2}{4} \right) \langle Fu' \rangle Y + \langle F^2 \rangle$$

$$\cot 2\theta = \infty$$

where the Keulegan-Carpenter parameter, $K$, is defined for simple harmonic kinematics by
Equations (7) demonstrate the parametric dependency of the translation of the coordinates of the origin \((X_0, Y_0)\) on \(K\) for simple harmonic kinematics. Equations (7) are similar to those given by Dean and Dalrymple which are neither dimensionally correct nor demonstrate an explicit parametric dependency on \(K\).

In order to demonstrate explicitly the parametric dependency of the eccentricity of the error ellipse on the parameter \(K\) for simple harmonic kinematics, not that: 

\[ a^2 = \left(\frac{\pi D}{2}\right)^2 <u'^2> \leq 0; \quad \beta^2 = (\rho D^2/4)^2 <u'^2> \geq 0; \quad R^2 = (8/3)<F|u'\mid u'|>^2/\alpha^4 + 2<\dot{F}\dot{u'}^2/a^2 + e^2 - \langle \dot{F}^2 \rangle; \quad \text{and} \quad H = 0; \]

which implies (Thomas) that the translated axes of the Dean error ellipse are parallel to the Cartesian axes, \(X = C_d\) and \(Y = C_m\), as illustrated in Fig. 2.

Dean and Dalrymple demonstrate that it is more illustrative for the case of simple harmonic data to write the conic section equation by completing the square of Eq. (4) in the following manner:

\[ \frac{(X-X_0)^2}{(\alpha/\beta)^2} + \frac{(Y-Y_0)^2}{(\alpha/\beta)^2} = 1 \]  \hspace{1cm} (9a)

The eccentricity of the error ellipse may be defined by the Dean eccentricity parameter, \(E\), which is given by

\[ E^2 = (\alpha/\beta)^2 = 3(K/2\pi^2)^2 \]  \hspace{1cm} (9b)

The eccentricity determines geometrically the condition of the data for identifying \(C_d\) and \(C_m\) may be evaluated from the ratio \(\alpha/\beta\). For \(K < 2\pi^2/\sqrt{3} = 11.40\), this eccentricity becomes \(E^2 < 1.0\); and \(R/\alpha = \) semi-minor axes parallel to the \(Y (=C_m)\) axis and \(R/\beta = \) semi-major axis parallel to the \(X (=C_d)\) axis. For \(K > 2\pi^2/\sqrt{3} = 11.40\), the ratio becomes \(E^2 > 1.0\) and \(R/\alpha = \) semi-major axis parallel to the \(Y (=C_m)\) axis and \(R/\beta = \) semi-minor axis parallel to the \(X (=C_d)\) axis. The data are relatively better conditioned for identifying the force coefficient that is parallel to the semi-minor axis (cf. Fig. 2 and Dean).

The eccentricity, \(e^2\), of the error ellipse determines geometrically the condition of the data for identifying \(C_d\) and \(C_m\). It is easily shown to be computed from (Thomas)

\[ e^2 = 1.0 - E^2; \quad \alpha/\beta < 1.0 \]

\[ -2; \quad E > 1.0; \quad \alpha/\beta > 1.0 \]  \hspace{1cm} (10a, b)

The parametric dependency of the eccentricity of the Dean error ellipse and the alignment of the axes are now shown to depend explicitly on the Keulegan-Carpenter parameter, \(K\), by the Dean eccentricity parameter, \(E\), defined in Eq. (9b).

There appear to be at least a set of physical data in which the significance of the Dean eccentricity parameter is obvious. The physical data set are the well-known replotted Keulegan-Carpenter force data for a circular cylinder (cf. Sarpkaya and Isaacson\(^1\) and Chakrabarti\(^2\)).

Fig. 2 (cf. Sarpkaya and Isaacson\(^1\) and Chakrabarti\(^2\)) demonstrates that the peak in the \(C_d\) graph and the trough in the \(C_m\) graph of the replotted Keulegan-Carpenter data occur approximately at a Dean eccentricity parameter of unity or \(K \approx 11.40\). A Dean eccentricity parameter of unity identifies the value of \(K\) at which the eccentricity of the error ellipse is zero and the semi-major and semi-minor axes of the Dean error ellipse are equal. Data with values of \(K < 11.40\) are relatively well-conditioned for identifying \(C_m\) (i.e., the semi-minor axis is parallel to the \(C_m\) axis for \(E < 1.0\)); while data with values of \(K > 11.40\) are relatively well-conditioned for identifying \(C_d\) (i.e., the semi-minor axis is parallel to the \(C_d\) axis for \(E > 1.0\)).

It is obvious in Fig. 2a that the values of \(C_m\) all collapse onto a single line for \(E < 1.0\) (or \(K < 11.4\)). However, this is not the case for \(E > 1.0\) (or \(K > 11.4\)). Conversely, in Fig. 2b, there is less correlation in \(C_d\) for \(E < 1.0\) (or \(K < 11.4\)); but the data are relatively better correlated for \(E > 1.0\) (or \(K > 11.4\)). The Dean eccentricity parameter, \(E\), identifies the peak in \(C_d\) and the trough in \(C_m\) with
data that are equally well-conditioned for identifying these two coefficients. It also delineates the two regions where the Reynolds parameter, $Re$, must also be considered in addition to $K$.

3. AMPLITUDE/PHASE ERROR ANALYSIS

The two-term Morison equation coefficients ($C_d$ and $C_m$) for a cylinder in waves depend on the correct measurements of amplitudes and phase shifts between the ambient wave kinematics and the measured force. Hudspeth, et al. used a regression analysis to develop an algorithm that illustrated how these force coefficients will change if incorrect amplitudes or phase shifts are introduced into the analysis due to: errors in the data acquisition; numerical data reduction techniques; or natural variations due to vortex shedding. The algorithm assumed that the two-term Morison equation modeled the measured forces exactly and that linear wave theory modeled the wave kinematics exactly. A least-squares analysis of the time-averaged, meansquared error between measured and predicted forces was used. Dimensionless variations in the force coefficients were shown to depend on two dimensionless parameters: 1) a dimensionless force amplitude ratio, $W$ (proportional to the Dean eccentricity parameter, $E$, and to the Keulegan-Carpenter number, $K$); and 2) a dimensionless velocity amplitude ratio, $V$, which is a function of the vertical elevation in the water column, $z$. Their algorithm combined both the effects of data conditioning and wave amplitude/phase that complemented the earlier development by Dean. Good agreement was obtained with laboratory data of wave forces on a vertical, sand-roughened cylinder wherein the force measurements were purposefully phase shifted with respect to the wave phase in small increments, up to $\pm 33.8$ degrees ($\pm 3\pi/16$ radians).

Variability observed in the values of $C_d$ and $C_m$ may be due to several causes: the accuracy of the two-term Morison equation; incorrect estimates or measurements of the wave kinematics; the influence from unknown roughnesses; measurement errors; poor conditioning of the data; wake encounter effects; or the inadvertent introduction of erroneous amplitudes or phase shifts into the data acquisition or the numerical analysis.

There are several possible causes for a phase shift error. For example, there may be a spatial separation between the wave profiler, the current meter (if used), and the pile on which the force is measured. The electronic or numerical filtering of data signals may introduce both a phase shift and an amplitude distortion. The sequential sampling of multiple data channels by analog-to-digital recorders introduces a small phase shift. If these potential amplitude and phase shift errors and the conditioning of the data for parameter estimation are not appropriately considered, variations in the values of the force coefficients will result.

Fig. 3 illustrates a typical experimental configuration. The wave staff and current meter (which may not be superimposed as shown) are located at the origin, while the pile is located at some distance from the origin. A phase shift in the measurements will result from this spacing which must be taken into account.

The “true” force is assumed to be represented exactly by the two-term Morison equation given by Eq. (2a). An erroneous phase shift, $\omega \tau$, between the “computed” force and the “computed” kinematics is denoted by $\omega(t+\tau)$ in Eqs. (6c,d).

Minimizing the mean-square error given by Eq. (1) with respect to the “computed” coefficients (denoted by superscript primes) according to

$$\frac{\partial \varepsilon^2}{\partial K_m} = 0; \quad \frac{\partial \varepsilon^2}{\partial K_d} = 0$$

yields the following 2 equations:

$$-K_d \langle U | U | \dot{u} \rangle - K_m \langle \dot{u} | u | \dot{u} \rangle + K_d \langle u | u | \dot{u} \rangle = 0$$

$$-K_d \langle U | U | u | u \rangle - K_m \langle \dot{U} | u | u \rangle + K_d \langle u^4 \rangle = 0$$

Equations (12) may be rearranged to give a dimensionless inertia coefficient ratio, $\varepsilon_m$, and a dimensionless drag coefficient ratio, $\varepsilon_d$, defined by the following:

$$\varepsilon_m = C_m = \frac{C_m}{C_m} \left( \frac{2}{\pi D} \right) \left( \frac{U | U | \dot{u}}{U | U | \dot{u}} \right)$$

$$\varepsilon_d = \frac{C_d}{C_d} \left( \frac{U | U | u | u}{U | U | u | u} \right)$$

It is not a trivial task to evaluate some of the
integrals in Eqs. (13a, b) that require absolute values of elementary transcendental functions. Both negative and positive phase shifts must be considered.

The dimensionless inertia coefficient ratio, $\varepsilon_m$, is given by

$$\varepsilon_m = V(z) \left[ \frac{\cos(\omega t) - \left( \frac{8W}{3\pi} \right) \sin(\omega t)}{|\omega t|} \right] ;$$

$$|\omega t| \leq \pi \quad \text{................................................. (14a)}$$

and the dimensionless drag coefficient ratio, $\varepsilon_d$, by

$$\varepsilon_d = \left( \frac{1}{3\pi} \right) V(z) V(z) \left[ \frac{2\pi + 3\sin(|2\omega t|) - 2|2\omega t|}{|2\omega t| - \pi} \cos(|2\omega t|) + \frac{32}{3W} \sin(|\omega t|) \right] ;$$

$$|\omega t| \leq \pi \quad \text{................................................. (14b)}$$

where the dimensionless velocity amplitude ratio, $V(z)$; the dimensionless O'Brien force amplitude ratio $W$; and Keulegan-Carpenter number, $K$, are defined with using the “true” dimensional amplitude of the horizontal component, $A(z)$ and the “computed” horizontal component, $a(z)$ of water particle velocity by

$$V(z) = \frac{A(z)}{a(z)} ; \quad W = \frac{C_d}{C_m} \frac{K}{\pi^2} ; \quad K = \frac{A(z)T}{D} \quad \text{................................................. (15, a, b, c)}$$

The O'Brien force ratio, $W$, is the ratio between the “true” drag force and the “true” inertia force. The magnitude of $W$ may be used to determine the conditioning of the data to estimate the force coefficients because it is directly proportional to the Dean eccentricity parameter, $E$; $\ldots$

$$W = \left[ \frac{C_d}{C_m} \right] \left[ \frac{2E}{\sqrt{3}} \right]$$

in which the Dean eccentricity parameter, $E$, is defined by Eq. (9b).

The dimensionless ratios defined by Eqs. (14a, b) incorporate not only the effects of amplitude/phase shift errors but also the conditioning of the data for estimating the force coefficients through the parametric dependency on the Dean eccentricity parameter, $E$. The parametric dependency on the two dimensionless parameters, $V$ and $W$ (or $E$ or $K$), will be evaluated separately.

Fig. 4 illustrates the parametric dependency of the dimensionless drag coefficient ratio, $\varepsilon_d$, on the dimensionless force amplitude ratio, $W$ (or $E$ or $K$), for a constant dimensionless velocity amplitude ratio $V(z) = 1.0$ (i.e., the “computed” velocity amplitude $=$ the “true” ambient velocity amplitude). For relatively large values of $W$ ($>4.0$), $\varepsilon_d$ is not sensitive to the magnitude of the phase shift near the origin, $|\omega t| \sim 0$. Relatively large values of $W$ (or $E$ or $K$) imply that the data are drag-dominated and are relatively well-conditioned for determining the drag coefficient, $C_d$. Note that if $C_d \approx 0.9$ and $C_m \approx 2.0$, then $K \approx 22W$. For relatively small values of $W$ ($\leq 0.1$), $\varepsilon_d$ is very sensitive to the magnitude of the phase shift near the origin. This implies that for small values of $W$ ($\leq 0.1$) (or $K \leq 2.2$), the data are relatively ill-conditioned for determining the drag coefficient, $C_d$. The slope, $S_d$, of the dimensionless drag coefficient ratio, $\varepsilon_d$, near the origin provides additional insight into the conditioning of the data for estimating $C_d$ and will be examined in detail later.

Fig. 5 illustrates the parametric dependency of the dimensionless inertia coefficient ratio, $\varepsilon_m$, on the dimensionless force amplitude ratio, $W$ (or $E$ or $K$), for a constant velocity amplitude ratio $V(z) = 1.0$. For relatively small values of $W$ ($<0.1$), $\varepsilon_m$ is not very sensitive to the magnitude of the phase shift near the origin, $|\omega t| \sim 0$. Relatively small values of $W$ (or $E$ or $K$) imply that the data are inertia-dominated and are relatively well-conditioned for determining the inertia coefficient, $C_m$. For relatively large values of $W$ ($>4.0$), $\varepsilon_m$ is very

![Fig. 4](image1.png)

![Fig. 5](image2.png)
sensitive to the magnitude of the phase shift near the origin. This implies that for large values of $W (> 4.0)$ (or $K > 88$), the data are relatively ill-conditioned for determining the inertia coefficient, $C_m$. The slope, $S_m$, of the dimensionless inertia coefficient ratio, $\varepsilon_m$, near the origin provides insight into the conditioning of the data for estimating $C_d$ and will be examined in detail later.

For small phase shifts ($|\omega\tau| \approx 0$) limiting values for Eqs. (14a and 14b) are given by the following:

$$\varepsilon_m \sim V[1 - (8W/3\pi)(\omega\tau)] ;$$
$$\varepsilon_d \sim V^2[1 + (32/9\pi)(\omega\tau/W)] \quad (16a, b)$$

Differentiating Eqs. (16) with respect to $\omega\tau$ gives

$$S_m = \partial \varepsilon_m / \partial (\omega\tau) \sim -(8WV/3\pi) \quad (17a)$$
$$S_d = \partial \varepsilon_d / \partial (\omega\tau) \sim (32/9\pi)(V^2/W) \quad (17b)$$

The slope, $S_m$, is negative near the origin, independent of $\omega\tau$, and proportional to the product $V(z)/W$. This confirms our earlier observation that for inertia-dominated data ($W < 0.1$), changes in $\varepsilon_m$ are relatively small and nearly independent of the phase shift near the origin. For data that are ill-conditioned for determining the inertia coefficient ($W > 4.0$), changes in $\varepsilon_m$ are relatively large near the origin.

The slope, $S_d$, is positive near the origin, independent of $\omega\tau$, and proportional to the ratio $V(z)^2/W$. This confirms our earlier observation that for drag-dominated data ($W > 4.0$), changes in $\varepsilon_d$ are relatively small and nearly independent of the phase shift near the origin. For data that are ill-conditioned for determining the drag coefficient ($W < 0.1$), changes in $\varepsilon_d$ are relatively large near the origin.

### 4. ERROR MATRIX CONDITION NUMBER

The Dean error ellipse methodology and the amplitude/phase error methodology provide geometric interpretations of the condition of the wave kinematic data to identify the drag and inertia coefficients, $C_d$ & $C_m$. Because both of these methods were derived from a least square error, standard techniques from error analyses are available to determine matrix condition numbers (Atkinson). These matrix condition numbers provide numerical measures of the sensitivity of the "computed" empirical force coefficients to small perturbation in the wave kinematic/force data. This numerical measure of the condition of the data may again be related to the two geometric methodologies by the Dean eccentricity parameter, $E$.

Minimizing the mean squared error defined in Eq. (1) with respect to $C_m$ and $C_d$ gives the following matrix equation:

$$AX = B \quad (18)$$

where the scaled matrices in Eq. (18) are given by

$$A = \begin{bmatrix} 4\pi^2/3K & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2/E\sqrt{3} & 0 \\ 0 & 1 \end{bmatrix} \quad (19a)$$

$$X = \begin{bmatrix} C'_d \\ C'_m \end{bmatrix} ; B = \begin{bmatrix} 16/3 \\ \langle F_\ast u_\ast \rangle \langle F_\ast u_\ast \rangle \end{bmatrix} \quad (19b, c)$$

where, $F_\ast = F/\rho a^3$, $u_\ast = u/a$, and $u_\ast = u/ao$. Matrix $A$ is Hermitian and unitary. It becomes a unit matrix with matrix condition numbers identically equal to unity when $K = 4\pi^2/3 \approx 13.16$ and $E = 2/E\sqrt{3} \approx 1.15$.

Note that the transverse lift force is stable and repeats itself exactly in the Maull and Milliner$^{(e)}$ data shown in Fig. 6 only when $K = 13.02$.

The four standard measures of the condition number of the error matrix $A$ defined in Eq. (19a) are summarized in Table 1. The condition number when $K = 11.40$ or $E = 1.0$ are also tabulated in column 4 of Table 1.

The four standard matrix condition numbers listed in column 1 of Table 1 are defined as follows (Atkinson):

$$\text{Cond}(A) = \text{Cond}(A)_m = \|A\| \cdot \|A^{-1}\|$$

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Table 1 Summary of condition numbers for error matrix $A$

<table>
<thead>
<tr>
<th>Matrix Condition Number</th>
<th>$K &lt; 13.16$</th>
<th>$K &gt; 13.16$</th>
<th>$K = 11.40$</th>
<th>$K = 13.16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Cond}(A)$</td>
<td>$3 \pi^2$</td>
<td>$4 \pi^2$</td>
<td>$2 \pi^2$</td>
<td>$\pi^2$</td>
</tr>
<tr>
<td>$\text{Cond}(A)_m$</td>
<td>$2 \pi^2$</td>
<td>$4 \pi^2$</td>
<td>$2 \pi^2$</td>
<td>$\pi^2$</td>
</tr>
<tr>
<td>$\text{Cond}(A)_d$</td>
<td>$2 \pi^2$</td>
<td>$4 \pi^2$</td>
<td>$2 \pi^2$</td>
<td>$\pi^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RUN 131</th>
<th>$K = 30.8$</th>
<th>STABLE TRANSVERSE FORCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUN 129</td>
<td>$K = 28.76$</td>
<td>IN-LINE FORCE</td>
</tr>
<tr>
<td>RUN 133</td>
<td>$K = 17.18$</td>
<td>SIMUL TEANS</td>
</tr>
<tr>
<td>RUN 145</td>
<td>$K = 13.02$</td>
<td>POWER TRANS</td>
</tr>
</tbody>
</table>

Fig. 6 In-line and transverse lift forces on a circular cylinder. Note the stable repeatability in the transverse lift force in Run 145. $K = 13.02$ (Maull and Milliner$^{(e)}$).
\[ \text{Con}(A)_2 = \left[ \frac{\text{Max} |\lambda|}{\frac{\lambda \sigma(A^*A)}{\text{Min} |\lambda|}} \right]^{1/2} \]

\[ \text{Con}(A)_b = \left[ \frac{\text{Max} |\lambda|}{\frac{\lambda \sigma(A)}{\text{Min} |\lambda|}} \right]^{1/2} \]

where \( \| \cdot \| \) = a matrix norm; \( A^{-1} \) = matrix inverse; \( \lambda \) = eigenvalue of the matrix; \( \sigma(\cdot) \) = spectral radius of the matrix; \( (\cdot) \); and \( A^* \) = complex conjugate transpose.

5. CONCLUSIONS

The condition of wave kinematic/force data to identify the empirical force coefficients, \( C_d \) and \( C_m \), used in the Morison wave force equation for small bodies has been evaluated by three methods; two geometric and one numerical. The two geometric methods were the Dean error ellipse methodology and the amplitude/phase error methodology. The Dean error ellipse demonstrates geometrically the condition of the data by the alignment of the axes of the error ellipse. A separate error ellipse is required for each value of the O'Brien force ratio, \( W \). The amplitude/phase methodology demonstrates geometrically the condition of the data by the magnitude of the slopes of contours of the force coefficient ratios passing through a zero phase error. Each of the separate graphs required by the Dean error ellipse methodology may be replaced by a single graph with contours of \( W \). Both of these two error methodologies may be related to the Keulegan-Carpenter parameter, \( K \), by the Dean eccentricity parameter, \( E \).

The Dean eccentricity parameter \( E = \sqrt{3} K / (2\pi^2) \) provides a geometric measure of the condition of wave force data on circular members for estimating force coefficients, \( C_m \) and \( C_d \). A set of physical data appear to illustrate the physical significance of the Dean eccentricity parameter. The variability in \( C_m \) for \( E > 1.0 \) (or \( K > 11.40 \)) and in \( C_d \) for \( E < 1.0 \) (or \( K < 11.40 \)) in the replotted Keulegan-Carpenter data may be explained by dividing the data into two parts determined by a Dean eccentricity parameter of unity. The axes of the Dean error ellipse are shown to be parallel to the \( C_m \) and \( C_d \) axes for simple harmonic kinematics \( (\i.e., \langle u|\ddot{u}\rangle = 0) \). The Dean eccentricity parameter, \( E \), has been incorporated into an error analysis that also includes errors in the amplitudes/phases of the kinematics.

Comparisons with synthetically phase-shifted laboratory data were quite good for phase-shifts \( |\omega t| < \pi / 8 \).

Four measures from standard matrix error analyses were used to compute the matrix condition numbers for the least square error. Each of the four error matrix condition numbers was identically equal to unity when \( K = 13.16 \) or \( E = 1.15 \). The only stable transverse lift force found in the Maull and Milliner data occurred at \( K = 13.02 \). The matrix condition numbers were equal to 1.15 for \( K = 11.40 \) and \( E = 1.0 \).

The Dean eccentricity parameter, \( E \), may be used to compare each of the three methods used to evaluate the condition of the wave kinematic/force data to identify the force coefficients, \( C_d \) and \( C_m \). It also connects each of the methods to the Keulegan-Carpenter parameter, \( K \).

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Hudspeth, R.T. · Leonard, J.W. · KUBOTA · KOTOGUCHI


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慣性力係数と抗力係数へのデータ条件の影響について

Hudspeth, R.T. · Leonard, J.W. · 久保田稔 · 事口寿男

本論文は、モリソンの波力方程式に用いられる慣性力係数 $C_m$ と抗力係数 $C_d$ を規定するデータの影響を、2種類の異なる実験結果より示している。つまり、単振動流の条件のもとで、Dean の偏心率変数 $E$ と Keulegan-Carpenter 数 $K$ との関係 ($E = \sqrt{3} K/2\pi$), および無次元流体力振幅比 $W$ と $K$ 数との関係 ($W = (C_d/C_m)(K/2\pi^2)$) より、慣性力係数と抗力係数を決定する際のデータの影響について述べている。