VERIFICATION OF THE UNIVERSAL STRESS TRANSFER MODEL

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Verification of the proposed universal stress transfer model is conducted. The model was proposed as the one for stress transfer across cracks in concrete with universal applicability. Thus the verification is done by applying the model on various cases of stress transfer experiments covering the classical cases which are the crack width constant crack deformational paths as well as the mixed mode deformational paths under high confinement, the specialized cases which are the cyclic mixed mode crack deformational paths, and the cases of stress transfer across concrete cracks having distinctive characteristic, i.e., high strength concrete, lightweight concrete, mortar and cracks in construction joints. Verified results are very satisfactory and therefore confirm the universal applicability of the proposed stress transfer model.

Keywords: stress transfer, crack, shear, contact density

1. INTRODUCTION

Development of the universal stress transfer model is reported in the previous two papers by the authors1,2. Complicated mechanisms of concrete stress transfer which are characterized by the effect of contact friction, the contact anisotropic plasticity and the contact fracturing were examined and became well understood using the framework of the Contact Density Model as the basic analytical tool. The original Contact Density Model was verified by Feenstra et al. with other models and regarded as being highly ranked with wide applicability3. The series of papers by the authors aims at further expanding the applicability up to the universal level. To this end, the authors have conducted qualitative analyses to grasp understanding of the mechanisms and also attempted to quantitatively model all of the inter-relating mechanisms in a unified fashion and succeeded in devising a model designed to have universal applicability on the problems of stress transfer across cracks in concrete. Verification of the proposed universal model is presented in this paper.

2. GENERAL FORMULATION OF THE UNIVERSAL STRESS TRANSFER MODEL

Before going further into the extensive verification of the universal model, it is preferable to have some background of the model by reviewing its general formulation.

The universal model is devised based upon the basic concepts of the original contact density model4 which idealizes that the shear and compressive stresses transferred across a concrete crack are derived from the integration of all contact forces \( R_c \) acting at all contact units constituting a crack plane [Fig. 1] and thus we have,

\[
\tau = \int_{-\pi/2}^{\pi/2} R_c(\omega, \phi, \theta) \sin \theta \, d\theta \\
\sigma' = \int_{-\pi/2}^{\pi/2} R_c(\omega, \phi, \theta) \cos \theta \, d\theta 
\]

As shown in Fig. 1, the resultant contact force \( R_c \) comes from the vector summation of the normal contact force \( N_c \) and the tangential contact force \( S_c \) on each contact unit constituting a crack surface. Both components of contact forces are derived from microscopic contact stresses in respective direction of a contact unit having inclination between \( \theta \) and \( \theta + d\theta \) as follows.

\[
N_c(\omega, \phi, \theta) d\theta = \sigma_c(\omega) K(\omega) dA_o \\
S_c(\omega, \phi, \theta) d\theta = \tau_c(\omega) K(\omega) dA_o
\]

where \( K(\omega) dA_o \) is the effective contacted area of the unit and \( dA_o \) is equated to the product of contact density term \( \Omega(\omega) d\theta \) and the whole surface area per unit crack plane \( A_o \).

The nonlinearity modeling of the contact stresses in normal and tangential direction to the contact unit is the main feature of the universal model5 and greatly contributes to its universality. The normal contact stress is formulated corresponding to the stress history in Fig.2 as

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\[ \sigma' = \sigma \left( \theta \right) \]

\[ \sigma' = k_n R_s \left( \omega_b - \omega_b' \right) \quad \text{for} \quad \omega_b \geq \omega_b' \]
\[ \sigma' = 0 \quad \text{for} \quad \omega_b < \omega_b' \]

where \( R_s \) is the elastic rigidity per length, \( k_n \) and \( k_c \) are the anisotropic plasticity and the contact fracturing parameter respectively.

The anisotropic plasticity model represented by the parameter \( k_n \) and the contact fracturing model represented by the parameter \( k_c \) are the two main models that account for the nonlinear characteristics of the normal contact stress model. The \( k_n \) model was proposed to be a function of \( \theta_s^* \), a variant of the resultant contact angle \( \theta_s \) [Fig.1], in such a way that the more the angle \( \theta_s \), or in other words, the steeper is a contact unit the lower the value of \( k_n \) factor.

On the other hand, the \( k_c \) model of contact fracturing was proposed to be a function of the accumulated effective frictional slip \( d\delta^* \) in which the term \( d\delta^* \) is a variant of the effective frictional slip which is the microscopic deformation of a contact unit in the tangential direction.

As for the contact tangential stress \( \tau_c \) in Eq.4, it was proposed to be governed by the path dependent frictional model in the form of

\[ d\tau_c = \left\{ \begin{array}{ll}
G_s d\delta & \text{when} \quad \tau_c \cdot d\delta \geq 0 \\
3G_s d\delta & \text{when} \quad \tau_c \cdot d\delta < 0
\end{array} \right. \]

where \( \tau_c \) is the tangential contact stress, \( G_s \) is the tangential contact stiffness, \( \mu \) is the frictional coefficient, \( \delta^* \) is the normal contact stress and \( \delta^* \) is the incremental effective frictional slip.

3. VERIFICATION ON LI’S PREVIOUS CASES

Li & Maekawa had devised the so-called contact density model and had investigated its applicability on various stress transfer experiments. The experiments used for the verification were concentrating in the category of classical cases of concrete stress transfer problems, i.e. cases involving crack deformation paths of single mode crack deformation and mostly were of constant crack width with shear slipping paths. This kind of deformational paths provide very high degree of confinement around contact points along a crack plane resulting in reasonably high transferred stresses across the crack. The paths in Li’s work are the monotonic and cyclic crack width constant loading paths, the reversed cyclic paths, the single mode step-type loading paths, and the mixed mode deformational paths under relatively high confinement. These paths are very useful for the basic model verifica-
tion on which the original model is based. In general, they are found in deformation of cracks at junction planes between different members but are rarely encountered in some practical reinforced concrete joint problems. At any rate, the universal model, being universal, must also be able to deal with these basic crack deformational paths. The universal model was applied to these paths and the verifying analytical results are presented in Fig. 4a ~Fig. 6.

4. VERIFICATION ON THE SPECIALIZED CASES

The so-called “specialized loading paths” are the crack deformational paths which involve the cyclic mixed mode of crack deformation. This means that there are both components of crack deformation, crack opening \( \omega \) and shear slip \( \delta \), happening at the same time. This is the kind of crack deformation often encountered in the discrete cracks induced in reinforced concrete members. Steel reinforcement provides confining compressive stress derived from steel elongation caused by crack opening and at the same time shear stress is provided by the concrete crack under deformation. The situation creates the kind of crack deformational paths that are of “mixed mode” in nature.

The specialized crack deformations were reproduced in experiments done by the authors using the same experimental set-up as that adopted by Li.

--- Experiment
---- Universal model

**Fig. 4a** Stress transfer responses from crack width constant deformational path \( \omega = 0.5 \text{ mm}, f' = 23.32 \text{ MPa} \).

**Fig. 4b** Crack width constant with \( \omega = 1.0 \text{ mm}, f' = 23.32 \text{ MPa} \).

**Fig. 4c** Reversed cyclic loading with \( \omega = 0.5 \text{ mm}, f' = 39.00 \text{ MPa} \).

**Fig. 4d** Reversed cyclic loading with \( \omega = 0.3 \text{ mm}, f' = 39.00 \text{ MPa} \).

**Fig. 5a** Step-type loading, \( f' = 22.05 \text{ MPa} \).

--- Experiment
---- Universal model

--- Universal model
pressive and shear force on the cracks. Shown in Fig. 7 a to 7 f is the series of cases which are successfully predicted by the universal model. In the analysis the measured deformation of cracks were control values and the output was the stress transferred along cracks.

It can be obviously seen that the universal model can precisely trace the stress transfer responses even under such complicated crack deformational paths in the controlled experiments. However, one may note that the experimental compressive stress transfer responses are still not precisely predicted especially along the specialized unloading paths marked by arrows in Fig. 7 c, 7 d, and Fig. 7 f. Although the magnitude and trend of experimental responses for compressive stress in these cases are very well anticipated, the not-decreasing trend in the unloading paths is still eluding.

To explain the discrepancy, first of all, it should be observed that so far the universal model has been using the crack deformation, i.e. crack opening and shear slip, as the input to calculate resulting transferred shear and compressive stresses. The nature of this analytical method as well as the real stress transfer behavior causes the model to be very sensitive to the input deformation especially to the deformation along the unloading specialized paths (crack closing with decreasing shear slip). Fig. 8 illustrates this sensitiveness by showing analytical results of the universal model on assumed deformational paths shown in Fig. 8 (a). The paths consist of the same crack width constant deformation and then unloading paths which are the specialized type but slightly different from each other as shown in the figure. The sensitiveness of the model can be understood by the fact that the analytical compressive stress responses in Fig. 8 (b) are so much different even the input deformations are just somewhat varied from each other.

To better verify the universal model on the complicated specialized paths especially for the better application along the unloading paths, the analytical method must be slightly modified by using compressive stress and shear slip as input information. The iterative calculation using modified Newton-Raphson method was utilized to compute resulting transferred shear stress and corresponding crack opening. Analytical outcomes using the technique of calculation are shown in Figs. 9 a, 9 b and 9 c corresponding to Figs. 7 c, 7 d and 7 f. The figures show that the analytical crack deformations obtained from model computation are deviated from the experimental ones at the beginning of deformational history but come closer to the values at subsequent state of the deformation within acceptable range of experimental accuracy. The discrepancy of experimental and analytical crack deformational paths in the beginning can be...
Fig. 7a Stress transfer responses of specialized crack deformational path, \( f' = 21.71 \) MPa.

Fig. 7b Stress transfer responses of specialized crack deformational path, \( f' = 28.18 \) MPa.

Fig. 7c Stress transfer responses of specialized crack deformational path, \( f' = 35.26 \) MPa.

Fig. 7d Stress transfer responses of specialized crack deformational path, \( f' = 33.60 \) MPa.
explained by the fact that, in the tests, there existed some initial crack opening under zero load condition. In the analysis, such condition is numerically unstable and thus the zero crack opening was set as the initial state.

5. CONCRETE CRACKS HAVING DISTINCTIVE CHARACTERISTICS

It was shown in Ref. 1) that the distinctive characteristic of high strength concrete crack compared with that of normal concrete was its different crack configuration associated with splitting of aggregates. Also it was recognized that the difference affected stress transfer behavior across crack of high strength concrete to a great extent. Therefore, it is of great interest to examine the universal model applicability on the concrete cracks having distinctive characteristics in comparison to those of normal concrete cracks based on which the model was developed. The purpose of investigation is to verify whether the universal model provides appropriate parameters which are well in harmony with concrete stress transfer nature so that it can be expanded to cover the whole range of stress transfer problems in concrete.

The concrete cracks with distinctive characteristic to be examined, beginning with the high strength concrete crack, are, lightweight concrete, mortar and artificial concrete cracks. These cracks have one characteristic in common, i.e. a relatively flat crack surface compared with that of normal concrete. The similarity of the surfaces can be evidenced in Fig. 10 (b) illustrating two-dimensional projections of the cracks along with their corresponding calculated contact density distributions.  

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Fig. 7 e Stress transfer responses of specialized crack deformational path, $f' = 33.60 \text{ MPa.}$

Fig. 7 f Stress transfer responses of specialized crack deformational path, $f' = 33.60 \text{ MPa.}$

Fig. 8 Sensitiveness of universal model for the unloading along specialized paths.

Fig. 9 a Analytical results of the specialized case in Fig. 7 e using transferred compressive stress and shear slip as input.
compared with those of normal concrete crack in Fig. 10 (a). It should be noted that the assumed contact density functions do not perfectly represent the calculated histograms of directional distributions. In reality, the experimental histograms depend much on the scanning interval of the profile of crack surface. The larger interval would give rise to the histograms having greater ratio of flatter units of the plane. Then, it is needed to verify the density function not only through the digital measurement of the surface based on a particular interval (1 mm) used, but also through the macroscopic behaviors of cracks in shear. It is possible, however, to qualitatively compare crack surfaces provided the same scanning interval is used and this renders useful information for the crack surface modeling. It can be said that the high strength concrete crack and cracks of concretes of the same category in Fig. 10 (b) possesses more amount of flatter contact planes than the crack of normal concrete in Fig. 10 (a).

The flatness of crack surfaces is a characteristic that is out of range of an assumption in the stress transfer model. It is expected that, apart from this characteristic, another distinctive ones will be found in lightweight concrete crack, mortar or the artificial crack. Applicability of the stress transfer model after modification of assumptions which are relevant to the distinctive characteristics will verify universality of the model.

(1) High Strength Concrete Crack

It was found that the flat crack configuration of high strength concrete profoundly affects stress transfer behavior of the crack. The assumptions in the universal model which take care of crack geometry are the assumed functions for contact density and for the effective ratio of contact. Relevant modification of the functions must result in an enhanced model applicable to the stress transfer of high strength concrete had the framework of the universal model be correct.

The modification is two folds. One is to replace the cosine function for the contact density function of normal concrete crack with that of truncated normal distribution which more appropriately reflects the density distribution of flat crack.
surfaces as shown in Fig. 10. The replacing function is proposed to be
\[ Q(\theta) = \frac{5}{6} \exp\left(-21(\theta/\pi)^2\right) \quad \text{for } \theta \text{ as radian} \]
(8)
where the integral of the function over the whole domain with respect to contact direction is unity according to the requirement stated in Ref. 6.

The other is to use an appropriate value of the maximum roughness for the effective ratio of contact function instead of using one half of maximum size of coarse aggregate for the roughness of normal concrete crack as originally proposed. The function is
\[ K(\omega) = 1 - \exp\left(1 - \frac{R}{\omega}\right) \geq 0 \]
(9)
in which the maximum roughness \( R \) is decided to be 3 mm as can be figured out from the crack surface projection in Fig. 10 (b). The smaller roughness is caused by splitting failure of coarse aggregates.

The enhanced model resulted from the modification adopting the two assumptions in Eq. 8 and Eq. 9 is well applied to the stress transfer of high strength concrete. The enhanced model resulted from the modification adopting the two assumptions in Eq. 8 and Eq. 9 is well applied to the stress transfer of high strength concrete. The enhanced model resulted from the modification adopting the two assumptions in Eq. 8 and Eq. 9 is well applied to the stress transfer of high strength concrete. The enhanced model resulted from the modification adopting the two assumptions in Eq. 8 and Eq. 9 is well applied to the stress transfer of high strength concrete. The enhanced model resulted from the modification adopting the two assumptions in Eq. 8 and Eq. 9 is well applied to the stress transfer of high strength concrete.

**Lightweight Concrete Crack**

Observing crack surface projections and judging from the directional distributions in Fig. 10 (b), one can see that the crack configuration of lightweight concrete is similar to that of high strength concrete. The same geometrical property of the two kinds of crack postulates the use of the same assumptions represented by Eq. 8 and Eq. 9 which govern geometrical modeling in the universal model. Therefore, the same enhanced model successfully applied to high strength concrete as shown in Fig. 11 is used to predict stress transfer responses of lightweight concrete crack and the analytical results are illustrated in Fig. 12.

It can be seen in Fig. 12 that analytical results of the enhanced model adopting the assumptions for high strength concrete crack geometry gives very high stress transfer responses when applied to lightweight concrete. The unusually high transferred stresses obtained from the analysis are quite natural because lightweight aggregates used as one of ingredients of the concrete are undoubtedly much softer than aggregates normally used in practice. The softer lightweight aggregates account for softer contact units which in turn will cause lowering of contact stresses of the units. The softness of contact units can be reflected by a low contact stress yielding level \( f' \) in the contact stress model. The same enhanced model with the same parameters and microscopic models as the ones used in the analysis of Fig. 11 except with lower rigidity \( R_s \) of 66.7 MPa/53 MPa/mm is used to reanalyze the lightweight concrete responses and the analytical outcomes are also shown in Fig. 12. They are in very good agreement with the experimental values.

**Mortar**

Crack configuration of mortar is very similar to those of high strength and lightweight concretes. Lack of coarse aggregates in mortar makes the crack rather flat compared with the crack of normal concrete as also shown in Fig. 10. Experimental results of mortar stress transfer conducted by Li is used for this quantitative analysis. The test was done on mortar with the maximum size of sand aggregate of 5 mm subjected to a crack width constant cyclic loading path. The crack configuration of mortar which is identical to that of high strength concrete can be represented by the same contact density function \( Q(\theta) \) in Eq. 8 and the same function for effective ratio of contact in Eq. 9. Consequently, the successful enhanced model for the analysis of high strength concrete shown in Fig. 11 may also be applicable to mortar. The experimental as well as analytical results from the enhanced model are shwon in Fig. 13. As it turns out, it is not as expected and the enhanced model gives higher stress transfer responses than those
from the experiment as indicated in the figure. Here the precise analytical tool, which in this case is the enhanced universal stress transfer model, gives an evidence indicating that some physical reality corresponding to some parameters in the model are neglected in the analysis. The neglected physical characteristic is the lack of coarse aggregates in mortar. Besides affecting geometrical modeling which is already taken into account by the modified contact density function, this factor also affects the intrinsic anisotropic property of contact stress-displacement relation in Eq. 5 as discussed in Ref. 2. Sand aggregates in mortar are too small in scale compared with the general crack configuration to produce the observable intrinsic anisotropic contact plasticity. The universal model must be enhanced by revising relevant factor which in this case is the unified \( K_r \) model\(^2\). The insignificant intrinsic anisotropic plasticity in mortar can be expressed by revising the \( K_r \) model to be the one shown in Fig. 14 in which the anisotropic parameter \( K_r \) is always unity regardless of the inherent inclination of load bearing contact units \( \theta_h \) shown in Fig. 1.

Using the enhanced model with the revised \( K_r \) model to analyze the mortar case, one can obtain very satisfactory analytical results which agree very well with the experimental ones as also illustrated in Fig. 13.

**4 Artificial Joints in Concrete**

In constructions, rehabilitations or maintenances of reinforced concrete structures, from time to time, some construction procedures necessitate the making of construction joints between old concrete already in place and newly cast concrete. The joints are required to carry or transfer some loads across themselves. Tension and compression transfer poses not so much problem in such joints but not in the case of shear force transfer. Consequently, there is a need to produce artificial roughness at the joint for the purpose of the shear force transferring through interlocking mechanism. To study behaviors of the construction joints, a precise analytical tool is necessary and thus the universal model for stress transfer in concrete can render itself useful in this aspect.

**a) Experiment**

An experiment to verify applicability of the universal model on the construction joint made as an artificial concrete crack was conducted. It was done on the so-called “push-off” concrete specimen shown in Fig. 15. In the specimen, confining compression was provided by reinforcing bar running across a crack plane at right angle and constant external tension was applied to the specimen through the PC bars attached at the
upper and lower parts of the specimen as shown in the figure. Crack was introduced at the middle of the specimen and loaded by applying shear force using the horizontal PC bars and the crack underwent crack deformation, crack opening and shear slip, which were measured. Strains in reinforcing steels producing confining compression at the crack position were also measured as well as the shearing force and the tension force applied through the PC bars. The test was conducted using two specimens of which properties are shown in Table 1 with D10 steel bars having $E_s = 1.88 \times 10^5$ MPa and $f_s = 400$ MPa. Crack in one of the specimen was the artificial type produced by chiseling and brushing hardened concrete surface and that in the other specimen was the natural crack produced by tension force.

b) Analysis

Analysis of the reinforced concrete push-off specimens was done by considering that the cracks were in equilibrium under confining compression provided by reinforcing steels crossing them and under transferred compression and shear force provided by the deformed concrete cracks. The crack deformation was related to strains in reinforcing steels by the modified bond-slip-strain model originally proposed by Shima, et al. The modified model is as follows.

\[ S \left( \frac{f'}{D} \right)^{2/3} = \varepsilon_s \left( 6 + 3500\varepsilon_s \right) \]

in which $S$ is the bond slip, $D$ is the diameter of reinforcing steel in the same unit as that of the bond slip, $f'$ is the concrete strength in MPa, and $\varepsilon_s$ is the strain in reinforcing steel.

Stresses in reinforcing steels providing confining compression could be computed from measured strains utilizing the established Kato's steel model. Shape of the artificially produced crack was projected and its directional density distribution was computed to confirm its geometrical property which is different to that of normal concrete but similar to the crack of high strength concrete as indicated in Fig.10. Hence the modified contact density function and the function of effective ratio of contact for high strength concrete proposed in Eq.8 and Eq.9, respectively, is used to represent the crack geometry of a construction joint also. Thus the same enhanced model which is applicable to high strength concrete must also be applicable to the artificial crack.

Fig.16 shows successful analytical results of the enhanced universal concrete stress transfer model combined with the simple model for bond-slip-strain. The figure illustrates the satisfactory application of the models on the construction joint crack. The figure also shows successful analytical result using the universal model on the stress transfer of a normal concrete crack produced by tensile stress across the shear section in the same push-off experimental set-up to confirm applicability of the analytical method on the experiment. The combined models successfully traces both the deformational behaviors and the ultimate strength of the reinforced concrete cracks. It also should be noted that the shear transfer performance of the construction joint is quite lower than that of a normal concrete crack in both stiffness and ultimate strength.
6. CONCLUSIONS—TOWARDS MODELING OF REINFORCED CONCRETE DISCRETE CRACKS

Versatility of the universal stress transfer model was verified by extensive experimental observations. The verifying experimental results included those from the classical crack deformational paths which were the paths mostly involving single mode crack deformation that produced relatively high confinement around contact points of a crack surface and the so-called specialized crack deformational paths which were the paths involving cyclic mixed mode crack deformation under rather low confinement. The verification using these cases were very satisfactory confirming that the model could anticipate any kind of crack deformational paths. Moreover, enhanced universal model was applied on stress transfer responses of concrete cracks having distinctive features, e.g., cracks of high strength and lightweight concrete, cracks of artificial construction joints or those of mortar. The successful application of the model which was enhanced by customizing some relevant parameters corresponding to respective physical features of the cracks further ensured the versatility of the model.

Concrete alone can not be of much use without being reinforced by tension-resistance steel bars. Similarly, the precise universal model for stress transfer in concrete alone is of no substantial value without being complemented by precise models of steel reinforcement and concrete-reinforcement interaction. The behaviors of reinforced concrete discrete crack are enormously complex and they are the subjects of investigations by many researchers in the recent decades. The complexities can be understood only when their constituents are completely comprehended. The universal model is an attempt to thoroughly understand one aspect constituting those complexities and it has done its mission successfully. The further development, which is now underway, is to combine together the precise model of concrete with the precise model of reinforcement and the precise model of bond-slip of embedding reinforcement in concrete. What achieved will be a realistic physical model for RC discrete cracks and it is only when we have the precise physical model of RC joint element in hand that we can gracefully step towards any simplification or any constitutive equation tailored to our needs in any practical application.

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応力伝達に関する一般化モデルの検証

本研究は、任意の載荷経路に適用可能な、ひびわれ面における一般応力伝達モデルの系続的な検証を行ったものである。一定ひびわれ幅での応力伝達のみならず、ひびわれ幅とせん断変位が同時に増加するケース、さらに繰り返し載荷を受けた場合、ならびに高強度コンクリート・軽量骨材コンクリート・モルタル・内線き目といった、普通強度のコンクリートとはひびわれ面形状が大きく異なるケースについても検証を実施し、高い適用性を有していることを明らかにした。