FURTHER STUDY ON TWO-SURFACE MODEL FOR STRUCTURAL STEELS UNDER UNIAXIAL CYCLIC LOADING

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In this paper, a general definition of the accumulated effective plastic strain concept is introduced so that the two-surface model proposed by the authors in Ref. 1) can be extended easily to multiaxial stress state by combining with a proper hardening rule. In the treatment of the bounding line, the ultimate tensile stress is used as the limiting value of the bounding surface radius. In addition, the reduction of the bounding line slope is considered. Finally, the model parameters are calibrated according to the experimental data and presented. The predictions for various kinds of steels with the present model show a good agreement with the experimental results.

Keywords: *plasticity, two-surface model, cyclic loading, plastic work.*

1. INTRODUCTION

Following experimental observations, the authors proposed a two-surface model¹⁾ for the uniaxial cyclic behavior of steels with yield plateau, such as SS400, SM490 and SM570. In order to extend this uniaxial model to the multiaxial stress state, further modification and refinement have been made in the present paper: firstly, the concept of the *accumulated effective plastic strain*²⁾ (as abbreviated to A.E.P.S.) is generalized to the multiaxial stress state by introducing a surface defined in the plastic strain space; secondly, the bounding surface is assumed to change in size with the plastic deformation but has some limiting value; thirdly, the bounding line slope is assumed to decrease with the plastic work.

2. EXTENSION OF A.E.P.S. IN MULTIAXIAL STRESS STATE

The concept of *accumulated effective plastic strain* (A.E.P.S.) under uniaxial condition in Refs. 1) and 2) is defined as the maximum amplitude of plastic strain that the material has ever experienced before and can be written as:

$$\bar{\epsilon}^p = \epsilon_{max}^p - \epsilon_{min}^p \quad \dots\dots\dots (1)$$

where ϵ_{max}^p and ϵ_{min}^p represent the arithmetic maximum and minimum values of the axial plastic strain respectively. Extending the concept of A.E.P.S. under the uniaxial case to the multiaxial stress state, a surface defined in the plastic strain space³⁾, called A.E.P.S. *surface*, is introduced and expressed as follows.

$$\phi = \phi(\epsilon_{ij}^p) = \frac{2}{3}(\epsilon_{ij}^p - \eta_{ij})(\epsilon_{ij}^p + \eta_{ij}) - \rho^2 = 0 \quad \dots\dots (2)$$

where ϵ_{ij}^p is the component of plastic strain; η_{ij} and ρ represent the center and radius of the surface respectively. During the plastic deformation, A.E.P.S. surface moves and changes in size conditionally. According to the definition of A.E.P.S. in the uniaxial case, the motion of A.E.P.S. surface may be defined as follows.

$$d\eta_{ij} = \begin{cases} \frac{1}{2}\epsilon_{ij}^p & \text{if } \phi(\epsilon_{ij}^p + d\epsilon_{ij}^p) > 0 \\ 0 & \text{otherwise} \end{cases} \quad \dots\dots\dots (3)$$

From the condition $d\phi = 0$, there exists:

$$\frac{2}{3}(\epsilon_{ij}^p - \eta_{ij})(d\epsilon_{ij}^p - d\eta_{ij}) - \rho d\rho = 0 \quad \dots\dots\dots (4)$$

Then substituting Eq.(3) into Eq.(4) yields:

$$d\rho = \begin{cases} \frac{1}{3}(\epsilon_{ij}^p - \eta_{ij})d\epsilon_{ij}^p/\rho & \text{if } \phi(\epsilon_{ij}^p + d\epsilon_{ij}^p) > 0 \\ 0 & \text{otherwise} \end{cases} \quad \dots\dots\dots (5)$$

It can be seen that, in the uniaxial case, ρ is one half of A.E.P.S. $\bar{\epsilon}^p$. Therefore, the equations in Ref. 1) related to A.E.P.S., such as the equations to calculate the elastic range and the reduction of the yield plateau, can also be used in the multiaxial stress state by replacing $\bar{\epsilon}^p$ with 2ρ .

3. RADIUS OF BOUNDING SURFACE

In Ref. 1), the heights of two bounding lines in tension and compression sides were assumed to change with A.E.P.S. linearly (i.e., Eq.(10) in Ref. 1)). However, since the bounding lines will become the bounding surface in the multiaxial stress state, it is important to know the radius of the bounding surface. Moreover, it is observed that the stress-strain

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(a) The example of steel SS400

(a) The example of steel SS400

(b) The example of steel SM490

(b) The example of steel SM490

(c) The example of steel SM570

(c) The example of steel SM570

Fig. 2 The Comparison between Experiment and Prediction by the Present Model

Fig. 3 The Comparison between Experiment and Prediction by the Previous Model

Table 1 The parameters of steels SS400, SM490 and SM570 in the present model

material	SS400	SM490	SM570
$E(kgf/cm^2)$	2.11×10^6	2.10×10^6	2.20×10^6
$\sigma_y(kgf/cm^2)$	2.80×10^3	3.64×10^3	5.35×10^3
E_{st}^p/E	2.49×10^{-2}	3.40×10^{-2}	1.02×10^{-2}
E_0^p/E	8.96×10^{-3}	1.01×10^{-2}	7.85×10^{-3}
$\omega \cdot \sigma_y$	3.08	4.0	2.67
$\bar{\kappa}_0/\sigma_y$	1.15	1.13	1.06
σ_u/σ_y	1.81	1.61	1.22
$\zeta \cdot \varepsilon_y^2$	9.89×10^{-4}	1.52×10^{-3}	8.04×10^{-3}

Note: (1) $\bar{\kappa}_\infty = \sigma_u$ is the limiting value of the bounding surface radius; (2) ε_y is the strain at the initial yield point, i.e., $\varepsilon_y = \sigma_y/E$; (3) see Table 2 in Ref. 1) for the other parameters.

stress-plastic strain loading curve up to 5% in tension side(DE in Fig. 1). As for the compression part, i.e., CD in Fig. 1, it is difficult to evaluate the bounding line $Y_0Y'_0$ because of a possible occurrence of buckling and therefore the same slope of the bounding line $Z_0Z'_0$ as in tension part is assumed in the determination of the model parameters.

5. EXAMPLES

In the uniaxial case, the present model is the same as the previous one except the calculation of the bounding surface radius and the slope of the bounding line. The present model is applied to predict the uniaxial cyclic experiment for steels SS400, SM490 and SM570. The comparison between the experiment and the prediction by the present model is shown in Fig. 2. At the same time, the prediction by the previously proposed model is shown in Fig. 3 for the same experiments. It can be seen that the prediction by the present model is accurate enough even for the yield plateau.

6. SUMMARY AND CONCLUSIONS

From the present study, it can be concluded that:

- (1) With the introduction of A.E.P.S. surface, the present uniaxial two-surface model can be easily extended to the multiaxial stress state.
- (2) By considering reduction of the bounding line slope, the present model is more accurate, especially when the plastic strain is relatively large.
- (3) The assumption that the radius of bounding line changes with plastic deformation and limits to the ultimate stress σ_u is in accord with the experimental results. Moreover, Eq.(6) is convenient in the case of multiaxial stress state.

The present model can be extended to the multiaxial stress state easily and will be presented in a different paper. The parameters newly introduced in the present model, such as ω , ζ , $\bar{\kappa}_0$ and $\bar{\kappa}_\infty$, are simply and clearly determined without increasing the experiment work.

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一軸繰返し荷重を受ける構造用鋼素材の二曲面モデルに関する補足的研究

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本論文では、文献 1) において著者らが提案した二曲面モデルを多軸応力状態に容易に拡張することができるように累積相当塑性ひずみの一般的な定義を導入した。境界線に対しては、鋼素材の引張極限応力を境界曲面の限界値とし、さらに、境界線の勾配は塑性仕事量により低減すると仮定した。モデルパラメータの決定法について説明し、鋼素材 SS400, SM490, SM570 に対するパラメータ値を示した。本提案モデルによるこれら鋼素材の予測結果と実験データとを比較し、モデルの有効性を検討した。