Practical Determination of an Effective Reduced Order in a Model Order Reduction of Dynamic FEM via Krylov Subspace

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A model order reduction has been utilized as a booster of FEM, mainly in the design of micro-electromechanical system, by using orthogonal basis vectors of the Krylov subspace. However, there is no clear procedure to determine an effective reduced order, which indicates the number of the orthogonal basis vectors, because of the lack of error definition. In this paper, a practical use of the model order reduction via Krylov subspace (KS-MOR) is proposed to simulate low frequency responses of structure dynamics accurately. One of the important approximation properties of KS-MOR is the Moment Matching (MM). The MM provides a certain accuracy of KS-MOR in frequency domain, but the accurate frequency range cannot be defined. Then, a stepwise iterative frequency response analysis is utilized to evaluate an effective reduced order before it is applied in the time transient analysis. In the frequency analysis, simple error estimation is performed to check the comparison between the KS-MOR results with different number of basis vectors. Finally, a couple of numerical example has been simulated to validate our proposed procedure.

Key Words: *dynamic FEM, model order reduction, frequency response analysis, moment matching*

1. Introduction

A model order reduction via Krylov subspace (KS-MOR) has been utilized as a booster tool for large scale dynamic FEM structural simulations. KS-MOR has a similar profile as the modal decomposition method since KS-MOR also generates a set of orthogonal basis vectors. The modal decomposition method can evaluate accurate vibration mode related to resonance behavior, but its numerical cost for solving eigen-value analysis is relatively large. On the other hand, KS-MOR generates an effective set of orthogonal basis vectors rapidly, but the modes are not the same as the eigen-vectors in the modal decomposition method, and there is no clear guideline to determine for the reduced order which indicates the number of orthogonal basis vectors.

KS-MOR can simulate time transient analysis¹⁾ and frequency response analysis by using the same reduced order system. There is a theoretical background to support the accuracy of KS-MOR. The features is called by ‘Moment Matching^{2),3),4)} (MM). The physical meaning of MM is that

KS-MOR approximation shows high accuracy around an expansion point in the frequency domain. The expansion point is usually selected at zero frequency, and it implies that the low frequency response can be simulated accurately. Another study by Rudnyi⁵⁾ suggested to using different expansion point located at the middle of the target frequency range.

In this paper, we try to estimate an efficient reduced order by using approximation property of MM. As far as authors know, there is no accurate error estimation in KS-MOR, and this induces the difficulty to decide the reduced order. Therefore, a simple error indicator is proposed to predict an efficient reduced order that is equal to the number of required basis vectors. To validate our proposed procedure, two numerical examples has been simulated. The first example is a simple column problem applied different frequency load. After the discussion with the first example, a 3-D bridge model with two type of bearing device is simulated. At the same time, efficiency in the simulation cost is discussed in these examples.

2. Model Order Reduction in Time Transient FE Analysis

Here, the general process of KS-MOR for time transient FE analysis is summarized. The Second Order Arnoldi (SOAR)² algorithm is utilized to generate the basis vectors in the Krylov subspace in this paper.

2.1 Linear Structural Dynamic System

First, we assume that governing equations in the linear structural FE simulation is denoted by

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{D}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \alpha(t)\mathbf{f}, \quad (1)$$

where $\mathbf{M} \in \mathfrak{R}^{N \times N}$, $\mathbf{D} \in \mathfrak{R}^{N \times N}$, $\mathbf{K} \in \mathfrak{R}^{N \times N}$ indicate the mass, damping and stiffness matrix respectively, while $\ddot{\mathbf{u}} \in \mathfrak{R}^N$, $\dot{\mathbf{u}} \in \mathfrak{R}^N$, $\mathbf{u} \in \mathfrak{R}^N$ and $\mathbf{f} \in \mathfrak{R}^N$ are the nodal acceleration, velocity, displacement and force vectors. The N indicates the number of total degree of freedoms (DOFs), and $\alpha(t)$ is a scalar load function in time.

2.2 KS-MOR Basis Vector Calculation

The Arnoldi and Lanczos algorithm are famous algorithms to generate a set of basis vectors in the Krylov subspace. As shown in the Eq. (1), the governing equation of FEM for structural dynamics is a second order time differential equation. For second order time differential problems, there are two processes to generate the basis vectors. One is the use of the projection onto the equivalent first order time differential equations. The other is the use of the Second Order algorithm like SOAR. The later process is selected in this paper, because the former process generates double size of matrix equation and its numerical cost becomes almost double.

Algorithm 1: SOAR² procedure for single input

1. $\mathbf{q}_1 = \mathbf{r}_0 = \mathbf{K}^{-1}\mathbf{f} / \|\mathbf{K}^{-1}\mathbf{f}\|$
2. $\mathbf{p}_1 = 0$
3. for $j=1, 2, \dots, n$ do
4. $\mathbf{r} = \mathbf{D}\mathbf{K}^{-1}\mathbf{q}_j + \mathbf{M}\mathbf{K}^{-1}\mathbf{p}_j$
5. $\mathbf{s} = \mathbf{q}_j$
6. for $i=1, 2, \dots, j$ do
7. $t_{ij} = \mathbf{q}_i^T \mathbf{r}$
8. $\mathbf{r} := \mathbf{r} - \mathbf{q}_j t_{ij}$
9. $\mathbf{s} := \mathbf{s} - \mathbf{p}_j t_{ij}$
10. end for
11. $t_{j+1,j} = \|\mathbf{r}\|$
12. if $t_{j+1,j} = 0$, breakdown
13. else
14. $\mathbf{q}_{j+1} = \mathbf{r} / t_{j+1,j}$
15. $\mathbf{p}_{j+1} = \mathbf{s} / t_{j+1,j}$
16. end if
17. end for

Algorithm 1 shows the flowchart of SOAR for the calculation of a set of orthogonal basis vectors. Note that two linear equations with N degree of freedoms should be solved in the procedure to generate a basis vector. This iterative process continues until a selected reduced order n , which should be decided by user of KS-MOR. The selection of n will be discussed in later.

2.3 Projection onto a Small System

Assume that the reduced order n is reasonably evaluated, displacement, velocity and acceleration $\mathbf{u}_n$ is determined by multiply N dimension of vector displacement, $\mathbf{u}$ with a transpose $N \times n$ of basis matrix, $\mathbf{Q}_n^T$. These basis vectors are stored at the columns of $\{\mathbf{Q}_n\}$. In summary, the change of state variables can be expressed with dimension of vectors as

$$\mathbf{u} \cong \mathbf{Q}_n \mathbf{u}_n, \quad \dot{\mathbf{u}} \cong \mathbf{Q}_n \dot{\mathbf{u}}_n, \quad \ddot{\mathbf{u}} \cong \mathbf{Q}_n \ddot{\mathbf{u}}_n. \quad (2)$$

By substituting Eq. (2) into Eq. (1) and multiplying with $\mathbf{Q}_n^T$ from left, a reduced order matrix equation can be defined as

$$\mathbf{M}_n \ddot{\mathbf{u}}_n(t) + \mathbf{D}_n \dot{\mathbf{u}}_n(t) + \mathbf{K}_n \mathbf{u}_n(t) = \alpha(t)\mathbf{f}_n, \quad (3)$$

where the KS-MOR reduced system matrixes are defined by

$$\begin{aligned} \mathbf{M}_n &= \mathbf{Q}_n^T \mathbf{M} \mathbf{Q}_n; \quad \mathbf{D}_n = \mathbf{Q}_n^T \mathbf{D} \mathbf{Q}_n; \\ \mathbf{K}_n &= \mathbf{Q}_n^T \mathbf{K} \mathbf{Q}_n; \quad \mathbf{f}_n = \mathbf{Q}_n^T \mathbf{f}. \end{aligned} \quad (4)$$

Here, $\mathbf{M}_n \in \mathfrak{R}^{n \times n}$, $\mathbf{D}_n \in \mathfrak{R}^{n \times n}$, $\mathbf{K}_n \in \mathfrak{R}^{n \times n}$ are $n \times n$ matrices, and $\mathbf{f}_n \in \mathfrak{R}^n$ is n vector.

2.4 Inverse Projection (Recovery from the Small System Solution)

The small system solution can be solved by the conventional time integration scheme. In this paper, the Newmark- β is utilized for this purpose. These small system matrixes become dense matrix, and then the direct solver by LU decomposition with full matrix storage is used to solve linear equations. After solving the small system solutions, the nodal solution is recovered by inverse projection as in Eq. (2).

Here, it is not necessary to apply the inverse projection at each time increment, and only several selected nodal solutions can be recovered to save the disk space. For example, only the surface nodal solution can be output at an interval in time.

3. Determination of an Effective Reduced Order Using a Stepwise Iterative Frequency Response Analysis

After the description of application into the frequency response analysis with KS-MOR, the important approximation property 'Moment Matching (MM)' is summarized. Then, a

determination process of the effective reduced order is proposed by using MM properties.

3.1 Frequency Response Analysis with KS-MOR

Taking the Laplace transform, the structure dynamic equation in time domain as in Eq. (1) can be represent in frequency domain as ⁶⁾

$$s^2 \tilde{\mathbf{M}} \tilde{\mathbf{u}}(s) + s \tilde{\mathbf{D}} \tilde{\mathbf{u}}(s) + \tilde{\mathbf{K}} \tilde{\mathbf{u}}(s) = \tilde{\alpha}(s) \mathbf{f}, \quad (5)$$

where $\tilde{\mathbf{u}}(s)$ and $\tilde{\alpha}(s)$ indicate the Laplace transform of $\mathbf{u}(t)$ and $\alpha(t)$ respectively. s is the complex number of $s = i\omega$, ω is the angular frequency with a condition of $\omega \geq 0$ and $i = \sqrt{-1}$. For an arbitrary loading $\mathbf{F}(s) = \tilde{\alpha}(s) \mathbf{f}$, the frequency response can be calculated by

$$\tilde{\mathbf{u}}(s) = [s^2 \tilde{\mathbf{M}} + s \tilde{\mathbf{D}} + \tilde{\mathbf{K}}]^{-1} \mathbf{F}(s). \quad (6)$$

Similarly to Eq. (3), substitute Eq. (2) into Eq. (5) and multiplying with $\mathbf{Q}_n^T$ from left, the reduced order equations for the frequency response are given by

$$s^2 \mathbf{M}_n \tilde{\mathbf{u}}_n(s) + s \mathbf{D}_n \tilde{\mathbf{u}}_n(s) + \mathbf{K}_n \tilde{\mathbf{u}}_n(s) = \tilde{\alpha}(s) \mathbf{f}_n, \quad (7)$$

$$\tilde{\mathbf{u}}_n(s) = [s^2 \mathbf{M}_n + s \mathbf{D}_n + \mathbf{K}_n]^{-1} \mathbf{F}_n(s).$$

3.2 Moment Matching Properties of KS-MOR

Now, a scalar transfer function $h(s)$ is defined as

$$h(s) = \mathbf{I}^T \mathbf{u}(s), \quad (8)$$

where $\mathbf{I}$ is an arbitrary output measurement vector, which selects output nodal values.

The transfer function can be re-written with an expansion point s_0 as

$$h(s) = \mathbf{I}^T [(s-s_0)^2 \tilde{\mathbf{M}} + (s-s_0) \tilde{\mathbf{D}} + \tilde{\mathbf{K}}]^{-1} \mathbf{F}(s), \quad (9)$$

where $\tilde{\mathbf{D}} = 2s_0 \tilde{\mathbf{M}} + \tilde{\mathbf{D}}$ and $\tilde{\mathbf{K}} = s_0^2 \tilde{\mathbf{M}} + s_0 \tilde{\mathbf{D}} + \tilde{\mathbf{K}}$ are modified matrixes with s_0 .

According to the reference⁶⁾, Eq. (9) can be represented in a power series expansion form with moments $(m_N)_i$ as

$$h(s) = (m_N)_0 + (s-s_0)(m_N)_1 + (s-s_0)^2(m_N)_2 + \dots$$

$$= \sum_{i=0}^{\infty} (s-s_0)^i (m_N)_i. \quad (10)$$

The moment $(m_N)_i$ is generated by

$$(m_N)_i = \mathbf{I}^T \mathbf{r}_i \text{ for } i \geq 0, \quad (11)$$

where the vector sequence $\mathbf{r}_i$ constructs n^{th} second order

Krylov subspace $\mathcal{S}_n(\mathbf{A}, \mathbf{B}; \mathbf{r}_0) = \text{span}\{\mathbf{r}_0, \mathbf{r}_1, \dots, \mathbf{r}_n\}$, and these vectors are given by

$$\mathbf{r}_0 = -\tilde{\mathbf{K}}^{-1} \mathbf{f}, \mathbf{r}_1 = \mathbf{A} \mathbf{r}_0,$$

$$\mathbf{r}_i = \mathbf{A} \mathbf{r}_{i-1} + \mathbf{B} \mathbf{r}_{i-2} \text{ for } i \geq 2, \quad (12)$$

where $\mathbf{A} = -\tilde{\mathbf{K}}^{-1} \tilde{\mathbf{D}}$ and $\mathbf{B} = -\tilde{\mathbf{K}}^{-1} \tilde{\mathbf{M}}$.

The transfer function in the small system is defined similar to the one of original system in Eq. (9) as

$$h_n(s) = \mathbf{I}_n^T [(s-s_0)^2 \mathbf{M}_n + (s-s_0) \tilde{\mathbf{D}}_n + \tilde{\mathbf{K}}_n]^{-1} \mathbf{F}_n(s), \quad (13)$$

where $\mathbf{M}_n$, $\mathbf{D}_n$ and $\mathbf{K}_n$ are the small system matrices defined in Eq. (4), and $\mathbf{I}_n$ indicates an output measurement vector with dimension n as

$$\mathbf{I}_n = \mathbf{Q}_n^T \mathbf{I}. \quad (14)$$

The moment expansion as Eq. (10) can be applied into the small system as

$$h_n(s) = \sum_{i=0}^{\infty} (s-s_0)^i (m_n)_i,$$

$$(m_n)_i = \mathbf{I}_n^T (\mathbf{r}_n)_i \text{ for } i \geq 0, \quad (15)$$

where $(\mathbf{r}_n)_i$ are the second order Krylov vectors as defined in Eq. (11) associated with the matrices $\mathbf{M}_n$, $\mathbf{D}_n$ and $\mathbf{K}_n$. According to the proof of MM^{23,4)}, the first n -th moments of the original system (10) and the reduced system (15) are matched. When the expansion point s_0 is located at zero frequency, the moment matching is simplified as

$$(m_N)_i = (m_n)_i \text{ for } i = 0, 1, 2, \dots, n-1. \quad (16)$$

The schematic image of the MM property in the frequency response is shown in **Fig.1**. The bold gray line shows the result with original FEM in the frequency response analysis. The dotted lines show the result with KS-MOR. As increasing the number of reduced order, the accurate frequency domain is expanding into a certain high frequency range from low frequency. The key point here is that the accurate frequency domain cannot be directly evaluated with respect to n . Then, the following frequency response analysis procedure is necessary to evaluate the accurate frequency domain numerically.

Fig.1 Schematic image of moment matching in the frequency response analysis

3.3 Stepwise Iterative Frequency Analysis to Evaluate an Effective Reduced Order

The MM implies that accurate frequency range around the expansion point is gradually expanding as the number of reduced order is increasing. In other words, an efficient number of reduced orders can be determined when a converged solution in the frequency response analysis is given within a target frequency range. Then, a stepwise iterative process by monitoring the differences from previous solutions with a lower reduced order is implemented as follows.

- [1] Input a target frequency ω_{target} and an interval m , which means an increasing number of reduced order to check the difference
- [2] Select output nodes to evaluate the error
- [3] Stepwise iterative frequency analysis using the small system equations in Eq. (6) within the target frequency
- [4] Check convergence of the frequency response analysis at each iterative procedure in step [3]

In the step [2], the most deformable nodal point during the frequency response analysis within a target frequency range is suitable for evaluating the error. In the step [3], the reduced order is increasing with the inputted interval m , and a stepwise iterative frequency analysis is implemented until a converged solution is evaluated. After the inverse projection, the difference from the previous solution with a lower reduced order is evaluated at the output nodes in the step [4]. Suppose that m and (j) indicates the inputted interval for evaluating the difference and the number of iteration respectively, the reduced order is given by $n = (j)m$ with $(j) = 1, 2, 3, \dots$. Within the target frequency range, a relative error is evaluated at each sampling frequency as

$$\varepsilon(\omega) = \frac{|\tilde{\mathbf{u}}^{(j)}(\omega) - \tilde{\mathbf{u}}^{(j-1)}(\omega)|}{|\tilde{\mathbf{u}}_0|} \quad (17)$$

The above error is normalized by the magnitude of the static solution $\tilde{\mathbf{u}}_0$ at 0 Hz. At each iteration, the maximum error $\varepsilon_{\max} = \max(\varepsilon(\omega))$ (in $0 < \omega < \omega_{\text{target}}$) is evaluated, and then this iterative procedure is continued until the maximum error

stepwise basis vector generation

Fig.2 Stepwise iterative frequency response analysis

become smaller than a threshold ε_{ET} .

$$\varepsilon_{\max} < \varepsilon_{\text{ET}} \quad (18)$$

Fig.2 shows the stepwise iterative process to get a converged solution of the frequency response analysis. The effective reduced order can be given by the number of basis vectors at the converged solution.

3.4 Time Transient Analysis Procedure with the Effective Reduced Order

The time transient analysis with the above defined effective reduced order is summarized here. **Fig.3** shows the whole procedure. As discussed in section 3.2, the MM implies that KS-MOR give an accurate approximation around expansion point in frequency domain. The accurate domain is evaluated numerically by using the stepwise iterative frequency response analysis. Only this iterative process is added in the original KS-MOR procedure.

Here, the most time consuming process is the calculation of basis vectors which include two linear solvers shown at 4th row in **Algorithm1**. Therefore, the additional computation cost of iterative frequency response analysis is negligible small.

Fig.3 Total procedure with an efficient reduced order

4. Numerical Examples and Validation of KS-MOR with an Effective Reduced Order

Here, two numerical examples are solved by KS-MOR with an effective number of reduced orders, which is evaluated by the proposed procedure. The first example is a simple column model acting a top surface load. In the first example, we have tested that the effective number of reduced orders depends on the loading frequency. Then, 3D bridge models with steel hinge and rubber bearing is solved to check the performance and efficiency.

Fig.4 A simple column model with 8 holes

4.1 A Simple Column Model

A simple 3D column with eight square holes shown in **Fig.4** is simulated by a reduced order FEM based on KS-MOR. Here, horizontal surface load paralleled in x -direction is applied on the top surface. The total amount of this surface load is 10N. The column is descritized into FE model with 40960 quadratic Hexagonal elements, and the total degree of freedom is 152019. The material is modeled by an isotropic elasticity, and its Young modulus (E), density (ρ) and poison ratio (ν) are 210 GPa, $7.95 \times 10^3 \text{ kg/m}^3$ and 0.3, respectively. The frequency of the surface load has been changed from 0 to 2000Hz in the frequency response analysis, and sine wave with 200 and 500Hz is applied as a time dependent loading parameter in the time transient analysis.

The frequency analysis is simulated for evaluating an effective number of reduced orders as discussed in Section 3. In the following time transient analysis, sine waves with 200 and 500Hz frequency will be applied. This means the target frequency in the frequency response analysis are 200 and 500Hz for each loading. The interval m for checking the difference from the previous result is selected by 5. After generating new 5 basis vectors, the same frequency analysis is simulated with the different small system matrixes defined in Eq.(4).

Fig.5(a) and **5(b)** show the frequency response results in the non-damping case and the cases with Rayleigh type damping, respectively. In these figures, only the results with 5, 10 and 50 basis vectors are plotted with different colors of line in each case, and KSMOR- n means the result with n basis vectors. Eigen

modes related to the response behavior has been shown at the top of **Fig.5(a)**. In the case of non-damping, loading frequency until 2000Hz may include the first 15th vibration modes in the structure.

In **Fig.5(a)**, the circle mark shows the reference solution by the commercial FEM software (MSC MARC). **Fig.5(a)** for non-damping shows that KSMOR-5, -10 and -50 represent the FEM results until 250, 850 and 1900 Hz, respectively. These accurate frequency domain may depend on the damping effect as shown in **Fig.5(b)**. It is clear that the accurate frequency domain is expanding to high frequency range from 0Hz, as increasing the number of reduced orders in both cases. The time transient analysis was implemented by Newmark- β method⁷⁾ with $\beta=0.25$ and $\delta=0.5$. The loading parameter $\alpha(t)$ in Eq. (1) and Eq. (3) is chosen by $\alpha(t) = \sin(2\pi ft)$. The loading frequency f is set to 200 and 500Hz. From the aforementioned frequency response analysis, 5 and 10 basis vectors are necessary to maintain the accuracy within the target frequency 200 and 500Hz, respectively.

Fig.5 Frequency response in the simple column model

Fig.6 Displacement histories in the loading direction with different loading frequencies

Fig.6(a) and **Fig.6(b)** show the displacement- y for non-damping case at the center of the top surface in the case of $f=200$ Hz and 500 Hz respectively. Only for the case of 500 Hz with 5 basis (KSMOR-5) small error appears after 0.5 second as shown in **Fig.6(b)**. In the lower loading frequency case ($f=200$ Hz), KSMOR-5 represent the original FEM result

accurately, and KSMOR-10 is required to get the accurate result in the case of higher loading frequency case ($f=500$ Hz). Note that these tendencies can be predicted by the frequency response results shown in **Fig.5(a)**.

4.2 3D Bridge Model with Steel Hinge and Rubber Bearing

Fig.7 Details in the 3D bridge models

A box-girder bridge model with two type of bearing system is simulated with the KS-MOR. **Fig.7** shows the dimension of

this bridge. The first bearing system is a steel hinge, and the other is a steel laminated rubber. The details and dimensions of these bearing systems are shown in **Fig.7** **Table 1** lists the number of element and nodes for each FEM model. The material properties for steel, rubber and concrete in these models are summarized in **Table 2**. The Rayleigh damping with $\alpha_R = 0.01$ and $\beta_R = 0.01$ is assumed to model the damping effect. These parameters should be carefully discussed in the future.

Table 1 Problem sizes in each FE model

	Steel hinge	Steel laminated rubber bearing
elements	390 956	392 036
nodes	483 808	485 728
DOFs	1 451 424	1 457 184

Table 2 Material properties

	Young's modulus [GPa]	Poisson's ratio [-]	Density [kg/m ³]
steel	210	0.30000	7874
rubber	0.01	0.49999	910
concrete	30	0.16666	2300

Here, enforced displacements are applied into all the bottom nodes of these models as shown in **Fig.7(a)**. The time histories for each direction are generated from observed accelerations at Takatori Station during Kobe's earthquake in 1995. The displacements histories in x , y and z direction are shown in **Fig.8**. Note that these displacement histories are adjusted to become nearly zero after 40 sec. The time increment is chosen by 0.01(sec.) in the Newmark- β method for both cases in the KS-MOR and the conventional FEM.

Fig.8 Kobe's earthquake excitation displacement time-history

In this example, equivalent loading vectors are defined by

$$\mathbf{f}_1 = -\mathbf{K}\bar{\mathbf{u}}_x, \mathbf{f}_2 = -\mathbf{K}\bar{\mathbf{u}}_y, \mathbf{f}_3 = -\mathbf{K}\bar{\mathbf{u}}_z, \quad (19)$$

where $\bar{\mathbf{u}}_x$, $\bar{\mathbf{u}}_y$ and $\bar{\mathbf{u}}_z$ are nodal vectors, in which each constraint direction (x , y and z) at constrained nodes has unit value as 1.0. This problem should be taken as a multiple loading problem discussed in our previous work¹⁾, and the Block version of SOAR⁸⁾ is utilized instead of the original SOAR.

First, the detail of our proposed pre-analysis to evaluate an effective number of reduced orders is summarized. The interval m , which is utilized to evaluate the difference at output node, is set to 6. The output node is located at the center of top surface in the upper structure as shown in **Fig.7**. The error tolerance for convergence is set to 1 % during a target frequency. The target frequency is selected by 5 Hz with refer to the FFT results of the input displacement history as shown in **Fig.9**.

Fig.9 FFT results of the Kobe earthquake's input displacement time-history

(a) steel hinge model

(b) steel laminated rubber bearing model

Fig.10 Frequency response at output node in each model

Fig.10(a) and **Fig.10(b)** shows the converged frequency response of these models. As we can see in these results, these different bearing systems may give quite different dynamic response. Using this frequency analysis, the effective reduced order of the steel hinge model and of the steel laminated rubber bearing model are given by 24 and 30, respectively.

Fig.11 show the time transient response results by using KS-MOR with the predicted number of reduced orders. In the both figures, a reference solution by the FEM without reduction is plotted by the circle mark. The black and red lines in **Fig.11** show the results from the steel laminated rubber bearing model and the steel hinged bearing model, respectively. Steel laminated rubber bearing model shows a larger deformation than results in steel hinged bearing bridge due to low stiffness in lateral displacement. In both models, KS-MOR results show good agreement with the FEM solutions.

Fig.11 Displacement in x,y, and z direction in each model

Finally, the CPU time in the KS-MOR calculation is compared with that in the FEM. In this research, a sheared memory parallel processing with OpenMP is utilized for solving linear equations by conjugate gradient method. The Kyushu University's high performance computer machine

Hitachi SR16000 is used for this computation, and two CPUs with 8cores are occupied. The CPU times in these calculations are listed in **Table 3**.

The CPU time for solving 4000 time steps is listed in the table. The CPU time in KS-MOR includes all the process, which includes 1) the calculation of basis vectors, 2) projection into small system, 3) stepwise iterative frequency response analysis, 4) time integration in small system and 5) inverse projection. KSMOR reduces the CPU time to 4.0 – 5.88 % compared to the corresponding FEM. The numerical costs in the steel laminated rubber bearing model is about 2-3 times larger than that in the steel hinge model, although these two FEM model has similar number of total degree of freedoms. In the steel laminated bearing, the Young's modulus in rubber and steel has 4 order differences. It may depend on the performance of our chosen iterative solver (diagonal scaling Conjugate Gradient method) for heterogeneous material problems.

Table 3 Comparison of CPU time

	Average time 1 solution (minutes)	Total Time (hours)
<i>Steel hinge</i>		
KSMOR-24	33	29.4
FEM	7.5	500
<i>Rubber bearing</i>		
KSMOR-30	65	68
FEM	25	1684

5. Conclusions

The one of the main disadvantages in the KS-MOR was the determination the number of reduced orders. In this paper, a practical procedure to evaluate an effective reduced order by using the frequency response analysis is proposed. Once we choose a target frequency domain, the number is automatically evaluated by using a stepwise iterative frequency response analysis.

The proposed procedure is supported by an approximation property called by Moment Matching (MM). The MM implicitly means that accurate frequency range is expanding gradually around the expansion point as the number of reduced order is increasing. Then, an efficient reduced order can be determined at the converged frequency response solution of the stepwise iteration within a target frequency range.

In the numerical examples, KS-MOR show an excellent efficiency in the CPU time, and its numerical cost is 17–24 times faster than the conventional FEM in the case of the Conjugate Gradient solver for large scale problem. In addition, KSMOR with the effective reduced order shows good agreements with the FEM solution.

The current KS-MOR can apply only to the linear structure dynamics simulation. In the practical

application of KS-MOR, we should consider a treatment of locally nonlinear problems, in which the most structural part can be treated as a linear problem.

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