PROBABLE MAXIMUM PRECIPITATION USING STATISTICAL METHOD FOR THE YODO RIVER BASIN

Nor Eliza Alias ¹, Pingping LUO ² and Kaoru TAKARA ³

¹Member of JSCE, Doctoral student, Graduate School of Engineering (Dept. of Civil and Earth Resources Engineering), Kyoto University, (Gokasho, Uji City, Kyoto 611-0011, Japan), permanent address: Universiti Teknologi Malaysia, UTM Skudai, 81310, Johor, Malaysia.

²Member of JSCE, JSPS Postdoctoral Fellowship, Disaster Prevention Research Institute (DPRI), Kyoto University (Gokasho, Uji City, Kyoto 611-0011, Japan)

³Fellow of JSCE, Professor, Disaster Prevention Research Institute (DPRI), Kyoto University, (Gokasho, Uji City, Kyoto 611-0011, Japan)

Probable maximum precipitation (PMP) is one of the keys for the designing of probable maximum flood (PMF) and flood disaster management. A basin-scale spatial distribution analysis of the extreme rainfall and PMP in the Yodo river basin are presented. The maximum 24-hour rainfall data from 1881 to 2011 were used. Spatial PMP and its isohyetal lines produced using Hershfield and Spline interpolation method agrees with the historical flood records. Highest PMP is observed around the Hikone and Shimogahara stations (above 900 mm), while the least at Yanagase and Torahime (below 500 mm). Estimation of the return periods (RPs) using non-linear regression and the generalized extreme value (GEV) distribution for the historical maxima suggests that the GEV overestimates RPs and non-linear regression (NLR) with PMP as upper bound stabilizes the estimates of RP by GEV. Spatial distributions of the 4000 years return periods extreme rainfalls show some similarities with the PMP's distribution.

Key Words: *Hershfield method, GEV, non-linear regression, extreme rainfall, spatial distribution*

1. INTRODUCTION

Japan is exposed to various kinds of water-related natural hazards. Typhoons and extreme rainfalls are common in most part of Japan. Probable maximum precipitation (PMP) is one of the keys for the designing of probable maximum flood (PMF) and flood disaster management (hazard map, dam operation, flood control, irrigation, etc.). World Meteorological Organization (WMO, 2009) defines the PMP as "the greatest depth of precipitation for a given duration meteorologically possible over a given size storm area at a particular location at a particular time of the year, with no allowance made for long-term climatic trends"¹⁾.

This research conducts extreme rainfall analysis and the PMP estimation for the Yodo river basin. The Yodo river basin extends over six prefectures of Mie, Shiga, Kyoto, Osaka, Hyogo and Nara with an area of 8,240 km². It includes Lake Biwa and several main rivers (Katsura, Seta-Uji, Kizu and

Yodo rivers) which flow towards the Osaka bay. The population has been increased from the Edo period (1700) to Taisho period (1925), from 2 million to 4.73 million inhabitants. In 2002 the population have reached 14 million^{2),3)}, thus proving the need for such research.

Various hydrological studies have been conducted within the Yodo river basin before. Luo et al. analyzed land use change under the paleo-environment (1843 to 1976) to explore the effects of human activities on hydrologic response⁴⁾. Kim et al. developed a numerical model for hydrological and climate change analysis. Combination of the Global Climate Model (GCM) and a hydrological model investigated the possible impacts of climate change from the hydrological viewpoint⁵⁾. Other researches were more focused towards the water quality analysis and water resources management especially in the Lake Biwa of Yodo river basin. Even though, several statistical researches have been conducted using data from stations within the Yodo

river basin^(6),7),8), extreme rainfall analysis using whole data of the basin has yet to be conducted.

The objective of this paper is to provide statistical analysis of the maximum annual 24-hour rainfall in the Yodo river basin from 1881 to 2011 in terms of the PMP values and to present spatial distribution of the PMP of the basin. The Hershfield statistical method is used for the point PMP estimation, while the PMP spatial distribution is constructed using the spline interpolation and isohyetal method. Return periods of possible extreme rainfalls are also presented and compared with the PMP values.

2. STUDY SITE AND DATA

We chose 62 stations for the extreme rainfall analysis and PMP estimation. 31 stations inside the Yodo river basin and 31 stations outside of the basin boundary. The stations outside the Yodo river basin were considered as additional margin for the PMP estimation and spatial distribution analysis for the Yodo river basin. The data were obtained from the Japan Meteorological Agency (JMA). Long historical data (1880 - 2011) belongs to the surface weather observation network, while the rest are from the Automated Meteorological Data Acquisition System (AMeDAS) (1976 - 2011) of JMA. **Fig.1** shows the locations of stations selected for the analysis. Distances among each stations are about 20 to 30 km.

Fig.1 Locations of the Yodo river basin and AMeDAS stations selected for the statistical analysis and PMP estimation.

3. METHODS

We use the Hershfield statistical method for estimating the PMP values. The method is being used widely^(9),10),11),12),13) and was considered in the manual on estimation of probable maximum precipitation by the World Meteorological Agency⁽¹⁾ after it had been modified and developed by Hershfield^(14), 15). It is also believed to be closely comparable to those obtained by the conventional moisture maximization and storm transposition methods⁽⁹⁾. The essence of the method is storm transposition, but instead of transposing the specific rainfall amount of one storm, an abstracted statistic K_m is transposed⁽¹⁾. The Hershfield technique for estimating the PMP value for a station uses the following equations.

$$X_{PMP} = X_n + S_n \times K_m \quad (1)$$

$$K_m = \frac{X_{max} - X_{n-1}}{S_{n-1}} \quad (2)$$

where, X_{PMP} is the PMP estimates for a station, X_n is the mean of the annual extreme series, S_n is the standard deviation of the annual extreme series, K_m is the frequency factor which depends on the availability of data period, X_{max} is the highest rainfall value at the station, X_{n-1} is the mean of the annual extreme series without the largest value, and S_{n-1} is the standard deviation of the annual extreme series without the largest value.

First, the parameters X_n , S_n and K_m are calculated. Then, K_m values for all stations are plotted against the X_n values respectively and an envelope curve is drawn. The new K_m value is picked up from the envelope line for each station's X_n . Finally, the PMP values for each station is calculated using Eq. (1) by replacing K_m with the new value.

According to the WMO (2009)⁽¹⁾ envelopment is a process for selecting the largest value from any set of data. Several researchers applied similar envelopment technique for the Hershfield statistical method^(13), 17). **Fig.2** shows the K_m plots against X_n and its envelope line. We compare two envelopes; one considers all the K_m values (PMP(1)) while the other exclude the two extreme K_m values (PMP(2)).

The two extreme K_m values comes from Hikone and Miyagawa. A more smooth an accurate envelope could be obtained if longer records and higher number of extreme rainfalls were recorded. Envelope PMP (1) can be considered as a first order estimation due to the available length of records and data.

Fig.2 K_m distribution of the 24-hr PMP for Yodo river basin.

Spline interpolation method in ArcGIS 10 Spatial analyst tool box were used for the interpolation of the PMP spatial distribution. The method was chosen due to its low interpolation error and its approach in preserving the point value whereby the interpolated surface goes through the data points¹⁶⁾.

4. RESULTS AND DISCUSSIONS

(1) Maximum 24-hour rainfall time series

Fig.3 illustrates the annual maximum 24-hour rainfall distribution of stations within the Yodo river basin from 1880 to 2011. The observed data ranges from 29 years to 131 years. The longest observation data were obtained from Kyoto, Osaka and Hikone stations which have 131, 129 and 118 years of observation respectively, while the others have only up to 36 years. This corresponds to the commencing of AMeDAS in 1974. Information of each station's data and the parameters for the PMP estimation are presented in Table 1.

(2) PMP estimates

Table 1 shows the PMP estimates and its rainfall statistical properties (X_n and X_{max}) of each station. By excluding the extreme K_m values, the PMP estimates (PMP_2) become very small, justifying the importance to include the extreme values for the PMP estimation. Therefore, values of PMP_1 were used throughout the study. In addition, Table 1 also shows higher X_{max} has higher PMP estimates indicating good correspondence between them.

The Hershfield statistical method uses a transposed frequency factor K_m for its PMP estimation; which considers the average maximum rainfall, X_n of one station instead of just the largest maximum rainfall, X_{max} . Thus, for this case Soni station has the highest estimated PMP value due to its high X_n even though Hikone's X_{max} is much

higher. Hikone station has 118 years data compared to Soni station with 36 years data, resulting into Hikone's X_n being much less than Soni's. Therefore, the PMP value in Soni is overestimated due to its less numbers of observations. More accurate PMP estimates could be obtained from longer data records.

Generally, both highest range of PMP value and maximum rainfalls are observed at Soni, Hikone and Shimogahara (PMP values higher than 1000 mm, and X_{max} higher than 300 mm), followed by Nabari, Ueno, Kasatorayama and Toyonaka (PMP values higher than 800 and X_{max} higher than 250 mm).

Table 1 Statistical properties of the annual maximum 24-hour rainfall and PMP values (mm) of the selected 62 stations.

Stations (Years)	X_n (mean)	X_{max}	PMP_1	PMP_2
Ueno (74)	106.53	286.7	881	429
Kasatoriyama (36)	158.85	259	846	414
Nabari (36)	103.17	295	910	441
Yanagase (36)	100.07	157	444	245
Imazu (36)	91.85	164	534	280
Torahime (33)	75.79	120	390	213
Arakawa (29)	110.62	223	788	391
Suijo (33)	100.03	249	728	364
Minamikomatsu (36)	110.47	188	572	301
Hikone (118)	98.00	596.9	1007	482
Omiachiman (36)	99.40	183	638	326
Shimogahara (32)	126.72	341	1060	501
Higashiomi (36)	90.58	174	560	291
Otsu (34)	106.50	158	550	291
Shigaraki (36)	100.58	235	698	352
Tsuchiyama (36)	121.18	231	794	394
Keihoku (36)	104.40	209	734	368
Hanasetoge (34)	122.76	215	758	379
Sonobe (36)	112.46	259	843	413
Kyoto (131)	105.29	288.6	738	369
Nagaokakyo (36)	113.19	239	766	382
Kyotanabe (36)	91.65	200	595	306
Nose (36)	104.01	221	725	363
Mino (33)	102.94	259	749	374
Hirakata (36)	90.56	172	587	302
Toyonaka (36)	96.65	272	837	410
Osaka (129)	90.75	250.7	632	322
Ikoma (36)	92.65	275	773	382
Hari (36)	97.01	220	636	325
Soni (36)	156.21	370	1140	524
Ouda (36)	96.00	235	746	371
Takami (30)	170.05	449	1366	598
Yoshino (21)	105.71	226	787	390
Ozu (27)	171.09	421	1159	524
Ibige (33)	157.53	354	963	457
Sekigahara (36)	143.28	245	817	403
Kamiishizu (36)	139.11	290	1048	494
Hokusei (36)	139.85	310	807	400
Kiraramine (32)	170.13	394	915	437
Yokkaichi (46)	127.41	295	857	419
Kameyama (36)	135.64	242	810	401
Hakusan (33)	142.59	324	1019	482
Kayumi (36)	194.57	498	1304	555
Miyagawa (34)	321.56	764	848	178
Imajo (36)	97.78	141	471	255
Tsuruga (111)	101.04	211.2	677	343
Mihama (36)	104.28	269	876	427
Oi (16)	105.75	218.5	880	429
Obama (36)	99.00	232	777	385
Tsu (111)	135.73	427	969	463
Miyama (33)	96.70	196	649	330
Shuchi (29)	97.83	206	716	359
Jubusan (34)	92.60	204	674	340
Sakai (36)	83.26	163	651	328
Kawachinagano (35)	92.93	213	693	348
Shitsukawa (36)	108.50	232	816	402
Sanda (36)	88.83	188	700	351
Najio (30)	109.93	269	873	426
Rokkasan (30)	100.10	175	650	331
Nara (59)	89.47	182.3	607	311
Tawaramoto (36)	79.75	191	599	305
Katsuragi (31)	85.47	196	574	295

Note: PMP_1 = PMP estimates calculated from the PMP(1) envelope
 PMP_2 = PMP estimates calculated from the PMP(2) envelope

Fig.3 Distribution of the annual maximum 24-hour rainfall observation in the Yodo river basin.

(3) Spatial PMP estimates

A more detail analysis of the PMP in a river basin could be presented using PMP spatial distribution. The PMP spatial distribution and 40-mm interval isohyetal lines for the Yodo river basin is presented in **Fig.4**. PMP values at any point in the river basin could be estimated from the isohyetal lines. The isohyetal lines clearly show the range of PMP values. The highest PMP observed is around the Hikone and Shimogahara stations which are from 900 to 1040 mm, while the lowest is at the north side of Lake Biwa around Yanagase and Torahime where the isohyetal lines are less than 500 mm. At the south-east of the basin, areas around Soni, Nabari Ouda and Ueno also show quite high PMP values (800 to 1300 mm).

Fig. 4 Yodo river basin main rivers, isohyetal map of the PMP and the PMP spatial distribution.

(4) Comparison of PMP with historical flood events

Extreme rainfalls have been proven to influence the discharge within the Yodo river basin⁴, therefore the spatial PMP distribution of the Yodo river basin is compared with previous flood records. **Table 2** presents flood disasters in the Yodo river basin according to the main rivers' maximum flow and the Yodo river's maximum water level. During every flood event the Kizu river frequently has the largest maximum flow compared to Katsura and Uji rivers. Generally, this agrees with the estimated spatial PMP distribution in **Fig.4** which shows areas around Kizu river having higher PMP range (800 mm to 1300 mm) compared to areas around Katsura and Uji rivers (600 mm to 800 mm).

(5) Return periods of extreme rainfalls estimates

Fig. 5 shows return periods for the annual maximum 24-hour rainfall for several stations with the highest PMP values within the Yodo River basin. Return periods based on the generalized extreme value distribution (GEV) were plotted and a non-linear regression equation (NLR) for each station were produced. The NLR equation produced is:

$$Y = \alpha X^\beta, 0 < Y < PMP \quad (3)$$

where Y =rainfall, X =return period, α, β = constants and a PMP as the upper boundary for Y . The parameters α and β are estimated by the non-linear regression method using the plots based on the GEV distribution. If we assume the PMP value to be the maximum (upper bounds) of this NLR lines, as an example Soni's PMP (1140 mm) can be plotted at a

Table 2 Flood disasters in the Yodo river basin¹⁾.

Date	Maximum flow (m ³ /sec)				Max. water level (m)
	Kizu River	Katsura River	Uji River	Yodo River	Yodo River
25/09/1952	5800	2700	1780	7800	6.97
27/09/1955	3850	810	670	4610	5.49
27/08/1957	3650	790	525	4030	5.07
14/08/1958	3900	2500	1270	6800	6.5
27/09/1959	6200	1700	885	7200	6.69
30/08/1960	770	2600	310	3840	4.7
28/10/1961	5220	2100	1000	7800	6.95
17/09/1965	5170	2500	900	6870	6.75
17/09/1972	3250	2320	810	5230	4.64
02/08/1982	3980	1950	990	6260	4.65

point of (X=7235, Y=1140) based on the GEV distribution. Then the optimized NLR line which stabilizes the GEV can be obtained as:

$$Y = 155.55X^{0.2368}, 0 < Y < 1140 \quad (4)$$

Table 3 shows the comparison between the return periods (RPs) estimated by using the GEV and NLR. Both GEV and NLR estimated closely the RPs for the maximum rainfall in most of the stations. However, for higher possible extreme rainfalls (800 mm and 1000 mm) the estimated RPs have bigger differences. GEV overestimates the RPs because GEV's upper bound is infinite. The NLR with upper bound given by PMP stabilizes the RP estimates. In other words, we can avoid (or reduce) overestimation by using NLR.

Fig. 6 shows spatial distribution of 500-, 2000- and 4000-year extreme rainfalls obtained by the GEV distribution ((b), (c) and (d), respectively) as well as PMP (a). Soni, Shimogahara, Nabari, Ueno and Hikone have PMP values around 1000 mm. If compared in **Fig.6** and **Table 3**, the PMPs for Soni, Shimogahara and Nabari are close to the extreme rainfall with 4000 years return period estimated by the GEV distribution. However, Ueno and Hikone are closer to an extreme rainfall with much higher return periods estimated by the GEV (near to 18000 years).

Table 3 PMP and estimates of return periods for historical maximum and possible extreme rainfalls by GEV and NLR.

Stations (Observed years)	PMP (mm)	X_{max} (mm)	Return Period (Years)			
			X_{max}	400 mm	800mm	1000mm
Soni (36)	1140	370	GEV	40	55	1237
			NLR	40	54	1008
Shimogahara (32)	1060	341	GEV	61	109	1636
			NLR	58	104	1284
Nabari (36)	910	295	GEV	71	194	1973
			NLR	65	174	1531
Ueno (74)	881	286.7	GEV	76	302	6430
			NLR	70	273	4257
Hikone (118)	1007	596.9	GEV	2282	464	7407
			NLR	1225	410	5748

Fig.5 Return periods of stations in the Yodo river basin. **Fig.6** (a) PMP spatial distribution compared with extreme 24-hr rainfall with return periods: (b) 500, (c) 2000 and (d) 4000 years using GEV distribution.

5. SUMMARY AND CONCLUSIONS

The PMP estimated using the statistical Hershfield methods corresponds reasonably with the X_{max} of each stations respectively. Generally the highest ranges of PMP value and maximum rainfalls are observed around Soni, Hikone, Shimogahara, Nabari and Ueno with a PMP value around 1000 mm. However, PMP value in Soni is overestimated due to its few numbers of observations. More accurate PMP estimates could be obtained from longer data records.

From the PMP spatial distribution and isohyetal lines the highest observed PMP is around the Hikone and Shimogahara stations which are from 900 to 1040 mm, while the least is around the Yamagase and Torahime stations where the isohyetal lines are shown to be less than 500 mm.

The PMP and maximum annual 24-hour rainfall spatial distribution and the PMP isohyetal lines of the Yodo river basin illustrate a good correspondence with the historical flood records. It is estimated that the area around Ueno station, Nabaki station and along the Kizu river has a high PMP value (800 mm to 1300 mm) compared to areas around Katsura and Uji river (600 mm to 800 mm) corresponding well to areas with the highest river discharge. Analysis on the GEV and NLR shows the GEV to overestimate the return periods because GEV's upper bound is infinite. The NLR with upper bound given by PMP stabilizes the return period estimates.

Generally, the spatial distribution of the PMP and extreme 24-hour rainfall presented in this paper will be useful as a background material in estimating the area with the most extreme rainfall that is possible to occur in a basin. It can also be used for the planning, designing and management of different type of hydraulic structures and future flood risk management. Comparisons as well as inputs to numerical models can also be conducted. Also, by including PMP analysis together with extreme rainfall return periods optimum decisions can be made easier. Such studies are crucial for basins with high population and exposed to various kinds of water related natural disasters.

ACKNOWLEDGMENTS: This study was supported by the Postdoctoral fellowship of Japan Society for the Promotion of Science (P12055), JSPS KAKENHI Grant Number 24-02055, and the Kyoto University Global COE program on "Sustainability/Survivability Science for a Resilient Society Adaptable to Extreme Weather Conditions". The authors wish to thank the Japan Meteorological Agency (JMA) for providing the data.

REFERENCES

- 1) World Meteorological Organization (WMO), Manual for Estimation of Probable Maximum Precipitation, Publication No.1045, World Meteorological Organization, Geneva, 2009.
- 2) Shiga Prefectural Government, Department of Lake Biwa and Environment, Water Policy Division: Towards Integrated Basin Management in Lake Biwa and Yodo River Basin-Past and Future, presented in the International Forum on ILBM-G Project 090307, 2007.
- 3) Kinki Regional Development Bureau, Ministry of Land, Infrastructure and Transport: 2002 Lake Biwa and Yodo River, 2002.
- 4) Luo, P., Takara, K., Apip, He, B., Nover, D. and Yamashiki, Y.: Land use change analysis and paleo-flood in the Kamo river basin, Kyoto, Japan, Journal of Japan Society of Civil Engineers, Ser.B1 (Hydraulic Engineering), Vol. 68, No.4, pp.I_127-I_132, 2012.
- 5) Kim, S., Tachikawa, Y., Takara, K. and Nakakita, E.: Hydrologic prediction under global warming at Tone and Yodo river basin using the output of Global 20-km mesh GCM, Annuals of Disaster Prevention Research Institute, Kyoto University, No.51 B, pp. 11 -20, 2008.
- 6) Takara, K. and Tosa, K.: Storm and Flood Frequency Analysis Using PMP/PMF Estimates, in Proceedings of International Symposium on Floods and Droughts, UNESCO-IHP, Nanjing, China, 18-20 October, pp. 7-17, 1999.
- 7) Takasao, T., Takara, K., and Shimizu, A.: A basic study on frequency analysis of hydrologic data in the Lake Biwa basin, Annuals of Disas. Prev. Res. Inst., Kyoto University, 29 B-2, pp. 157-171 (in Japanese), 1986.
- 8) Takara, K. and Stedinger, J.R.: Recent Japanese Contributions to Frequency Analysis and Quantile Lower Bound Estimators, Stochastic and Statistical Methods in Hydrology and Environmental Engineering, Vol. 1, pp. 217-234, 1994.
- 9) Deshpande, N. R., Kulkarni, B. D., Verma, A. K. and Mandal, B. N.: Extreme rainfall analysis and estimation of Probable Maximum Precipitation (PMP) by statistical methods over the Indus river basin in India, Journal of Spatial Hydrology, 8(1), pp. 22-35, 2008.
- 10) Koutsoyiannis, D.: A probabilistic view of Hershfield's method for estimating probable maximum precipitation. Water Resour. Res., 35(4), pp. 1313-1322, 1999.
- 11) Durbude, D. G.: Estimation of probable maximum precipitation for planning of soil and water conservation structures, Journal of Water Conservation, 7(No.3), pp. 31-35, 2008.
- 12) Desa M, M. N., Noriah, A. B., & Rakhecha, P. R.: Probable maximum precipitation for 24 h duration over southeast Asian monsoon region—Selangor, Malaysia, Atmospheric Research, 58(1), pp. 41-54, 2001.
- 13) Desa M, M. N., & Rakhecha, P. R.: Probable maximum precipitation for 24-h duration over an equatorial region: Part 2-Johor, Malaysia, Atmospheric Research, 84(1), pp. 84-90, 2007.
- 14) Hershfield, D.M.: Rainfall Frequency Atlas of the United States. Technical Paper No. 40, Weather Bureau, United States Department of Commerce, Washington, DC, 1961.
- 15) Hershfield, D.M.: Estimating the probable maximum precipitation, Journal of Hydraulics Division: Proceedings of the American Society of Civil Engineers, 87: pp. 99-106, 1961.
- 16) Blöschl, G. and Grayson, R.: Spatial Observation and Interpolation. In R. Grayson & G. Blöschl (Eds.), Spatial Patterns in Catchment Hydrology: Observations and Modelling, Cambridge University Press, UK, 2000.
- 17) Metreata, S.: Dynamic - Statistical Model for the Determination of Probable Maximum Flood, presented for BALWOIS 2006, Conference on Water Observation and Information System for Decision Support, presentation no. A_098, 2006.

(Received September 30, 2012)