The aim of this paper is to propose the method for evaluating the shear carried by steel fibers of reinforced concrete with steel fibers (RSF) beams. Ten RSF beams with various steel fiber volume fraction, stirrup ratio and specimen size were tested by a four-point bending test. The crack surface displacement was examined by the image analysis. Length of the diagonal crack was measured. The tension softening curves were used to calculate the stress transferred across the diagonal crack. The contribution of steel fibers on the shear resistance of RSF beams has been investigated based on the fracture mechanics. The shear carried by steel fibers calculated from the proposed method showed good agreement with experimental values.

Key Words: shear carried by steel fibers, reinforced concrete beams with steel fibers, crack surface displacement, diagonal crack length, tension softening curves, image analysis

1. INTRODUCTION

It is known that shear failure of reinforced concrete is a sudden behavior that occurs without a warning. Therefore, the standard specifications for concrete structures of Japan Society of Civil Engineers (JSCE) have required a large amount of reinforcing steel bars in structures in order to prevent the shear failure. This causes the congestion of reinforcing steel bars in the structures. One way to improve the shear performance without adding more reinforcing steel bars is the use of steel fibers as a shear reinforcement because the addition of steel fibers to concrete can clearly enhance the ultimate shear strength of reinforced concrete beams.

The shear capacity of RSF members can be predicted by the equation proposed in the design guidelines of JSCE for RSF piers. In the design guidelines, the increment of the shear strength due to steel fibers is expressed as a value \( \kappa \), which is defined as a ratio of the shear carried by steel fibers to the shear carried by concrete, and \( \kappa \) is proposed to equal to 1.0. However, Watanabe et al. concluded that all of the experimental values of \( \kappa \) were more than 1.0 when the volume fraction of steel fibers to full concrete volume (SF) was varied from 0.3% to 1.0%. Fiber contribution is underestimated in the design guidelines. In addition, there was a combination effect between steel fibers and stirrups, which would relate with the diagonal crack behavior that are crack surface displacement and length of diagonal crack. There were few researches on the influences of crack surface displacement and length of diagonal crack on the shear carried by fiber reinforced concrete beams. On the other hands, the image analysis system developed by Higashi et al.
is used in this study for measuring crack surface displacement of specimens at the peak load. The system can measure crack surface displacement in the wide region with high accuracy.

A number of investigators have proposed several empirical and semi-empirical equations to predict the shear strength of fiber reinforced concrete beams. Sharma\(^9\) proposed the empirical equation for predicting the shear strength of RSF beams based on the experimental results. While Narayanan & Darwish\(^6\) suggested that the ultimate shear strength of RSF beams could be predicted based on the split-cylinder strength, the dowel action and the contribution of fibers pullout forces along the inclined crack, which is represented by the fiber matrix interfacial bond factor and fiber factor. Khuntia et al.\(^16\) recommended using compressive strength of concrete and post cracking tensile strength to evaluate the fiber contribution. The available studies, however, have been performed without providing stirrups and the softening behavior of fiber reinforced concrete has not been considered. In addition, the number of steel fibers at cross section and the fiber matrix interfacial bond stress, which have not been satisfactorily quantified yet, are required in those studies. To overcome this shortcoming, the tension softening curve is suggested in this study. The tension softening curves can be used to evaluate the tensile stress transferred across the crack without requiring the number of fibers per unit area.

The objective of this study is to evaluate the shear carried by steel fibers of RSF beams using the crack surface displacement, diagonal crack length and the tension softening curves. Unlike the existing equations, the shear carried by steel fibers is evaluated by concerning the softening behavior of concrete reinforced with steel fibers. The parameters affecting the shape of diagonal crack, which are steel fiber content, stirrup ratio and specimen size, are considered. The crack surface displacements of the diagonal crack are measured and transformed to the tensile stress using the tension softening curves. The length of diagonal crack is also measured for computing the area of stress transfer. The shear carried by steel fibers is computed and compared with the experimental result in order to confirm the validity of the method. The findings in this study provide the understanding of the effectiveness of steel fibers in the shear resistance mechanism of reinforced concrete members with steel fibers and will be useful for formulating more precise predictive equation for the shear capacity of RSF members.

2. PROPOSED METHOD

Before introducing the proposed evaluation method for shear carried by steel fibers of RSF beams, the calculation methods of shear capacity, shear carried by concrete and stirrups are defined.

(1) Calculation method of the shear contribution in RSF beams

Considering the force acting at the diagonal crack in a RSF beam subjected to point loads (Fig. 1), it can be seen that the shear force is resisted by the shear carried by concrete \(V_c\), the shear carried by stirrups \(V_s\) and the shear carried by steel fibers \(V_f\). Consequently, the shear capacity \(V\) of reinforced concrete beams with steel fibers is simplified as:

\[
V = V_c + V_s + V_f
\]

where, \(V_c\) is the shear carried by concrete (kN), \(V_s\) is the shear carried by stirrups (kN), and \(V_f\) is the shear carried by steel fibers (kN).

The shear carried by concrete \(V_c\) can be calculated from Equation (2) as proposed in JSCE design guidelines\(^{12}\).

\[
V_c = 0.2 \cdot \sqrt{f'_c \cdot \frac{1000}{d} \cdot \frac{\sqrt{100p_w \cdot b_w \cdot d}}{}}
\]

where, \(f'_c\) is compressive strength of concrete (N/mm\(^2\)), \(d\) is effective depth (mm), \(p_w\) is longitudinal reinforcement ratio (%), and \(b_w\) is web thickness (mm).

Up to date, the value of \(V_c\) which is the shear carried by concrete at the peak load, in RSF beams has not been satisfactorily quantified. However, \(V_c\) is
expected to be reduced at the peak load because of wide crack surface displacement. Thus, the value of \( V_c \) obtained from Equation (2) is not so far for expressing the contribution of concrete and longitudinal bars.

The shear carried by stirrups (\( V_s \)) was calculated from the strains of stirrups, which diagonal crack passed. The strains of stirrups were measured by strain gauges.

\[
V_s = \begin{cases} \sum A_u E_s \varepsilon_s & (\varepsilon_s < \varepsilon_y) \\ \sum A_u f_{wy} & (\varepsilon_s \geq \varepsilon_y) \end{cases}
\]  

(3)

where, \( A_u \) is cross section area of stirrups (mm\(^2\)), \( E_s \) is the elastic modulus of stirrups (N/mm\(^2\)), \( \varepsilon_s \) is the stirrup strain, \( \varepsilon_y \) is the yielding strain of stirrup, and \( f_{wy} \) is the yield strength of stirrups (N/mm\(^2\)).

In this study, the increase in tensile strength of concrete (\( f_t \)) was considered as the contribution of steel fibers (\( V_f \)). Therefore, the equation of \( V_c \), which was for no-fiber reinforced concrete, was used to determine the contribution of concrete itself. The contribution of steel fiber was evaluated by subtracting the contribution of concrete and stirrups from the shear capacity. As a result, the experimental value of shear carried by steel fibers (\( V_{fexp} \)) can be obtained from Equation (4).

\[
V_{fexp} = V_{exp} - V_c - V_s
\]  

(4)

where, \( V_{exp} \) is the experimental value of the shear capacity (kN).

Besides, according to JSCE design guidelines \(^{12}\), the shear capacity of RSF members can be predicted by using Equation (5).

\[
V_{cal} = (1 + \kappa) \cdot V_c + V_s
\]  

(5)

where, \( V_{cal} \) is the calculated value of shear capacity of RSF members (kN) and \( \kappa \) is coefficient representing the effect of fibers (\( \kappa = 1.0 \)).

Thus, the experimental value of \( \kappa (\kappa_{exp}) \) can be calculated by Equation (6).

\[
\kappa_{exp} = \frac{V_{exp} - V_c - V_s}{V_c} = \frac{V_{fexp}}{V_c}
\]  

(6)

(2) Proposed evaluation method

The post cracking behavior is crucial in RSF beams. After cracking, the tensile stress (\( \sigma \)) can transfer along the length of the diagonal crack due to the bridging effect of steel fibers and this stress becomes the shear contribution from steel fibers. The values of \( \sigma \) transferred across the diagonal crack; however, relate to the crack surface displacement (\( u \)). \( u \) is the displacement of crack on the principal tensile strain direction (\( \theta \)), which is the direction of crack’s movement. \( u \) consists of the displacements from crack opening and crack sliding as shown in Fig. 2. Crack opening displacement is the crack width perpendicular to crack surface, while sliding displacement is the displacement in the tangential direction. The relationship between \( u \) and \( \sigma \) is represented by the tension softening curve. As a result, the tension softening curve is an effective tool for evaluating \( \sigma \).

Investigation procedures of tensile stress are shown in Fig. 3. The distribution of \( u \) along the height of diagonal crack (Fig. 3 (b)) was measured by the image analyzing system developed by Higashi et al.\(^{13}\) and converted to distribution of \( \sigma (\text{Fig. 3 (d)}) \) using the tension softening curve. The tension softening curve will reflect the characteristic of each steel fiber. If other types of steel fibers are used, the slope of tension softening curve will be changed\(^{17}\). Consideration of the tension softening curve will cooperate to predict the shear carried by those steel fibers.

In order to evaluate the shear carried by steel fibers precisely, the specimens were modeled as many
elements with a height of 20 mm (Fig. 1) corresponding to the interval of red targets in the image analysis. Crack surface displacement \( (u_i) \), length \( (L_i) \), angle of principal tensile strain \( (\theta_i) \), and angle of a diagonal crack \( (\beta_i) \) of each element were investigated. The direction of principal tensile stress of the element has been assumed to be the same as the direction of principal tensile strain. Because of existence of reinforcing bars, both directions are not identical. However, the difference between them will not significantly affect the results.

The shear force carried by steel fibers is equal to the stress multiplied by the area of crack surface normal to direction of \( \sigma \). The vertical component of force with consideration of \( \theta \) is the shear carried by steel fibers. The stress and shear forces were calculated for element by element. As seen in Fig. 1, the force along the diagonal crack is obtained from the portion below the compression zone to the tip of diagonal crack in the tension zone and this zone is called the region of interest. The summation of forces in region of interest is the shear carried by steel fibers. Consequently, the shear carried by steel fibers \( (V_{f/cal}) \) can be expressed as follows:

\[
V_{f/cal} = \sum_{i=1}^{n} \left( \sigma_i \cdot b_w \cdot L_i \cdot \cos(\theta_i + \beta_i - 90) \cdot \sin \theta_i \right)
\]  

(7)

where, \( n \) is the number of elements in the region of interest (discussed later), \( \sigma_i \) is the tensile stress of element \( i \) \( (N/mm^2) \), \( L_i \) is the length of diagonal crack of element \( i \) \( (mm) \), \( \theta_i \) is the angle of principal tensile strain of element \( i \) \( (degree) \), and \( \beta_i \) is the angle of diagonal crack of element \( i \) \( (degree) \).

According to Equation (7), \( V_{f/cal} \) increases with the increase in \( \sigma_i \) and \( L_i \). This means, indirectly, \( V_{f/cal} \) increases when \( u_i \) reduces.

It can be seen that the main factors affecting the shear carried by steel fibers is the shape of diagonal crack (i.e. crack surface displacements and length of diagonal crack). The shear carried by steel fibers will change depending on the reinforcement ratio, which are steel fiber volume fraction\(^6,13\), stirrup ratio\(^13\), and specimen size\(^5\). As a result, they became the parameters in this paper.

Even the description of design procedure for the evaluation of the shear carried by steel fibers is not the target of this study, the finding in this paper is essential for improving the more precise prediction for the shear capacity of RSF members. This study evaluated that stress transferred across the diagonal crack was affected by \( SF \), \( r_w \) and specimen size, which the current equation\(^{12}\) has not considered. These results will contribute to the evaluation method for the shear capacity of RSF beams in near future.

### Table 1 Mix proportion of concrete.

<table>
<thead>
<tr>
<th>Maximum size of coarse aggregate (mm)</th>
<th>Water-cement ratio W/C (%)</th>
<th>Sand-aggregate ratio s/a (%)</th>
<th>Unit weight (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>35</td>
<td>53.1</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>471</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>917</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>790</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5.2</td>
</tr>
</tbody>
</table>

### Table 2 Characteristics of steel fibers.

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Diameter (mm)</th>
<th>Density (kg/m³)</th>
<th>Tensile strength (N/mm²)</th>
<th>Elastic modulus (kN/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.62</td>
<td>7850</td>
<td>1050</td>
<td>210</td>
</tr>
</tbody>
</table>

### Table 3 Results of notched beams.

<table>
<thead>
<tr>
<th>Case</th>
<th>SF (%)</th>
<th>Compressive strength (N/mm²)</th>
<th>Tensile strength (N/mm²)</th>
<th>Young's modulus (kN/mm²)</th>
<th>Fracture Energy (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>59.6</td>
<td>2.7</td>
<td>24.3</td>
<td>2.03</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>56.6</td>
<td>4.1</td>
<td>33.3</td>
<td>3.26</td>
</tr>
</tbody>
</table>

Fig. 4 Outline of notched beams.

Fig. 5 Steel fibers.
3. TENSION SOFTENING CURVES

Tension softening curves were obtained from the bending tests of notched beams conducted according to the standard of Japan Concrete Institute (JCI)\textsuperscript{18}). The tension softening curve is one of the fracture mechanics parameters. It can describe the post-cracking behavior and express the resistance of concrete against crack development. Hence, it is effective to explain the mechanism of RSF beams. The tension softening curve is defined as the relationship between tensile stress and crack opening displacement in the fracture zone. The test programs and results of notched beams test are described in this section.

(1) Bending test of notched beams

The notched beams with dimension of $100 \times 100 \times 400$ mm were subjected to three point bending test as shown in Fig. 4. The notched width was 5 mm and height was 30 mm. The displacement transducers were used to measure the displacement of notched beams during loading. Four notched beams were tested for the estimation of a tension softening curve.

(2) Material

a) Concrete

The detail of mix proportion is summarized in Table 1. The materials used in the concrete mixes were high-early strength cement, fine aggregates, coarse aggregates and superplasticizer, which was high-performance air-entraining water-reducing agent. The concrete was designed with an average 7-day age strength of 50 N/mm$^2$.

b) Steel fibers

The steel fibers with hooked-end were used. The characteristics of steel fibers and steel fibers shape are presented in Table 2 and Fig. 5. The volume fractions of steel fibers to full concrete volume ($SF$) in this study were 0.5\% and 1.0\%.

(3) Results of notched beams test

Bending tests of notched beam specimens with the $SF$ of 0.5 and 1.0\% were conducted in order to estimate the tension softening curves. The fracture energy and the tension softening curves were obtained by the equation and software proposed by JCI\textsuperscript{18}). The average data of load-displacement curves from four notched beam specimens was used to determine the tension softening curve.

Table 3 lists the concrete properties and fracture energy of notched beams. Fracture energy increased with the increase in steel fiber volume fraction. The load–displacement curves are shown in Fig. 6. The curves present that the maximum load increased with the increase in steel fiber volume fraction. By considering at the same displacement, the increase in steel fiber volume fraction can improve the load resistance in the post peak region.
The tension softening curves are shown in Fig. 7. The results show that the fiber reinforced concrete with 1.0% of steel fibers can resist higher tensile stress across the crack than that having 0.5% of steel fibers.

Conventionally, tension softening curves represent the relationship of crack opening displacement $w$ and tensile stress $\sigma$. Since the steel fibers are randomly distributed in the concrete, the pullout forces arise in any directions. If the total displacement of crack surface in a tensile test (Fig. 8 (a)) is equal to the total displacement of crack surface in RSF beams (displacement between the black dots as presented in Fig. 8 (b)) (i.e. $w = u = u_0$), they can transfer the same amount of stress $\sigma_0$. Thus, $u$ is equivalent to $w$ and is used for calculating the stress from the relationship of tension softening curves. In addition, the sliding displacement was insignificant.

Moreover, the stress $\sigma$ is the average stress in 20×20 mm element due to steel fibers on the direction of principal tensile strain, which is the significant direction because crack opened in this direction. As a result, this stress takes into account the variation of orientation of steel fibers and is used to evaluate the capacity.

The relationships between the crack surface displacement ($u$) and tensile stress of concrete ($\sigma$) can be obtained by the regression analysis from the tension softening curves. The bilinear curves for $\sigma(u)$ of 0.5 and 1.0% of steel fiber volume fraction are given by Equation (8) and Equation (9), respectively.

$$\sigma = \begin{cases} 4.5 - 58.6u & \text{for } u < 0.06 \text{ mm} \\ 1.0 - 0.24u & \text{for } u \geq 0.06 \text{ mm} \end{cases} \quad (8)$$

$$\sigma = \begin{cases} 7.0 - 40.2u & \text{for } u < 0.13 \text{ mm} \\ 1.85 - 0.55u & \text{for } u \geq 0.13 \text{ mm} \end{cases} \quad (9)$$

4. EXPERIMENTAL PROGRAMS OF RSF BEAMS

(1) Specimens cases

A total of ten RSF beams were tested by four symmetrically concentrated loads as illustrated in Fig. 9. The experimental cases can be classified into two series as shown in Table 4. Series-I considered the effect of steel fiber volume fraction ($SF$) and stirrup ratio ($r_w$). Series-II investigated the effect of specimen size or size effect.

The eight beams in Series-I were prepared with varying $SF$ ($SF = 0.5$ or $1.0\%$) and $r_w$ ($r_w = 0.00-0.30\%$). All eight beams had the same size.

Series-II consisted of three specimens with dif-

![Fig. 9 Outline of specimens.](image)
different size. The beams were geometrically similar in three dimensions. The original specimen was SF10-r30. The size of SF10-r32-S and SF10-r30-L were reduced to 0.75 times and enlarged to 1.5 times of the original size in SF10-r30, respectively. The effective depth ($d$) was 187.5, 250 and 375 mm. The constant variables were $SF$ and $rw$. $SF$ was 1.0% while $rw$ was 0.30%.

Specimens were named according to steel fiber volume fraction, stirrup ratio and size of specimens; e.g., SF10-r32-S corresponded to the specimen with $SF = 1.0\%$, $rw = 0.30\%$ and large size.

(2) Material

The mix proportion of concrete and characteristics of steel fibers used in RSF beams were same with those in the notched beams. The details of reinforcement are summarized in Table 5. Two high strength deformed bars were used as tensile reinforcing bars. Stirrups were deformed steel bars. Two round bars were used as compression bars.

(3) Specimen fabrication

The details of RSF beams are illustrated in Fig. 9 and summarized in Table 4. All beams were rectangular cross sectional beams designed to fail in shear. The shear reinforcement ratio was varied (i.e. $rw = 0.00, 0.12, 0.18, 0.24$ and $0.30\%$). The spacing of stirrups ($s$) in each specimen was changed corresponding to stirrup ratio as shown in Table 4. In order to control a shear span of failure, the number of stirrups in both shear spans was different. The less number of stirrups was considered as the test shear span. For example, in Fig. 9 (a), the right side was the test shear span.

(4) Loading method

Specimens were subjected to a four-point bending. The locations of loading and supporting points are illustrated in Fig. 9. Specimens were placed on the roller supports. Teflon sheets and grease were inserted between a specimen and supports in order to prevent the horizontal friction. Loads were applied to the beams through steel plates to decrease stress concentration. Moreover, anchor plates and nuts were used to ensure the sufficient anchorage of the tensile bars and prevent anchorage failure.

(5) Measurements

a) Load, deflection and strain

During the loading test, the applied load was measured. Mid-span deflection was measured using displacement transducers. One strain gauge was placed on the tensile reinforcing bar at mid-span. The strain gauges were attached on the stirrups at the region where the diagonal crack was expected to occur. The locations of strain gauges are shown in Fig. 9.
b) Image analysis and crack surface displacement

The image analysis was conducted in order to measure the crack surface displacements of specimens. Higashi et al.\textsuperscript{15)} have developed the real-time image analyzing system for the crack surface displacements on the surface of specimens.

In order to conduct the image analysis, white color was sprayed on the surface of specimens. Red targets with diameter of 5 mm were attached on the specimen surface with an interval of 20 mm. During the loading test, photos of the specimen were taken by every 5 kN of the shear force by using three digital cameras fixed on tripods. The white light-emitting diodes (LED) were also used for increasing accuracy of image analysis. A picture of loading test is shown in \textbf{Fig. 10}. The coordinates of targets were investigated by the image analysis.

The crack surface displacements were calculated by using the displacements of targets obtained from the image analyzing system. Crack surface displacement \((u)\) is defined as the relative displacement of targets on the principal tensile strain direction, which is the direction of crack’s movement.

The calculation procedures of \(u\) are presented in \textbf{Fig. 11}. First, the displacements of the targets in \(x\) and \(y\) directions are examined. The displacements of targets are the difference of coordinate of targets between first photo and any photos. Second, the relative displacement between two targets that the crack passed \((\Delta x_1, \Delta y_1)\) (\textbf{Fig. 11 (a)}) is calculated by using Equation (10) and Equation (11). Third, the angle of principal tensile strain \((\theta)\) is calculated at center of square that the crack passed (\textbf{Fig. 11 (b)}) by using Equation (12). Meshing geometry is given based on the targets and displacements of the targets are transformed to the strain of the element by using the shape function of finite element method. The values of \(\varepsilon_x\), \(\varepsilon_y\) and \(\gamma_{xy}\) in Equation (12) are measured by the image analysis. Finally, the crack surface displacements on the direction of principal tensile strain, \(u\), are determined by Equation (13) as shown in \textbf{Fig. 11 (c)}.

\begin{align}
\Delta x &= \Delta x_1 - \Delta x_2 \quad \text{(10)} \\
\Delta y &= \Delta y_1 - \Delta y_2 \quad \text{(11)} \\
\theta &= 0.5 \tan^{-1} \left( \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \right) \quad \text{(12)} \\
u &= \Delta x \cdot \cos \theta + \Delta y \cdot \sin \theta \quad \text{(13)}
\end{align}

where, \(\Delta x\) is the relative displacement in \(x\) direction,
diagonal crack lengths and angles of diagonal crack at the peak load.

5. RESULTS AND DISCUSSIONS

The result of ten RSF beams with different steel fiber volume fraction, stirrup ratio and specimen size is discussed from this section. Table 6 lists the summary of the experimental results.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Specimen properties</th>
<th>Properties of diagonal crack</th>
<th>Experimental results</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SF$ (%)</td>
<td>$r_w$ (%)</td>
<td>$f'_c$ (N/mm$^2$)</td>
<td>$f_t$ (N/mm$^2$)</td>
</tr>
<tr>
<td>SF05-r00</td>
<td>0.00</td>
<td>54.1</td>
<td>2.81</td>
<td>1.40</td>
</tr>
<tr>
<td>SF05-r18</td>
<td>0.18</td>
<td>45.0</td>
<td>2.59</td>
<td>0.69</td>
</tr>
<tr>
<td>SF05-r30</td>
<td>0.30</td>
<td>59.3</td>
<td>3.17</td>
<td>1.25</td>
</tr>
<tr>
<td>SF10-r00</td>
<td>0.00</td>
<td>64.9</td>
<td>3.90</td>
<td>1.54</td>
</tr>
<tr>
<td>SF10-r12</td>
<td>0.12</td>
<td>53.0</td>
<td>3.91</td>
<td>1.12</td>
</tr>
<tr>
<td>SF10-r18</td>
<td>0.18</td>
<td>46.6</td>
<td>3.45</td>
<td>1.10</td>
</tr>
<tr>
<td>SF10-r24</td>
<td>0.24</td>
<td>48.3</td>
<td>3.26</td>
<td>0.94</td>
</tr>
<tr>
<td>SF10-r30</td>
<td>0.30</td>
<td>55.3</td>
<td>3.05</td>
<td>0.67</td>
</tr>
<tr>
<td>SF10-r32-S</td>
<td>0.32</td>
<td>52.0</td>
<td>3.41</td>
<td>0.62</td>
</tr>
<tr>
<td>SF10-r30-L</td>
<td>0.30</td>
<td>55.9</td>
<td>3.02</td>
<td>1.56</td>
</tr>
</tbody>
</table>

$f'_c$: compressive strength, $f_t$: tensile strength, $u_{avg}$: average crack surface displacement of diagonal crack in the region of interest at the peak, $L$: total crack length in the region of interest at the peak, $\theta_{avg}$: average angle of principal tensile strain in the region of interest at the peak, $\beta_{avg}$: average angle of diagonal crack in the region of interest at the peak, $V_{exp}$: ultimate shear force, $V_c$: shear carried by concrete, $V_s$: shear carried by stirrups, $V_{fexp}$: experimental values of shear carried by steel fibers, $k_{exp}$: experimental values of coefficient representing the effect of steel fibers, $V_{fcal}$: calculated values of shear carried by steel fibers.

![Fig. 12 Load-deflection curves of RSF beams.](image)

1) Load-deflection relationships of RSF beams

Figure 12 presents the load-mid span deflection of RSF beams. These diagrams show clearly that shape of the load-deflection curves of the corresponding beams is basically similar in all specimens.

All beams exhibited similar linear behavior from the initial loading up to the first flexural crack. After the first flexural crack, the curve still increased linearly until the initiation of the diagonal crack occurred in a shear span. The diagonal crack was observed around the middle height of the specimens.
This load did not significantly change among specimens in Series-I. Consequently, the increase of $SF$ and $rw$ did not affect the diagonal cracking strength of RSF beams. In Series-II, the load level that diagonal crack occurred was different regarding to the specimen size. However, the shear stress that diagonal crack initiated was not varied among the specimens in Series-II. After the initiation of the diagonal crack, the diagonal crack propagated to supports and to the top of specimens. In addition, the appearance of the diagonal crack reduced the slope of load-deflection curves of RSF beams. Most of the stirrups, which the diagonal crack passed, showed the yielding strain in this stage. The propagation of the diagonal crack length was slow and stopped when the diagonal crack reached to the compression zone (the position of neutral axis). From this stage, the rate of load increment became slight and the load-deflection curves showed nonlinear behavior. Then, the load reached to the peak and suddenly dropped due to the crushing of the concrete in compression zone. Similar process of shear failure was observed in all beams.

(2) Diagonal cracking behavior

Figure 13 presents the diagonal crack at the peak load. It is obvious that only one critical diagonal crack was remarkable in each specimen. Moreover, the diagonal crack was not observed near the top and bottom fibers. This study focuses on the tensile force along this diagonal crack. Figure 14 shows crack patterns of specimens after the loading test. The mode of failure of all specimens was shear failure.

In this study, specimens were divided as many elements with a height of 20 mm corresponding to the interval of targets of the image analysis. Part of element that the crack passed is shown in Fig. 13 (g). The number of elements of the specimens in Series-I was 15 elements. In addition, there were 11 elements in SF10-r32-S and 21 elements in SF10-r30-L. However, the region of interest for calculating the shear carried by steel fibers is only some part as shown in Fig. 13. As mentioned before, the region of interest is from the height below the compression zone until the tip of the diagonal crack in the tension zone, which is the same position with the location of tensile bars except the case of $SF = 0.5\%$. In speci-
mens with 0.5% of steel fibers, the diagonal crack propagated into the concrete cover zone and stopped at 30 mm from the bottom fiber because of the less shear reinforcement. Hence, the number of elements in the region of interest was 13 elements for specimens with 0.5% of steel fibers, 11 elements for specimens with 1.0% of steel fibers, 7 elements for SF10-r32-S, and 16 elements for SF10-r30-L. The crack surface displacement ($u_i$), length ($L_i$), angle of principal tensile strain ($\theta_i$), and angle of the diagonal crack ($\beta_i$) of each element (see Fig. 1) were investigated based on the pictures taken at the peak load. The results of these values are discussed in this section.

a) Crack surface displacement ($u$)

Figure 15 plots the crack surface displacement

![Figure 15](image_url)

Fig. 15 Development of crack surface displacement ($u$) and deflection.

Fig. 16 Distribution of crack surface displacement along the height of diagonal crack at the peak load.
measured at the middle height of the specimens \((u_{mid})\) in the top horizontal axis together with load-deflection curve. Dash lines show the load that the diagonal crack occurred in each specimen from visual observation. Blue lines present the load that stirrups yielded. After initiation of a diagonal crack, values of \(u_{mid}\) measured by the image analysis increased with the increase in the load. Then, \(u_{mid}\) increased drastically before the peak load. This increase in \(u_{mid}\) corresponded to the load when the load-deflection curve began to show nonlinear behavior. Thus, this finding implies that the sudden increment of \(u_{mid}\) related to the peak and led the failure of RSF beams. The crack surface displacement at other heights also showed the same behavior. The significant growth of the crack before load reached to the peak; however, tended to decrease when stirrup ratio increased. This finding can be seen by comparing Fig. 15 (a)-(b) and Fig. 15 (c)-(d). This result implies that stirrups can prevent the growth of the diagonal crack.

The value of crack surface displacement along the height of diagonal crack \((u_i)\) at the maximum load is shown in Fig. 16. Unlike flexure cracks, the diagonal crack surface displacement was larger at around the middle height of specimens compared with the crack surface displacement at the top and bottom of specimens because there was compression zone at top of specimens and restraint by longitudinal reinforcing bars at the bottom fiber. In order to consider effect of specimen size on crack surface displacement, the crack surface displacement distribution of the specimens in Series-II is plotted in Fig. 16 (c). The height of elements \((h_i)\) was normalized by the specimen’s height and expressed as a ratio in Y-axis. It was found that the crack surface displacement was greater in the larger specimen. Nonetheless, the parabolic shape of distribution of \(u\) was also observed though the size of specimens was changed.

Table 6 shows the average value of crack surface displacements \((u_{avg})\) of diagonal crack in the region of interest at the peak load. The results present that \(u_{avg}\) tended to increase with the increase in SF and specimen size. On the other hand, \(u_{avg}\) tended to decrease with the increase in \(r_w\). However, SF05-r30 showed different tendency. This tendency may come from the orientation of steel fibers in the specimen.

Since \(u\) at the peak load is a key to investigate the shear carried by steel fibers in this research, it is important to measure \(u\) at the peak load. Nevertheless, \(u\) increased drastically just before the peak load. Thus, some pictures were taken near the peak load with short time interval in order to capture behavior at the peak exactly. Furthermore, the picture at the maximum load was compared with other pictures around the peak to confirm that the differences between these pictures were insignificant. As a result, the deviation from the difference of taken-time of the picture at the peak does not affect the discussion in this paper.

b) Diagonal crack length (\(L\))

The crack length \((L)\) is the length of a diagonal crack in the region of interest as seen in Fig. 13. Table 6 summarizes \(L\) of all specimens. \(L\) decreased with the increase in the SF and \(r_w\). By considering the values of \(L/d\), the results of diagonal crack length of specimens with different size can be discussed. \(L/d\) increased slightly with the increase in the beam depth as presented in Fig. 17. This is because the shape of diagonal crack changed from straight line to bilinear shape when the specimen size became larger. This behavior can be seen in Fig. 14.

c) The angle of principal tensile strain (\(\theta\)) and diagonal crack (\(\beta\))

The angle of principal tensile strain \((\theta)\) and the angle of diagonal crack \((\beta)\) were measured at the same position with \(u\). Table 6 presents the average values of these angles in the region of interest. \(\theta_{avg}\) and \(\beta_{avg}\) did not change significantly among these specimens.
(3) Shear capacity and shear carried by steel fibers observed in the experiment

Table 6 summarizes the shear forces from the experiments and the shear forces calculated from Equation (2) to Equation (4) and Equation (6). Most of the stirrups that diagonal crack passed were yielded. The shear capacity of RSF beams from the experiment \( V_{exp} \) was higher than the calculated value obtained from Equation (5), which is the existing equation for predicting the shear capacity of RSF beams proposed by JSCE design guidelines \(^{12}\), in most specimens. As a result, \( \kappa_{exp} \) was more than 1.00 even in the specimens with 0.5% of steel fibers except SF05-r30. \( \kappa_{exp} \) was varied from 0.98 to 1.43. The values of \( V_{exp} \) and \( V_{fexp} \) were varied depending on \( SF, r_w \) and \( d \).

(4) Effect of steel fiber volume fraction and stirrup ratio

Eight RSF beams (Series-I) were tested with a viewing of investigating the influences of \( SF \) and also \( r_w \). Figure 18 shows the effects of \( SF \) and \( r_w \) on the shear capacity \( V_{exp} \) and Fig. 19 presents the effects of \( SF \) and \( r_w \) on the shear carried by steel fibers of RSF beams \( V_{fexp} \). Among the specimens in Series-I, \( V_{exp} \) showed the maximum value in SF10-r30, while the maximum value of \( V_{fexp} \) was observed in SF10-r00.

The influence of steel fiber volume fraction can be investigated by comparing \( V_{exp} \) and \( V_{fexp} \) of specimens with various \( SF \) but same \( r_w \). It was found that the increase in \( SF \) from 0.5 to 1.0% enhanced the values of \( V_{exp} \) and \( V_{fexp} \). \( V_{exp} \) increased 15, 8, 1% in the beams which \( r_w = 0.0, 0.18, 0.30\% \), respectively. Besides, the increase in \( V_{fexp} \) was 23, 19, 10% in specimens having \( r_w = 0.0, 0.18, 0.30\% \). The increase of \( V_{fexp} \) is because the more stress can transfer across diagonal crack when steel fiber volume fraction increased as seen in the results of tension softening curves. The increase of \( V_{exp} \) and \( V_{fexp} \) due to the effect of \( SF \); however, became less significant with the increase in \( r_w \). This implies that the effect of steel fibers as shear reinforcement is dominant in case of smaller \( r_w \).

On the other hand, beams with the same \( SF \) but different in \( r_w \) showed another tendency. That is, the value of \( V_{exp} \) in RSF beams will not necessarily increase with the increase in \( r_w \) as shown in Fig. 18 because the value of \( V_{fexp} \) became less significant with the increase in \( r_w \) (Fig. 19). It is because the increase in \( r_w \) led the shorter diagonal crack length resulting in the decrease in \( V_{fexp} \). Nonetheless, it also led the smaller crack surface displacement resulting in higher \( V_{fexp} \). Due to these positive and negative effects, among the specimens having same \( SF \) but various \( r_w \), the value of \( V_{fexp} \) showed the maximum value in SF10-r00 and the minimum value in SF10-r18.

(5) Size effect

The influence of specimen size is discussed based on the results of specimens in Series-II. The shear strength of RSF beams \( (v_{exp}) \) and the contribution of shear stress due to steel fibers \( (v_{fexp}) \) are given by Equation (14) and Equation (15), respectively.

\[
v_{exp} = \frac{V_{exp}}{b_w \cdot d} \quad \text{(14)}
\]

\[
v_{fexp} = \frac{V_{fexp} - V_c - V_s}{b_w \cdot d} \quad \text{(15)}
\]

Figure 20 shows results of \( v_{exp} \) and \( v_{fexp} \). Although \( V_{exp} \) and \( V_{fexp} \) increased with the increase in the
specimen size, \( V_{exp} \) slightly decreased. It can be concluded that the size effect in RSF beams was very slight when \( d \) was in the range from 187.5 to 375 mm. Furthermore, the difference of the shear stress carried by steel fibers (\( V_{exp} \)) was insignificant as presented in Fig. 20; hence, there was no size effect on the shear carried by steel fibers in case of \( 187.5 \leq d \leq 375 \) mm. The reason is that though \( u \) increased with the increase in the specimen size, \( L/d \) also increased resulting in larger crack surface area that the stress can transfer. Consequently, the average shear stress transferred across the crack did not decrease even in the large specimen.

6. CALCULATED SHEAR CARRIED BY STEEL FIBERS AND COMPARISON

The stress across the diagonal crack and the shear carried by steel fibers were evaluated based on the fracture mechanics as mention in Section 2(2). The tensile stress transferred across the diagonal crack was calculated from the crack surface displacement and the equations of the tension softening curves (Equation (8) and Equation (9)). The tensile stress along the height of diagonal crack (\( \sigma_i \)) is shown in Fig. 21. The larger stress can be resisted at top and bottom parts of specimens due to the smaller crack surface displacement. The difference of tensile stress among specimens having \( SF = 0.5\% \) was slight even though stirrups ratio was varied (Fig. 21 (a)). This is because the slope of tension softening curve, which \( SF = 0.5\% \), is almost flat when \( u \geq 0.06 \) mm. Therefore, \( \sigma_i \) converted from \( u \) did not change significantly. The result was expected that SF05-r30 would provide the smallest \( u \); however, SF05-r30 showed the variation that may be caused by the orientation of steel fibers in the specimen. Therefore, SF05-r30 revealed the relatively large \( u \). As a result, instead of SF05-r30, SF05-r18 showed largest stress due to the smallest \( u \). With the increase in steel fiber volume fraction from 0.5 to 1.0\%, the values of stress across diagonal crack increased and the effect of stirrups became significant as shown in Fig. 21 (b). This is because, comparing with 0.5\% of steel fibers, the tension softening curves of \( SF = 1.0\% \) can resist more stress at the same \( u \) and the slope of the curve when \( u \geq 0.13 \) mm is steeper than the slope of \( SF = 0.5\% \) resulting in the significant stress converted from \( u \). Besides, as shown in Fig. 21 (c), the stress transferred across crack decreased in a larger specimen due to the increase of \( u \) when the specimen became larger as shown in Fig. 16 (c).

The shear carried by steel fibers was calculated by Equation (7). The number of elements in the region of interest was different depending on \( SF \) and \( d \). There were 13 elements for specimens with 0.5\% of steel fibers, 11 elements for specimens with 1.0\% of steel fibers, 7 elements for SF10-r32-S, and 16 elements for SF10-r30-L.

The results of the experimental value (\( V_{exp} \)) given by Equation (4), the calculated shear carried by steel fibers (\( V_{cal} \)) using Equation (7), and the comparison between \( V_{exp} \) and \( V_{cal} \) are shown in Table 6. Moreover, Fig. 22 shows the variation of \( V_{exp}/V_{cal} \). A total of ten RSF beams were used to confirm the applicability of the proposed method. The average (avg.) and the coefficient of variation (C.V.) of the ratio of the experimental to the calculated value (\( V_{exp}/V_{cal} \)) are 1.01 and 9.9\%, respectively. The comparison demonstrates that \( V_{cal} \) provides the good agreement with \( V_{exp} \). This method can examine the shear carried by steel fibers of RSF beams though \( SF \), \( r_w \), and \( d \) are varied and can be applied in case of RSF beams without stirrups. This finding implies that the shear carried by steel fibers can be evaluated by using the tension softening curves.

7. CONCLUSIONS

1) The crack surface displacements of the diagonal crack around the middle height of RSF beams were larger than those at top and bottom fibers because there was the compression zone at the top fiber and restraint by the longitudinal steel bars at the bottom fibers.

2) The crack surface displacement of the diagonal crack at the peak load trended to increase with the increase in the steel fiber volume fraction and specimen size and decrease with the increase in the stirrup ratio except the specimen with steel fiber volume fraction of 0.5\% and stirrup ratio of 0.30\%. This different tendency may be caused
by the orientation of steel fibers inside the specimen.

3) With the increase in steel fiber volume fraction and stirrup ratio, the length of diagonal crack decreased. In addition, the ratio of diagonal crack length to effective depth was greater in the larger specimen size. It is because the shape of diagonal crack changed from straight line to bi-linear curve.

4) Increasing the steel fiber volume fraction from 0.5% to 1.0% enhanced the shear capacity of RSF beams because the value of shear carried by steel fibers increased due to the significant stress transferred across the diagonal crack even if the crack length became shorter.

5) The shear capacity will not necessarily increase with the increase in stirrup ratio in case of RSF beams. It is because the shear carried by steel fibers reduced due to the decrease in diagonal crack length. This implies that the addition of steel fibers is more beneficial for specimens with less stirrup ratio.

6) The size effect on shear capacity of RSF beams was very slight in case that effective depth (d) was varied from 187.5 to 375 mm. Moreover, there was no size effect in shear carried by steel fibers. The reason is that the ratio of diagonal crack length to effective depth increased in the larger specimen size resulting in the larger area of stress transfer. Hence, the same values of stress can be transferred even the crack surface displacement (u) was larger.

7) The tensile stress transferred across the diagonal crack was investigated using the crack surface displacement and the tension softening curves. The larger stress can be transferred at top and bottom fibers of specimens due to the smaller crack surface displacement.

8) The evaluation method for shear carried by steel fibers in RSF beams was proposed. The shear carried by steel fibers calculated by using the tension softening curves showed good agreement with experimental values.

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