PROPOSED PREDICTIVE EQUATION FOR
DIAGONAL COMPRESSIVE CAPACITY OF
REINFORCED CONCRETE BEAMS

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The current standard specifications of JSCE for the diagonal compressive capacity of RC beams only consider the effect of the compressive strength of concrete and are not applicable to high strength concrete. This research aims to investigate the effect of various parameters on the diagonal compressive capacity and propose a predictive equation. Twenty five I-beams were tested by three-point bending. The verification of the effects of concrete strength, stirrup ratio and spacing, shear span to effective depth ratio, flange width to web width ratio and effective depth was performed. The diagonal compressive capacity had a linear relationship to stirrup spacing regardless of its diameter. The effect of spacing became more significant with higher concrete strength. Thus, the effect of concrete strength and stirrup spacing was interrelated. On the other hand, there were slight effects of the other parameters on the diagonal compressive capacity. Finally, a simple empirical equation for predicting the diagonal compressive capacity of RC beams was proposed. The proposed equation had an adequate simplicity and can provide an accurate estimation of the diagonal compressive capacity than the existing equations.

Key Words : diagonal compressive capacity, web crushing, high strength concrete, predictive equation

1. INTRODUCTION

Recently, the development of high strength concrete with the compressive strength ($f'_{c}$) more than 100 N/mm\textsuperscript{2} and high strength reinforcing bars with the yield strength ($f_{y}$) more than 685 N/mm\textsuperscript{2} has been successful\textsuperscript{11}. Such advanced materials have been applied into the construction practice since designers can take advantages from their high strength properties and propose more economical infrastructures by reducing material volumes, allowing the increase in the span length of concrete bridges and the reduction of cross-sectional area of members. However, in case of reinforced concrete (RC) beams, the combination of thin web T- or I-shaped cross section and high strength shear reinforcing bars will lead to an uncommon type of shear failure known as the diagonal compression failure. It is caused by the crushing of web concrete prior to the yielding of stirrups. In addition, the diagonal compressive capacity of RC beams using high strength materials could be less than the typical shear capacity, which is assumed to be the summation of the contributions of concrete and shear reinforcements.

The research on the mechanism of diagonal compression failure is insufficient since it is usually avoided because of its brittle phenomenon. On top of that, there are some concerns regarding the applica-
bility of the existing predictive equations for the diagonal compressive capacity to high strength concrete since most of them were based on experimental results of beam specimens with normal strength concrete. The design equation for the diagonal compressive capacity of RC beams in the current Japan Society of Civil Engineers (JSCE) standard specifications for concrete structures only considers the effect of the compressive strength and limits the applicability to concrete with \( f'_c \) up to 50 N/mm\(^2\). From these reasons, further study on the diagonal compressive capacity of RC beams, especially in high strength region is required.

The objective of this research is to investigate the mechanism of thin web RC beams using high strength materials failing in diagonal compression failure and develop an accurate and simple procedure for predicting diagonal compressive capacity of RC beams. An experimental study on the diagonal compressive capacity of 25 I-shaped RC beams was conducted in this research. Some of the specimens were presented in previous research by the authors. The effect of compressive strength \( f'_c \), stirrup ratio \( r_w \), spacing of stirrups \( s \), shear-span to effective depth ratio \( a/d \), flange width to web width ratio \( b_f/b_w \) and effective depth \( d \) on the diagonal compressive capacity was investigated. Subsequently, an empirical equation for predicting the diagonal compressive capacity was proposed. Finally, the accuracy of existing equations by Placas & Regan, JSCE standard specifications, ACI Building Code and Eurocode on the prediction of the diagonal compressive capacity was verified and compared to the proposed equation in order to evaluate its validity.

2. REVIEW OF THE EXISTING EQUATIONS FOR DIAGONAL COMPRESSIVE CAPACITY

1) The equation by Placas and Regan

Placas and Regan proposed an empirical equation for evaluating the diagonal compressive capacity as the following:

\[
V_{\text{Placas}} = (1.04 + 0.21r_w)\sqrt{f'_c b_w d} \tag{1}
\]

Factors involving the diagonal compressive capacity in this equation are \( f'_c \) and the stirrup ratio \( r_w \). Although there is no upper limit of \( f'_c \) stated in this equation, the experimental evidences used to derive this equation are approximate 35 N/mm\(^2\).

2) JSCE Standard Specifications

In JSCE standard specifications, only \( f'_c \) is considered as the influential parameter of diagonal compressive capacity. Because this formula was originally proposed for application to normal strength concrete, the equation is only valid for concrete with \( f'_c \) not exceeding 50 N/mm\(^2\).

\[
V_{\text{JSCE}} = 1.25\sqrt{f'_c b_w d} \tag{2}
\]

3) ACI Building Code 318M-08

In the design recommendation for shear in ACI building code (ACI 318M-08), the contribution of shear reinforcement \( V_s \) is limited to be \( 0.66f'_c b_w d \) for preventing from the crushing of concrete in web. Therefore, the diagonal compressive capacity can be expressed as

\[
V_{\text{ACI}} = V_c + V_s \leq V_c + 0.66\sqrt{f'_c b_w d} \tag{3}
\]

where; \( V_c \) is the contribution of concrete and is equal to 0.17 \( f'_c b_w d \).

The validity of the ACI Code is limited to concrete having the compressive strength not greater than 70 N/mm\(^2\) because of a lack of test data and practical experience.

4) Eurocode 2 1992-1-1:2004

In Eurocode 2, the strut inclination method based on a truss model was used to design members with shear reinforcement. Designers can select the values of angle of concrete strut \( \theta \) freely from 21.8 to 45 degrees. In this paper, to study the most extreme cases, \( \theta \) was assumed to be 45 degrees. Hence, the maximum shear force which can be sustained by the member, limited by the crushing of the compression struts, becomes the following:

\[
V_{\text{EC2}} = 0.45v f'_c b_w d \tag{4}
\]

where; \( v \) is a coefficient that takes into account the increase of fragility and the reduction of shear transfer by the aggregate interlock with the increase of \( f'_c \). It may be taken to be 0.6 for \( f'_c \leq 60 \) N/mm\(^2\) and the maximum between 0.9-\((f'_c/200)\) and 0.5 for \( f'_c > 60 \) N/mm\(^2\).

3. EXPERIMENTAL PROGRAM

1) Specimen details

The experimental program prepared 25 RC beams with I-shaped cross section. Three-point bending tests were conducted by a 2000kN capacity hydraulic testing machine. The summary of test variables and details of specimens are provided in Table 1 and Fig. 1. The web width \( b_w \) and effective depth \( d \) of all specimens were 40 mm and 220 mm, except for
### Table 1 List of the experimental cases.

| Specimen | Series | \(f'_{c}\) [N/mm²] | \(b_{y}\) [mm] | \(b_{w}\) [mm] | \(d\) [mm] | \(a/d\) | \(h_{b}/b_{w}\) | \(\rho_{w}^{*2}\) [%] | \(D^{*3}\) [mm] | \(f_{r}^{*4}\) [N/mm²] | \(\phi_{r}^{*5}\) [%] | \(s^{*6}\) [mm] | \(f_{sy}^{*7}\) [N/mm²] |
|----------|--------|-----------------|----------------|----------------|---------|--------|----------------|----------------|----------------|----------------|----------------|---------|----------------|----------------|
| A06-s160\(^{3}\) | 2 | 35 | 660 | 3.0 | \(1214\) | 0.63 | 7.1 | 160 | 1425 |
| A1-s160\(^{5}\) | 2 | | | \(930\) | 2.0 | 80 | \(1275\) | | |
| A2-s80\(^{6}\) | 2,3 | | | | \(1198\) | 2.0 | 80 | \(1350\) | | |
| A3-s55\(^{5}\) | 2 | | | \(880\) | 4.0 | | \(55\) | | |
| A2-ad40\(^{8}\) | 3 | | \(890\) | 4.5 | | \(1275\) | | | |
| B2-s80\(^{8}\) | 2 | \(65\) | | | | \(930\) | 2.9 | 55 | | |
| B3-s55\(^{5}\) | 2 | | | \(660\) | 3.0 | | | \(930\) | 2.9 | 55 | | |
| C1.2-s150\(^{9}\) | 1 | | 250 | 40 | 220 | 660 | 3.0 | 6.25 | \(1214\) | 12.7 | \(160\) | \(955\) |
| C1.5-s120\(^{9}\) | 1 | | \(100\) | | | \(1204\) | 1.5 | 9.35 | \(120\) | \(953\) | | |
| C1.8-s100\(^{9}\) | 1,5 | | \(250\) | 40 | 220 | 660 | 3.0 | 6.25 | \(1214\) | 12.7 | \(160\) | \(955\) |
| C2-s160\(^{9}\) | 1 | | \(250\) | \(100\) | \(220\) | 660 | 3.0 | 6.25 | \(1214\) | 12.7 | \(160\) | \(955\) |
| C2-s90\(^{10}\) | 1,3 | | \(100\) | \(220\) | \(120\) | 660 | 3.0 | 6.25 | \(1214\) | 12.7 | \(160\) | \(955\) |
| C2-s50\(^{10}\) | 1 | | \(250\) | \(100\) | \(220\) | 660 | 3.0 | 6.25 | \(1214\) | 12.7 | \(160\) | \(955\) |
| C3-s60\(^{10}\) | 1 | | \(250\) | \(100\) | \(220\) | 660 | 3.0 | 6.25 | \(1214\) | 12.7 | \(160\) | \(955\) |
| C4-s45\(^{10}\) | 1 | | \(250\) | \(100\) | \(220\) | 660 | 3.0 | 6.25 | \(1214\) | 12.7 | \(160\) | \(955\) |
| C2-ad35\(^{10}\) | 3 | | | \(770\) | \(3.5\) | | \(1350\) | | |
| C2-ad40\(^{10}\) | 3 | | | \(880\) | \(4.0\) | | \(1350\) | | |
| D-s60\(^{3}\) | 2,4 | | \(250\) | \(120\) | \(220\) | 660 | 3.0 | \(930\) | \(3.5\) | 12.7 | \(90\) | \(955\) |
| D-s90\(^{4}\) | 2 | | \(250\) | \(120\) | \(220\) | 660 | 3.0 | \(930\) | \(3.5\) | 12.7 | \(90\) | \(955\) |
| D-s160\(^{4}\) | 2 | | \(250\) | \(120\) | \(220\) | 660 | 3.0 | \(930\) | \(3.5\) | 12.7 | \(90\) | \(955\) |
| E-s60\(^{3}\) | 2 | 165 | | | \(660\) | 3.0 | \(930\) | \(3.5\) | 12.7 | \(90\) | \(955\) |
| D-f150 | 4 | 130 | 150 | | | \(1204\) | 3.0 | 9.53 | \(60\) | \(953\) | | |
| D-f500 | 4 | 130 | 150 | | | \(1204\) | 3.0 | 9.53 | \(60\) | \(967\) | | |
| C-s100L | 5 | \(100\) | 343 | 58 | \(319\) | \(953\) | 6.25 | 8.6 | 31.8 | \(1187\) | 2.2 | 12.7 | \(100\) | \(931\) |

\(\ast\) upper flange width, \(\ast\) longitudinal reinforcement ratio \((=100.A_{b}/b_{w}d)\), \(\ast\) nominal diameter of longitudinal bar, \(\ast\) stirrup ratio \((=100.A_{w}/b_{w}s)\), \(\ast\) nominal diameter of stirrups, \(\ast\) spacing of stirrups, \(\ast\) yield strength of stirrup.

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**Fig. 1** Dimensions and re-bar layout of beam specimens.
C-s100L which has $b_w = 58 \text{ mm}$ and $d = 319 \text{ mm}$. The main parameters were the compressive strength of concrete ($f'_c$), the stirrup ratio ($r_w$) with different spacing ($s$), the shear span to effective depth ratio ($a/d$), the flange width to web width ratio ($b_f/b_w$) and the effective depth ($d$). The experimental cases can be classified into five series. First is the case for the effect of $r_w$ and $s$ in high strength concrete beams with various nominal diameters of stirrups ($7.1, 9.53, 12.7 \text{ mm}$). Second is the case for the effect of $s$ with different $f'_c$. Third is the effect of $a/d$ ratio. Forth is the effect of $b_f/b_w$ by changing upper flange width. The last is the effect of $d$ or the size effect. Mix proportions for concrete used in the experiment are listed in Table 2.

All specimens were designed to be symmetric and be able to resist against the flexure failure and the diagonal tension failure by using the high strength reinforcing bars ($f_y, f_{wy} > 930 \text{ N/mm}^2$) as both of tensile and shear reinforcements. Stirrups were anchored around a tensile bar by the semi-circular hook with 15 mm hook radius. In addition, the combination of thin web cross section with dense stirrups will cause specimens to exhibit the diagonal compression failure. In order to avoid the local failure, the web width outside support was increased to that of the bottom flange. Anchor plates and nuts were used to ensure the sufficient anchorage of the tensile bars and prevent the anchorage failure.

### (2) Instrumentation and test procedures
For all specimens, the applied load, mid-span deflections and strains of concrete, tensile bars and stirrups were measured. Concrete strain gauges were attached at the top fiber of the mid span. Strain gauges were attached at the mid span to measure the strain of longitudinal bars whereas at the distance of $d/2$ from the top fiber for all stirrups in the shear spans. The additional strain gauges were provided in D-f150 and D-f500 as shown in Fig. 2. As for D-f500, embedded strain gauges were installed inside the concrete in the flange portion to measure the strain distribution in the flange. Besides, both surfaces of all specimens were painted by white color to ease the drawing and observation of cracks during the experiments. Pictures were taken by two digital single-lens reflex cameras for both shear spans.

### (3) Method for evaluating beam and arch actions by strain distribution measurement
In RC beams, the total shear force ($V$) is equal to the derivative of the moment as shown in Eq. (5).

$$V = \frac{dM}{dx} \quad \text{and} \quad M = T \cdot jd \quad (5)$$

### Table 2 Mix proportion of concrete.

| Mix | Target $f'_c$ [N/mm²] | $G_{max}$*1 [mm] | $W/B$*2 [%] | $s/a$*3 [%] | $W$*4 | $C$*5 | $L$*6 | $SF$*7 | $S$*8 | $G$*9 | $SP$*10 | $V$*11 |
|-----|------------------------|------------------|------------|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A   | 35                     | 15               | 60         | 45.3       | 175 | 292 | 249 | -  | -   |  718 |  857 |  8.12 | 0.35 |
| B   | 65                     | 15               | 25         | 48.3       | 165 | 660 | -   | -  | -   |  753 |  790 |  9.9  | 0.17 |
| C   | 100                    | 15               | 20         | 44.6       | 150 | 675 | -   |  75 | 672 |  861 | 22.5 | -    |
| D   | 130                    | 15               | 16         | 38.2       | 150 | 856 | -   |  95 | 526 |  851 | 45.7 | -    |
| E   | 165                    | 15               | 16         | 38.2       | 150 | 844 | -   |  94 | 526 |  851 | 28.1 | -    |


![Fig. 2 Location of the stain gauges of D-f150 and D-f500.](image-url)
where, \( T \) and \( jd \) are tensile force in the longitudinal reinforcement and the moment lever arm, respectively. Consequently, Eq. (5) becomes

\[
V = d(T \cdot jd) \frac{dx}{dx} = jd \frac{dT}{dx} + T \frac{d(jd)}{dx}
\]

(6)

The former term is called ‘beam action’ while the latter component represents ‘arch action’. These actions are combined together to resist the total shear force. Although the beam action normally controls the shear strength of a beam when \( a/d \) is comparatively large, the situation in case of RC T- or I-beam can be changed because of the presence of top flange\(^8\). In Series 4 (effect of \( \frac{bf}{bw} \)), the changing of governing mechanism will be used to explain the effect of \( \frac{bf}{bw} \) on the diagonal compressive capacity. A method for evaluating the beam and arch actions in the experiment was adapted from the study of Pansuk and Sato\(^8\). To evaluate the value of these two actions between the two known sections of the beam, Eq. (6) can be rewritten as,

\[
V = jd \frac{\Delta T}{\Delta x} + T \frac{\Delta (jd)}{\Delta x}
\]

(7)

By measuring the strain distribution of a section with an assumption that the stress-strain relationship of concrete is elastic in compression and the stress of concrete in tension is negligible, the value of \( T \) and \( jd \) of that section can be obtained at each load level. The locations of strain gauges which are used for evaluating beam and arch actions for D-f150 and D-f500 are given in Fig. 2. To calculate Eq. (7), \( jd \) and \( T \) are taken to be the average of the two sections while \( \Delta T \) and \( \Delta jd \) equal to \( T_1 - T_2 \) and \( jd_1 - jd_2 \), respectively.

(4) Data analyzing methods

The shear force when the first flexural crack occurred \( (V_{cr}) \) can be determined by the relationship between the applied shear force and the longitudinal bar strains and visual observation during the loading tests. From the pictures taken at the peak load, the crack spacing in horizontal direction \( (s_{ci}) \) and the crack angle \( (\beta_i) \) were measured at the middle height of the web. The example of \( s_{ci} \) and \( \beta_i \) measurement of a crack is presented in Fig. 3. The average of \( s_{ci} \) of cracks in the shear span \( (s_{c,avg}) \) and the average of \( \beta_i \) of cracks in B-region \( (\beta_{avg}) \) will be used in the later discussion. It is noted that the B-region is the portion outside the distance approximately \( d \) away from the loading point and supports.

4. EXPERIMENTAL RESULTS

(1) General behavior of diagonal compression failure

Load-deflection relationships of representative specimens in Series 1 and 2 are illustrated in Fig. 4. Firstly, specimens behaved in elastic manner until the first flexural crack occurred in the bottom flange near the mid span, which is reflected in the graph as a rate of inclination decreases. After the first flexural crack, the load-deflection curve remained to advance linearly with the continuous initiation of diagonal cracks at the web concrete in the order from the loading point to the support. It indicates that stirrups were still functional even their hook length was short. In the pre-peak region, the deflection increased with a relatively small increase in applied load as the web concrete began to crush. Afterwards, the applied load reached to the peak. After the peak load, applied load rapidly decreased. The experimental results are summarized in Table 3. Data of the stresses of longitudinal bars and stirrups revealed no yielding at the peak load. It implies that the failure mode was neither the flexure failure nor the diagonal tension failure. The web concrete crushed at the peak load and splitting cracks along the member axis near tensile bars
did not initiate at that time; hence, the cause of failure was not by anchorage failure of both tensile bars and stirrups. It can be concluded by considering these observations that the failure mode of all specimens was designated as the diagonal compression failure. The diagonal compression failure in which the web concrete crushed before the yielding of stirrups exhibited the brittle mode. Crack patterns at the end of loading for representative specimens in Series 1 and 2 are demonstrated in Fig. 5. The thicker lines and the shaded areas represent the wider width crack and the crushing areas, respectively.

(2) Effect of stirrup ratio \( r_w \) with different diameter of stirrup \( \phi \)

Series 1 consists of eight specimens: C1.2-s150, C1.5-s120, C1.8-s100, C2-s160, C2-s90, C2-s50, C3-s60 and C4-s45. The dependence of \( f'_c \) is considered by \( f'_c^{1/2} \) in the design equation of JSCE\(^2\) and the predictive equation by Placas & Regan\(^5\). The relationship between \( f'_c \) and \( v_{exp}/f'_c^{1/2} \) is drawn in Fig. 6, where \( v_{exp} \) is the shear strength (=\( V_{exp}/b_w d \)). The values are nearly horizontal in the range of \( f'_c \geq 60 \) N/mm\(^2\). Therefore, the shear strength normalized by \( f'_c^{1/2} \) is used to eliminate the effect of \( f'_c \) in this study. The relationships between \( r_w \) and \( v_{exp}/f'_c^{1/2} \) for the specimens with \( f'_c = 100 \) N/mm\(^2\) are demonstrated in Fig. 7. With the increase in \( r_w \), the diagonal compressive capacity increases when using the nominal diameter of stirrups (\( \phi_d \)) equal to 9.53 mm. A linear relationship agrees well with the test results. However, in the case of constant value of \( r_w = 2% \) with different \( \phi_d \) and \( s \), \( v_{exp}/f'_c^{1/2} \) differs significantly. From Table 3, \( s_{avg} \) among these three specimens (C2-s50, C3-s60 and C4-s45). The dependence of \( f'_c \) is considered by \( f'_c^{1/2} \) in the design equation of JSCE\(^2\) and the predictive equation by Placas & Regan\(^5\).

### Table 3 Experimental results.

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<tr>
<th>Specimen</th>
<th>( f'_c ) N/mm(^2)</th>
<th>( r_w ) %</th>
<th>( s ) mm</th>
<th>( a/d )</th>
<th>( b/h_w )</th>
<th>( f_c ) N/mm(^2)</th>
<th>( \sigma_{u_{max}}^{1} ) N/mm(^2)</th>
<th>( f_{wo} ) N/mm(^2)</th>
<th>( \sigma_{u_{max}}^{2} ) N/mm(^2)</th>
<th>( s_{avg} ) [mm]</th>
<th>( \beta_{avg} ) [degree]</th>
<th>( V_{cr} ) [kN]</th>
<th>( V_{exp} ) [kN]</th>
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The difference in the diagonal compressive capacity with the change in \( s \) can be explained by two mechanisms resulting from the confinement effect by stirrups. One is smaller diagonal crack width \((w)\) caused by providing closer shear reinforcements; therefore, the critical average stress in web concrete \((\sigma_{2\text{max}})\) would be greater because \(w\) affects the diagonal compressive capacity as reported by Schäfer et al.\(^9\) and Reineck\(^10\). The other is the localization of compressive strut. From crack patterns after the loading in Fig. 5, in case of the closer-spacing specimens, cracks distribute more finely and the crushing area at web distributes more widely than that of the wider-spacing specimens (C1.2-s150 and D-s160). As well, \(sc_{\text{avg}}\) shows a decreasing trend with smaller stirrup spacing. It implies that the failure localization occurred in C1.2-s150 and D-s160 as higher \(sc_{\text{avg}}\) were observed. Figure 9 explains the model of compressive strut formation under different stirrup spacing.

---

**Fig. 5** Crack patterns at the end of loading for representative specimens in Series 1 and 2.

**Fig. 6** \(f_c'v_{\text{exp}}/f_c'^{1/2}\) relationship.

**Fig. 7** Effect of \(r_w\) \((f_c'=100 \text{ N/mm}^2)\).

**Fig. 8** Effect of \(s\) \((f_c'=100 \text{ N/mm}^2)\).
that the stress can distribute along the beam axis. However, further study by some analytical approach is required to clarify the actual stress flow.

(3) Effect of stirrup spacing $s$ with different compressive strength $f_c$

In Series 2, the additional 10 specimens (A06-s160, A1-s160, A2-s80, A3-s55, B2-s80, B3-s55, D-s60, D-s90, D-s160 and E-s60) were combined with Series 1 in order to discuss the effect of $s$ with different $f_c$. Figure 10 displays $v_{exp}/f_c^{1/2}$ as a function of $s$ for specimens with different $f_c$. It is confirmed that the effect of $s$ depends on $f_c$. There is a certain variation in $v_{exp}/f_c^{1/2}$ by the change in $s$ when using normal strength concrete while the greater effect of $s$ can be observed in the high strength concrete beams. As for beams with $f_c > 65 \text{ N/mm}^2$, the diagonal compressive capacity reduces as $s$ increases in a linear relationship. With higher $f_c$, the inclination of trend line increases with a limitation when $f_c$ approximates 100 N/mm$^2$. It can be explained by considering that the diagonal compressive capacity depends on not only $f_c$ but also the width of compressive strut. If the width of compressive strut becomes smaller with higher $f_c$, the area loss of the struts reduces the diagonal compressive capacity; Thus, the diagonal compressive capacity is limited.

Figure 11 plots the shear strength $v_{exp}$ (= $V_{exp}/b_wd$) as a function of $f_c$. Since $v_{exp}/f_c^{1/2}$ is nearly constant regardless of $s$ when using $f_c = 35 \text{ N/mm}^2$, the specimen A3-s55 is used as a representative of specimen with $f_c = 35 \text{ N/mm}^2$ and $s = 60 \text{ mm}$ in this figure. It can be observed that $v_{exp}$ increases with the increase in $f_c$. The incremental rate of $v_{exp}$ is different when using different $s$.

(4) Effect of shear span to effective depth ratio $a/d$

As a longer span is more preferable in the practical use of high strength concrete girders, this research focused on the range of $a/d \geq 3.0$. From the phenomenal point of view, lower $a/d$ can increase the angle of diagonal compressive strut ($\theta$) and drastically
change the shear resisting mechanism. However, in the focused range of \(a/d\), the effect of \(a/d\) on \(\theta\) is in- substantial. Figure 12 plots the value of \(v_{exp}/f'_{c}^{1/2}\) of the beams with \(f'_{c} = 35\) and 100 N/mm\(^2\) (A2-s80, A2-ad40, A2-ad45, C2-s90, C2-ad35 and C2-ad40) as a function of \(a/d\). As for the case of \(a/d \geq 3.0\), the normalized shear strength becomes almost constant. The effect of \(a/d\) can be neglected in this study.

(5) Effect of flange width to web width ratio \(b_f/b_w\)

Series 4 comprises of three specimens (D-s60, D-f150 and D-f500). The web width was designed to be constant and \(b_f/b_w\) was varied by increasing upper flange width. From Table 3, slight difference in \(s_{c,avg}\) can be noticed. The normalized shear strength against \(b_f/b_w\) is drawn in Fig. 13. It can be seen that the normalized shear strength shows nearly constant value with the increasing of \(b_f/b_w\). Hence, there is no effect of \(b_f/b_w\) on the diagonal compressive capacity of RC I-beams within this range. It can be explained by two reasons.

Firstly, the strain distribution along the top flange (the perpendicular direction to beam axis) of D-f500 at the mid span (see Fig. 2) is presented in Fig. 14. The positive value indicates tensile strain. The strain concentration can be observed at the zone near the center of the beam. This tensile strain is resulted from the development of high diagonal compressive stress in the web. As the distance from the beam center increases, the strain decreases and becomes comparatively small at some distance. Thus the outside flange portion resists less stress and can be considered as an ineffective area. Since the increase in upper flange width becomes ineffective, the capacity is constant.

Secondly, the results of beam and arch actions between section 1 and 2 in Fig. 2 calculated by Eq. (7) are demonstrated in Fig. 15. The calculation will be accurate in case that the summation of the both actions approximates the applied shear force. In both specimens, the beam action controls the shear resistance at the beginning stage. After that, the arch action gradually increases with the increase in the total shear resistance. Finally, the arch action becomes dominant in both specimens. Although the ratio between beam and arch actions is different in the early stage because of the difference in upper flange width, it becomes similar just before the peak. This result reveals that the mechanism at failure does not change with wider upper flange width. As a result, the diagonal compressive capacity becomes constant. It is noted that the measured region included both B-region and D-region. Based on the experimental observation that the failure occurred in the B-region, the result of this measurement will not be considered in the equation formulation.

(6) Effect of effective depth \(d\) or size effect

Size effect was examined by comparing between C1.8-s100 and C-s100L. Size of C-s100L was magnified by a 1.45 scale factor in three dimensions. The \(f'_{c}, p_{w}, a/d, b_f/b_w, s\) as well as \(s\) in this series were designed to be constant. As a result of series 1, \(s\) was chosen to be constant, rather than \(r_{w}\). From Table 3, \(s_{c,avg}\) is wider in the larger specimen and approximate 1.45 times larger. Figure 16 plots the shear strength \(v_{exp}\) as a function of \(d\). It exhibits slight variation of the shear strength with the change of \(d\). Therefore, the effect of \(d\) from 220 to 319 mm on the diagonal compressive capacity cannot be observed.

(7) Angle of diagonal crack in B-region

Since it was observed from crack patterns that the crushing area, which is corresponding to the failure region, were mostly found in B-region, the measurement of diagonal crack angle was focused on only in B-region. The relationships between the average diagonal crack angle in B-region at the peak load (\(\beta_{avg}\)) and \(f'_{c}, r_{w}, s\) and \(a/d\) are illustrated in Fig. 17. It can be
seen that the values of $\beta_{avg}$ are scattering in the range between 27 to 47 degrees. A clear tendency between $\beta_{avg}$ and each factor influencing the diagonal compressive capacity cannot be observed. The average value of $\beta_{avg}$ is 36.6 degrees with the coefficient of variation 13.5%.

5. EQUATION PROPOSAL

(1) Development of the equation

A free body diagram cut at a web in vertical direction is illustrated in Fig. 18. The forces acting on the cutting surface are:

1) Compressive force of concrete fiber ($C'$)
2) Diagonal compressive force of strut in web ($D'$)
3) Tensile force of longitudinal reinforcement ($T$)

Considering the equilibrium in a vertical direction, the shear force, $V$ must be resisted by the vertical component of $D'$. Since it is difficult to indicate the exact width of compressive struts from experiments, this research assumes a uniform distribution of diagonal compressive stress in web. Subsequently, the diagonal compressive stress in web, $\sigma_2$ is equal to $D'/(b_wjd\cos\theta)$. With the assumption that the diagonal compressive force causes the failure of the member without yielding of stirrups, the following expression can be obtained.

$$V_{max} = D'_{max} \sin \theta = \sigma_{2_{max}} b_w jd \cos \theta \sin \theta \quad (8)$$

where; $V_{max}$ is the shear force carried at web crushing stage. $\theta$ is the angle of inclination of concrete struts to the longitudinal reinforcement. $\sigma_{2_{max}}$ is the critical compressive stress in the web concrete.

With the use of trigonometry, Eq. (9) is proposed based on Eq. (8).

$$V_{max} = \frac{1}{2} \sigma_{2_{max}} b_w jd \sin 2\theta \quad (9)$$

The $\sigma_{2_{max}}$ of the diagonally cracked concrete tends to be less than compressive strength of the same concrete obtained in a standard cylinder test since cracked concrete is subjected to tensile strains in the direction perpendicular to the compression\textsuperscript{[12]}. In this paper, an appropriate formula for $\sigma_{2_{max}}$ was empirically generated by using the trends of the experimental results presented in the chapter 4. The data
indicated that \( f'_c \) and \( s \) had a major effect on the diagonal compressive capacity, whereas the influence of \( a/d, b/h_w \) and \( d \) was insignificant. Hence, only the effect of \( f'_c \) and \( s \) was taken into account in the development of predictive equation.

As mentioned in section 4(2), the relationship between the diagonal compressive capacity and \( s \) demonstrated the inverse variation. Section 4(3) indicated that the effect of \( f'_c \) and \( s \) was interrelated and the diagonal compressive capacity increased as \( f'_c \) increased with the different increment ratio depending on the value of \( s \). The 3-D expression of \( v_{exp}/f'_c, s \) relationship was used to formulate a function of the \( \sigma_{2, max} \) having the minimal variation and corresponding to the relationship observed in the experiments. Also, the \( \sigma_{2, max} \) was considered to have adequate simplicity and was developed through a surface-fitting technique as the following equation:

\[
\sigma_{2, max} = \left(4.54 - \frac{s}{79.6}\right) \sqrt{f'_c} \quad \text{(10)}
\]

where; \( f'_c \) and \( s \) are in N/mm\(^2\) and mm, respectively.

The residual plot of \( v_{exp}/f'_c, s \) demonstrated in Fig. 19 manifests the accuracy of the fitting surface.

As for the angle of compressive strut (\( \theta \)), this study assumed it to be the same as that of the average crack angle in B-region (\( \beta_{avg} \)). Even though it is widely accepted that the angle of compressive strut is variable; however, it was chosen to be constant in this research since an obvious tendency of \( \beta_{avg} \) cannot be discovered as explained in section 4(7). Various researchers have proposed indeterminate truss systems representing coincident behavior of tied arch and beam\(^2-14\). Considering these models, when \( a/d \geq 3.0 \), the arch action have insignificant effect on \( \theta \) which was also observed from the experiment. Hence, the average value of \( \beta_{avg} \) (36.6 degrees) was selected as the constant value of \( \theta \). By substituting Eq. (10) into Eq. (9), \( id = (7/8)d \) as recommended in JSCE\(^2\) and \( \theta = 36.6 \) degrees, the diagonal compressive capacity is given by the following equation.

\[
V_{cal} = \left(1.9 - \frac{s}{190}\right) \sqrt{f'_c} b_w d \quad \text{(11)}
\]

where; \( f'_c \) and \( s \) are in N/mm\(^2\) and mm, respectively.

The range of application of this equation are 45 mm \( \leq s \leq 160 \) mm, \( 0.6\% \leq r_w \leq 4\% \), 19 N/mm\(^2\) \( \leq f'_c \leq 165 \) N/mm\(^2\), \( 3.75 \leq b/b_w \leq 12.5 \) and 2.5 \( \leq a/d \leq 4.5 \).

It is noted that, in the case that \( s \) exceeds 361 mm, the term in the parenthesis will become a minus value. However, most of the design standards allow the maximum stirrup spacing to be not larger than 300 mm. Therefore, the mentioned case will not occur in practice. The strength reduction factor \( v \) of Eurocode 2\(^7\) (Eq. (4)) and the proposed equation (= \( \sigma_{2, max}/f'_c \)) against \( f'_c \) is illustrated in Fig. 20. In case of the proposed equation, when \( f'_c \) increases from 20 N/mm\(^2\) to 165 N/mm\(^2\), \( \sigma_{2, max}/f'_c \) decreases from 0.89 to 0.31 when using \( s = 45 \) mm and from 0.57 to 0.20 when using \( s = 160 \) mm. However, in Eq. (4), \( v \) becomes constant when \( f'_c > 80 \) N/mm\(^2\). Eurocode 2 may be derived from specimens with \( s = 45 \) mm as close relationship between Eq. (4) and Eq. (11) when \( s = 45 \) mm can be seen in the range of \( f'_c = 60-80 \) N/mm\(^2\).

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Table 4 List of detail of the specimens and results of the other authors.
(2) Accuracy of the existing equations

The experimental results of the total of 35 beams including Rangan\(^5\) and Leonhardt & Walther\(^6\) were gathered (see Table 4). In order to clarify the applicability of the existing equations presented in Section 2 to high strength concrete, the equations were applied beyond their limitation of compressive strength. All specimens had \(f'_{c}\) from 19 to 165 N/mm\(^2\) and were reported to fail in the diagonal compression failure. The ratio between the shear capacity from experiments and results obtained by the existing equations were calculated. The average of these ratio \((\text{avg.})\) with a coefficient of variation \((\text{C.V.})\) is presented in Fig. 21.

Figure 21(a) compares the diagonal compressive capacity from test results with those obtained by the equation by Placas & Regan\(^5\) (Eq. (1)). The average of \(V_{\exp}/V_{\text{Placas}}\) equals to 0.95 with a C.V. of 17.2%. It implies that Placas’s equation can evaluate an average value of the diagonal compressive capacity including the case of \(f'_{c} > 100\) N/mm\(^2\). On the contrary, this study revealed that the evaluation for \(r_w\) is not appropriate and this equation exhibits large variation as C.V. = 17.2 %.

JSCE standard specifications (Eq. (2)) demonstrate the average of \(V_{\exp}/V_{\text{JSCE}}\) = 1.14. Figure 21(b) indicates that JSCE equation underestimates the capacity in almost all specimens. The specifications may be conservative because of safety reason. In the same way as Placas’s equation, the results calculated by Eq. (2) shows large variation as C.V. = 18.4%. Evaluating the accuracy of JSCE standard within its limitation of concrete strength, the average of \(V_{\exp}/V_{\text{JSCE}}\) = 1.09 with C.V. of 21.0 % is obtained. It also gives slight difference of accuracy.

The accuracy of ACI Building Code (Eq. (3)) is represented in Fig. 21(c). It can be seen that the ACI 318-08 does not correlate with the test data as average = 1.71 and C.V. = 18.6%. Eq. (3) highly underestimates the diagonal compressive capacity throughout the range of data and is the most conservative among these four equations. As well, the acquired average. = 1.66 and C.V. = 19.8% when verifying the accuracy of ACI code inside its regulation for compressive strength reveal similar accu- rateness.

Similarly, the evaluation of the Eurocode 2 (Eq. (4)) is shown in Fig. 21(d). The average of \(V_{\exp}/V_{\text{EC2}}\) is 0.77 with a C.V. of 27.6 %. The selection of 45-degree \(\theta\) results the prediction by Eurocode 2 in severe overestimation, especially in the case of high strength concrete. Likewise, Eq. (4) does not
consider the presence of stirrups. Thus its result gives huge scattering. Afterwards, the prediction of Eurocode 2 within its cover range of $f'_c$, is assessed. The average is 0.82 with $C.V.$ of 24.1%. Equation (4) performs better in this range of application; however, its accuracy is unsatisfactory.

From the reasons mentioned above, the current equations are not accurate enough to evaluate the diagonal compressive capacity of RC beams, especially in high strength concrete. Besides, the evaluation for $r_{sw}$ in the existing equations is not appropriate as explained in section 4(2).

### (3) Accuracy of the proposed equation

The validity of the developed equation (Eq. (11)) was established by the comparison between the predictions from the proposed equation with the experimental results. In Fig. 22, the accuracy of the proposed equation was evaluated with the same database used in section 5(2). The average of $V_{exp}/V_{cal}$ is 1.0 with a coefficient of variation of 10.5%. The proposed equation offers a better correlation with this set of data and can predict diagonal compressive capacity accurately compared to the other existing equations.

It should be noted that, as the proposed equation is an empirical type, it can be applied only in the range of the study. This equation can be implemented to beams with the compressive strength from 19 to 165 N/mm², the stirrup ratio in the range from 0.6 to 4%, the stirrup spacing in the range from 45 to 165 mm, the flange width to web width ratio from 3.75 to 12.5, the shear-span to effective depth ratio from 2.5 to 4.5, effective depth from 220 to 563 mm and web width from 40 to 150 mm. In addition, it is required that yielding of stirrups must not occur.

As for stirrup arrangement, the proposed equation covers both one-legged stirrup, as used in this study and the study of Rangan (5), and two-legged type, as used by Leonhardt & Walther (6). As for practice approach to real structures, stirrup spacing may exceed the applicable range of the proposed equation. Wahrave and Lehwalter reported significant size effect in shear compression failure of short beams (2); hence, there is a possibility that size effect in diagonal compression failure can be occurred. In JSCE standards, shear capacity of box section beams can be considered as the same as beams with $b_w$ equal to the total web widths of box section beams (3). In this sense, the proposed equation can also be extended to box section beams. Further investigation, however, is required to validate the applicability of the proposed equation to these cases.

In conclusion, the proposed equation has adequate simplicity with wider range of application and provides a high accuracy on the prediction of diagonal compressive capacity of RC beams than the current equations.

### 6. CONCLUSIONS

1. With insufficient confinement effects provided by stirrups, the localization of compressive stress in struts occurred. Thus, the stirrup spacing is prominent, not the stirrup ratio, for evaluating the diagonal compressive capacity of RC beams.
2. The greater effect of the stirrup spacing on the diagonal compressive capacity of RC beams is observed in higher strength concrete. Thus, the effect of compressive strength of concrete and the stirrup spacing is interrelated.
3. The diagonal compressive capacity increases as compressive strength increases with different incremental ratio depending on the stirrup spacing.
4. The ratio of shear span to effective depth from 3.0 to 4.5, the ratio of flange width to web width from 3.75 to 12.5 and the effective depth from 220 mm to 319 mm have almost no influence on the diagonal compressive capacity of RC beams.
5. Based on the experimental results, a predictive equation considering the compressive strength of concrete and the stirrup spacing for evaluating diagonal compressive capacity of RC beams is developed.
6. The proposed equation has sufficient simplicity and provides higher accuracy (avg. = 1.00) and lower variation ($C.V. = 10.5\%$) on the prediction of diagonal compressive capacity of RC beams. The proposed equation can be implemented to beams with the compressive strength: 19-165 N/mm², stirrup ratio: 0.6-4.0%, stirrup spacing: 45-165 mm, flange width to web width ratio: 3.75-12.5, shear-span to effective depth ratio: 2.5-4.5, effective depth: 220-563 mm, web width from 40 to 150 mm.
width: 40-150 mm and both one-legged and two-legged stirrups.

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(Received February 10, 2011)