SHEAR CARRYING CAPACITY OF SEGMENTAL CONCRETE BEAMS WITH DRAPEED EXTERNAL TENDONS

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This study presents an investigation of the shear behavior of segmental concrete beams with draped external tendons. Deviator force and transfer mechanism of prestressing force from an anchorage that affect the shear failure mechanism have been examined based on the results of experimental procedure and FEM with different location of deviator and inclined angle of draped tendons. The modified model proposed by authors has been extended for segmental concrete beams with draped external tendons with considering the effect of deviator force and transfer mechanism of prestressing force from the anchorage. The results from the extended modified model for draped external tendons had a good agreement with experimental results.

Key Words: draped external tendon, inclined angle, deviator force, segmental concrete beams, shear failure mechanism, shear carrying capacity, modified model, extended modified model

1. INTRODUCTION

Precast segmental construction is widely used in bridge structures, because of substantial cost and time saving in construction. In the beginning, the internal tendon technology has been used for segmental technology where tendons are located inside a concrete cross section. However, problems such as corrosion in internal tendons or leakage at epoxy joints may cause the damage in segmental bridges. So, the application of external prestressing to precast segmental structures has been used as an innovative method in segmental construction technology.

Both straight tendon profile and draped tendon profile have been applied in a segmental concrete beam with external tendons. The difference compared to the internal tendons is that the prestressing force from external tendons acts on the structure only at deviators and an anchorage at the ends of a beam. A straight external tendon profile was used in the authors’ previous study1). It was concluded that the effect of deviator force (uplift force at a deviator) in a shear span on the shear capacity was insignificant.

The draped profile has been used to alleviate congestion in anchorage zones, to reduce concrete stresses at transfer and to provide a vertical component for shear in the high shear and low moment zone in a simply supported beam. However, test data to investigate the effect of draped external tendons on shear behavior of segmental concrete beams are very limited. Several researchers reported the experimental results focusing on the shear behavior of...
monolithic concrete beams with draped internal tendons\textsuperscript{2,3}). Some studies focused on the flexural behavior of monolithic concrete beams with draped external tendons\textsuperscript{4}).

In most current codes\textsuperscript{5,6)}, the contribution of draped tendons on the shear carrying capacity has been considered via the vertical component of prestressing force without considering the effect of location of deviators and anchorages. The effects of deviator force and the transfer mechanism of prestressing force from the anchorage\textsuperscript{7)} by not only inclined angle of draped external tendons but also the location of deviator on the behavior of segmental joint, the shear behavior of segmental beams and supplementary load carried by draped external tendons are necessary to be evaluated. In addition, the simplified truss model\textsuperscript{8)} was modified for segmental concrete beams with straight external tendon profile in the authors’ study, so-called modified model\textsuperscript{1)}.

The modified model has not considered the effect of deviator force and the transfer mechanism of prestressing force from the anchorage in segmental beams with draped tendons. The supplementary load carried by draped tendons, therefore, has not been evaluated in the modified model yet.

This paper focuses on the shear mechanism and shear carrying capacity of segmental concrete beams with draped external tendons. The effects of deviator force and transfer mechanism of prestressing force from different locations of anchorages and deviators on the shear transfer mechanism across an opened segmental joint, the shear failure mechanism of segmental beams are examined by the experiment and the finite element analysis. Based on the comprehension of shear transfer mechanism, the modified model is extended to predict the shear capacity.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{detail_of_test_beams.png}
\caption{Detail of test beams.}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Beams & $a/d$ & $a_d$ (deg.) & $a_{de}$ (mm) & Lower fiber stress, $\sigma_l$ (N/mm$^2$) & Upper fiber stress, $\sigma_u$ (N/mm$^2$) & Compressive strength, $f'_c$ (N/mm$^2$) & Tensile strength, $f_t$ (N/mm$^2$) \\
\hline
SJ10-19 & 3.5 & 0 & 483 & 20.0 & -0.2 & 65 & 66.4 \\
D09-200 & 9.0 & 9.0 & 200 & 19.6 & -1.1 & 69.3 & 65.1 \\
D09-600 & 9.0 & 9.0 & 600 & 19.6 & -0.9 & 69.3 & 65.1 \\
D14-600 & 14.2 & 14.2 & 600 & 19.2 & -2.0 & 69.3 & 65.1 \\
\hline
\end{tabular}
\caption{Detail of test beams.}
\end{table}

Note: For beams with draped tendons, batches A and B correspond to end segments and middle segments, respectively.

For beams SJ10-19, batches A and B correspond to larger segment and smaller segment, respectively.
of segmental concrete beams with draped external tendons.

2. TEST PROGRAM

(1) Detail of specimens

Figure 1 shows the detail of specimens in this study. Three simply supported concrete beams designed to fail in shear with $a/d$ ratio of 3.5 were used. The distance, $a$, from a loading point to a joint position used in these beams was $1.0d$; where $d$ was the effective depth of the beam at deviators. The concrete stress at the upper and lower fibers of the span center of each beam was about 0 N/mm$^2$ and 19 N/mm$^2$ (positive means compression). The inclined angle of tendons was 9.0 deg. and 14.2 deg. Location of a deviator from the loading point, $a_{de}$ was 0.5$d$ (200 mm) and 1.5$d$ (600 mm). The name of beams is D09-200, D09-600 and D14-600. Thus, D09-200 means that the inclined angle was 9 deg. and $a_{de}$ was 200 mm. SJ10-19 in the previous study$^1$ is used as a reference beam. Table 1 tabulates the details of test beams and SJ10-19.

(2) Materials

a) Concrete and epoxy

The match-cast method was used for casting the segmental beams in order to provide a perfect matching at the segmental joints. In this method, the end segments of each beam were cast first with a wood shear key as an end formwork. Two days later the formworks were removed and the end segments themselves were used as the end formworks for the middle segment. The design compressive strength of concrete, $f'_c$ was specified as 65.0 N/mm$^2$ at 28 days. The actual compressive and tensile strengths of concrete are tabulated in Table 1. The tensile and bending strengths of the epoxy resin used to connect the segmental joints were about 25 N/mm$^2$ and 40 N/mm$^2$, respectively.

b) Prestressing tendons and deviators

Prestressing steel bars with type of 19-wire were provided including two draped external tendons and one unbonded internal tendon in the top flange as shown in Fig. 1(a). The tendons used for D09-200 and D09-600 had a nominal diameter of 17.8 mm. Since it requires larger strain in the draped tendons in D14-600, the tendons with a nominal diameter of 17.8 mm and 21.8 mm were used for the unbonded internal tendon and the draped external tendons, respectively. The mechanical properties of the tendons are given in Table 2. The tendons were introduced after the assembly of concrete segments with epoxy and stretched for three days before testing.

Figure 2 shows four bolts embedded in the web of a beam for each steel deviator. The bolts were tried to arrange in the smallest distance to ensure that the deviator force ideally transfers at the center of the deviator. Four teflon layers with thickness of 0.8 mm were also provided to diminish the friction force between a deviator and an external tendon.

c) Reinforcements

The arrangement of reinforcement in the beam is shown in Fig. 1(b). The non-prestressed steel bars were deformed bars. In all beams, six deformed bars with a nominal diameter of 13 mm (D13), and eight...
deformed bars with a nominal diameter of 10 mm (D10) were provided as internal longitudinal reinforcement at the bottom and the top flange, respectively. Stirrups were also provided at the web and the top flange with an interval of 400 mm and 100 mm, respectively. So, the stirrup ratio in the web is about 0.16 percent. It is larger than the minimum amount of vertical stirrups in JSCE Standard Specifications, i.e. 0.15 percent. The average mechanical properties of the reinforcements are given in Table 3. Steel meshes by steel bars with nominal diameter of 13 mm were also provided to resist the local stress caused by prestressing force as shown in Fig. 1(b).

3. DESCRIPTION OF MODELS

DIANA system was used in nonlinear finite element method (FEM). A concrete beam is modeled by means of four-node quadrilateral isoparametric plane stress elements with the mesh as shown in Fig. 3. The behavior of concrete in compression and tension is modeled according to the model proposed by Thorenfeldt et al. and Hordijk, as shown in Figs. 4 and 5, respectively. The smeared crack model is applied for the crack model of concrete. Constant shear retention is applied to model the shear behavior of concrete. After cracking the shear retention factor, \( \beta \), is adopted at 0.01 in this study. Longitudinal reinforcements are modeled by means of the embedded reinforcement element. Tendons are modeled by the two-node truss element. A bilinear elasto-plastic constitutive model appropriately represents the stress-strain relationship of reinforcing bars and tendons. The properties of concrete, reinforcing bars and external tendons are shown in Tables 1, 2 and 3.

To represent the interfacial behavior between external tendons and deviators or the concrete beam, the two-node interface element is applied. In order to neglect the friction between tendons and deviators or concrete at the beam ends the stiffness of the interface element is adopted from Sivaleepunth et al. Two-node interface element is also provided in some positions to model the unbonded internal tendon as shown in Fig. 3.

The flat joint model has been applied to reproduce the real geometry of a joint by using the two-line interface element as shown in Fig. 3. The discrete crack model has been selected for the interface elements of segmental joints. In the interface problem, the initial stiffness is generally assumed as the initial dummy stiffness. The values of the initial stiffness have to exhibit a sufficiently high value to reproduce the continuous geometry of segmental concrete beams before the joint opening. However, values of the initial stiffness have to be small enough to avoid numerical problems during the analysis. For these conflicting reasons, the initial stiffness was assumed to be the same value for the normal and tangential modulus to make the analysis simple. The normal stiffness, \( k_n \), and tangential stiffness, \( k_t \), in the elastic stage for the flat joint model in segmental beams with draped external tendons were assumed as

![Fig. 3 Mesh for finite element discretization.](image-url)

![Fig. 4 Compression model for concrete.](image-url)

![Fig. 5 Tension model for concrete.](image-url)
10^5 N/mm^3. The tensile strength of the epoxy applied in segmental joints is actually higher than that of the concrete of beams. Thus, the tensile strength of concrete, \( f_t \), is selected in modeling the segmental joints. The value of fracture energy, \( G_F \), is found by considering the average compressive strength of concrete and the maximum size of coarse aggregate, i.e. \( G_{\text{max}} = 20 \) mm, as recommended in JSCE Standard Specifications^5).

4. RESULTS AND DISCUSSION

(1) Generals about testing beams

D09-600 was first tested. For the safety, the applied load was stopped when the stress in the draped tendons became about 80 percent of the yield strength of tendons. D09-600, therefore, had not reached the actual failure stage. D14-600 was secondly tested and the applied load was provided until the beam was totally failed. D09-200 was lastly tested. When the stress in the draped tendon reached the yield strength, the applied load was stopped for the safety. However, at this stage, D09-200 had not been totally failed.

(2) Crack patterns and joint opening

Figure 6 presents the crack pattern of tested beams. Some flexural cracks first formed in the maximum moment zone between the loading points. Then flexural shear cracks occurred. Despite the difference in the location of deviators and the inclined angle \( \alpha \), one diagonal crack was observed to be formed from the deviator to the loading point in a shear span. After the occurrence of the diagonal crack from a deviator, the dominant diagonal crack occurred from the lower corner of the segmental joint toward the loading point. In D14-600 at the ultimate stage dominant diagonal cracks penetrated into the top flange, and then crushing occurred near the loading point. In D09-200 and D09-600, before stopping the applied load the dominant diagonal

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cracks were observed to penetrate into the top flange. Some cracks were seen on the top flange near the loading points. Based on the crack observation and the load and deflection curves discussed later, it seems that these two beams, D09-200 and D09-600, were close to the ultimate state when the applied load was stopped.

Joint opening was measured at 5 levels in both segmental joints of each beam as shown in Fig. 7. Figure 8 shows the joint opening observed in the segmental joint on the right of D09-200 that was the critical segmental joint. The segmental joint exhibited closing at the levels 1 and 2. At the stopping applied load, the joint opening in the levels 3 and 4 was very small, 0.04 and 0.09 mm, respectively. The joint opening observed in level 5 at the stopping applied load was 11.13 mm.

Figures 9 and 10 show the joint opening observed in the critical segmental joint that was on the left of D09-600 and the right of D14-600. The segmental joint also exhibited closing in levels 1 and 2. In D09-600 and D14-600, the measured data at level 3 was 0.28 and 0.17 mm that were the crack width of the diagonal crack from the deviator. In both D09-600 and D14-600 the joint opening was not observed between the diagonal crack from the deviator and the dominant diagonal crack in a shear span. It means that the diagonal crack from the deviator propagated across the segmental joint in the contact area. In the stopping applied load of D09-600, the joint opening at level 4 was 7.57 mm. The joint opening at level 5 could not be recorded, because the failure in the lower shear key was occurred as shown in Fig. 6. At the ultimate stage of D14-600, the joint opening of levels 4 and 5 was 6.25 and 7.75 mm, respectively. It was noted that the joint opening of SJ10-19 was 10.5 mm at the ultimate stage. The smaller inclined angle \( \alpha \) was the larger joint opening was.

(3) Load-deflection curves

Figure 11 presents the load-deflection curves of the test beams and SJ10-19 in the previous study. Even though there was a difference of location of deviators and the inclined angle \( \alpha \), all the segmental beams exhibited the similar linear elastic behavior in the beginning. Linear behavior was prolonged until the first flexural crack occurred with the load, \( P_{cr} \), as tabulated in Table 4. The load at first flexural crack was affected insignificantly by the location of deviators and the inclined angle \( \alpha \), and was close to that of segmental beam with straight external tendons

Table 4 Test results.

<table>
<thead>
<tr>
<th>Beams</th>
<th>( \alpha ) (deg.)</th>
<th>( P_{cr} ) (kN)</th>
<th>( P_{dcr} ) (kN)</th>
<th>( P_{sh} ) (kN)</th>
<th>( P_{u} ) (kN)</th>
<th>( V_u = P_u / 2 ) (kN)</th>
<th>( V_{p,EXP} ) (kN)</th>
<th>( V_{p,Cal} ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D09-200</td>
<td>9.0</td>
<td>280.0</td>
<td>338.4</td>
<td>360.2</td>
<td>489.2</td>
<td>244.60</td>
<td>17.65</td>
<td>60.87</td>
</tr>
<tr>
<td>D09-600</td>
<td>9.0</td>
<td>287.3</td>
<td>327.3</td>
<td>364.1</td>
<td>431.6</td>
<td>215.80</td>
<td>-11.15</td>
<td>60.58</td>
</tr>
<tr>
<td>D14-600</td>
<td>14.2</td>
<td>285.6</td>
<td>408.1</td>
<td>417.5</td>
<td>523.4</td>
<td>261.70</td>
<td>34.75</td>
<td>102.96</td>
</tr>
<tr>
<td>SJ10-19(^1)</td>
<td>0.0</td>
<td>290.5</td>
<td>-</td>
<td>381.7</td>
<td>453.9</td>
<td>226.95</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Note: \( P_{cr} \) is load at the first flexural crack; \( P_{dcr} \) is load at the diagonal crack formed from deviator; \( P_{sh} \) is load at the dominant diagonal crack; \( P_{u} \) is the peak load; \( V_{u} \) is shear carrying capacity; \( V_{p,EXP} \) is shear carrying supplemented by draped tendons in the experiment to compare with shear carrying of SJ10-19; \( V_{p,Cal} \) is vertical component of prestressing force proposed by current codes\(^{5,6}\).
The loads at the diagonal crack occurrence from a deviator and the dominant crack were not affected by the location of a deviator for D09-200 and D09-600. In D14-600 with higher inclined angle and area of draped external tendons, the load at the diagonal crack occurrence from a deviator was higher than that of D09-200 and D09-600. The load at the dominant crack of D14-600 was also higher than that of D09-600 and D09-200 about 13 percent. The maximum applied load of D09-600, where the stress in the draped tendons reached 80 percent of the yield strength, was smaller than the peak load of SJ10-19. However, the maximum load of D09-200, where the stress in the draped tendons reached the yield strength, was higher than the peak load of SJ10-19 about 8 percent. The peak load of D14-600 was higher than that of SJ10-19 about 15 percent.

(4) Failure mechanisms

The linear elastic behavior of segmental beams was not affected by the location of a deviator, inclined angle $\alpha$ and the area of draped tendons. It was expressed by the load at the first flexural crack to be similar. Compared with the same area of external tendons, the load at the dominant diagonal crack of segmental beams with inclined angle of 9 deg., D09-200 and D09-600, was smaller than that of the segmental beam with straight tendons, SJ10-19. It was because that the segmental beams with draped tendons exhibited reduced stiffness much lower than beams with straight tendons. The load at the dominant diagonal crack of D14-600 with higher inclined angle and tendon area was higher than those of D09-200, D09-600 and SJ10-19. It means that the area of draped tendons also affected the stiffness of segmental beams with draped external tendons.

The diagonal crack from the deviator to the loading point was not formed in segmental beams with straight external tendons. In segmental beams with draped external tendons the diagonal cracks from deviators, however, were formed before the occurrence of the dominant diagonal crack from the segmental joint to the loading point. It was proven that a deviator force was arisen and transferred from the deviator to the loading point. The deviator force transfer from a deviator to the loading point contributed affect the shear transfer mechanism in the segmental beams with draped external tendon. MacGregor et al. have well explained the local behavior of a segment near an opening joint. As a segmental joint opens, a diagonal crack is formed from the segmental joint to the loading point. A compressive strut is formed from the loading point to the lower corner of the segment. Another inclined strut runs across the segmental joint from the loading point to another segment. The formation of the dominant diagonal crack from the segmental joint to the loading point demonstrated that the local behavior of the segmental joint affected significantly the behavior of segmental beams with draped external tendons.

Although there was a difference in the location of a deviator and an inclined angle $\alpha$, the failure mode was designated as the shear compression failure mode in the tested beams. First, the dominant diagonal crack was formed toward the loading point.

\[ \sigma_l = 19.6 \text{ N/mm}^2, f'c = 65.1 \text{ N/mm}^2, d = 400 \text{ mm}, \]
\[ a/d = 3.5, a/d = 1.0, a_{de} = 600 \text{ mm}, \alpha = 1.3 \text{ deg.} \]
\[ \frac{1}{nS_xS_y} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \geq 0.95 \]
\[ x, y : \text{Coordinates of } i^{th} \text{ black dot} \]
\[ \bar{x}, \bar{y} : \text{Mean of } x \text{ and } y \]
\[ n : \text{Number of data considered} \]
\[ S_x, S_y : \text{Standard deviation of } x \text{ and } y \]

Black dots are Gauss’s points; Dash line is evaluated line.

**Fig. 12** Principal compressive stress at peak load.

**Fig. 13** Evaluation of inclination of concentrated stress flow in terms of $m$ value.
Finally, the failure took place with the crushing of concrete near the loading point in the shear span. D09-200 and D09-600 did not reach the peak. The maximum load of D09-200 was larger than that of D09-600, since the applied load was stopped where the stress in draped tendon of D09-200 was larger than that of D09-600. Both the maximum load of D09-200 and the peak load of D14-600 were larger than the peak load of SJ10-19. It means that the shear carrying capacity of segmental concrete beams with draped external tendons was higher than that of segmental beams with straight external tendons. The contribution for the shear carried by draped tendons was smaller than the vertical component of prestressing force as proposed by most current codes\(^5,6\) as shown in Table 4. That is because that the vertical component of prestressing force acts on the shear span only at the deviator. Therefore, the contribution of the vertical component of prestressing force on the shear resistance depends on the resistance of reinforced concrete where it transfers.

The nonlinear FEM analysis results were compared with experimental results in order to validate the FEM model. Figure 11 shows a good agreement of the load-deflection curves between the experimental and FEM analysis results. Figure 12 shows the principal compressive stress of all the beams at the peak load. It can be seen that one concentrated stress flow is arisen from the deviator to the loading point in all beams. The effect of deviator force on the shear transfer mechanism of segmental beams with draped external tendons was confirmed. Figure 12 also shows that the transfer mechanism of prestressing force from the anchorage\(^7\) consists of two concentrated stress flows. One transfers the prestressing force from the anchorage to the support. The other transfers the prestressing force toward the loading point. It means that not only the deviator force but also the transfer mechanism of prestressing force from the anchorage affects the shear transfer mechanism in segmental concrete beams with draped external tendons.

5. EXTENDED MODIFIED MODEL FOR DRAPE TENDONS

(1) Division of the model with the effect of location of a deviator and an inclined angle

To predict the shear capacity of segmental concrete beams with draped external tendons, the modified model\(^1\) for straight external tendons has been extended for segmental concrete beams with draped external tendons. The extended modified model for draped tendons has been proposed based on the effect of deviator force, transfer mechanism of prestressing force from the anchorage and local behavior of segments near the opened segmental joint\(^1,13\). The extended modified model in this paper has just considered Model 1 of the modified model\(^15\) where \(a/d\) is smaller than 1.25.

Figure 13 marks the location of the maximum absolute value of \(\sigma_r\) at each height of Gauss’s point in the location of the concentrated stress flow from the loading point, \(\theta\). The marked Gauss’s points with high values of correlation coefficient (≥0.95) are used to determine the inclination angle of the concentrated stress flow. The correlation coefficient is calculated based on a set of X-Y coordinates with the equation shown in Fig. 13. This concentrated stress flow from the loading point is evaluated in terms of \(m\) value, where \(m = \cot \theta\) and \(\theta\) is an angle of the stress flow inclining from the beam axis.

Figures 14, 15 and 16 show the concentrated stress flows observed from the contour figure at 90% of the shear carrying capacity\(^9\) of segmental beams. The concentrated stress flow marked by each dashed line shows the one probable stress flow. The parameters were an inclined angle \(\alpha\) of draped tendons and location of a deviator. As the inclined angle was of 0 deg., \(m\) value from FEM was approximate to the predicted \(m\) value from the modified model for segmental beams with straight external tendons in the authors’ previous study\(^11\) as shown in Eq. (1).

\[
m = 1.395 \left( \frac{a_l}{d} \right)^{0.3} \left( \frac{a}{d} \right)^{0.75} (\sigma_r)^{-0.25} \tag{1}
\]

For the segmental beam with \(a_{de}\) of 200 mm as shown in Fig. 14, \(a_l > a_{de}\), the compression stress flow from the deviator to the loading point and its width were progressively formed with the increase in the inclined angle \(\alpha\). The \(m\) value of the compression stress flow from the loading point across the segmental joint from FEM, however, was approximate to the predicted \(m\) value from the modified model for straight external tendons\(^1\) as shown in Eq. (1). The segmental beams with \(a_{de}\) of 600 mm and 800 mm as shown in Figs. 15 and 16, \(a_l < a_{de}\), the compression stress flow from the loading point tended to move to its location in segmental beams with straight tendons as the inclined angle \(\alpha\) reduced to zero.

Two compression stress flows from the anchorage of draped tendons were observed with inclined angle \(\alpha\) larger than 2 deg. regardless of deviator location as shown in Figs. 14, 15 and 16. It was recognized that the deviator force and transfer mecha-
nism of prestressing force from the anchorage of draped tendons had the significant effect on the shear capacity of segmental beams as the inclined angle $\alpha$ was larger than about 2 deg. The draped tendon with inclined angle less than 2 deg. was found to have little effect on the shear capacity in monolithic beams with draped internal tendons as explained by MacGregor et al. 2). 

As inclined angle $\alpha$ was larger than 2 deg., the shear failure mechanism can be acceptable to divide into two types of failure pattern. Figure 17 shows the extended modified model for segmental concrete beams with draped external tendons with inclined angle larger than 2 deg. Model I is applied to a segmental beam with $\alpha_j > a_{dc}$ as shown in Fig. 17(a). Model II is applied to a segmental beam with $\alpha_j < a_{dc}$ as shown in Fig. 17(b). In both Models for draped tendons, the diagonal compression members [1] and [2] are modeled by the concentration area in the red part from the support that is distributed widely within the shear span as shown in Figs. 14, 15 and 16. In both Models in Fig. 17, the diagonal compression members [12] and [13] are provided to express the stress flows marked from the anchorage as shown in Figs. 14, 15 and 16.

In Model I with $\alpha_j > a_{dc}$, the member [4] is provided from the deviator to the loading point. The end node of the member [3] is determined by the distance $md$, where $m$ value is from Eq. (1), in the horizontal distance from the loading point. The tension members [11] and [14] from the draped tendon are formed to balance the prestressing force. The model can be...
only existed with the appearance of the tension members [7] and [9] to balance the compression forces from the compression members [1], [2], [3] and [4]. It is reasonable to model the tension member [7] from the internal longitudinal steel bars. The tension member [9] cannot be modeled by internal longitudinal steel bars because it was cut by a segmental joint. The tension force in the member [9] can be mainly resisted by the compression of concrete as shown in Fig. 17 (a).

In Model II with \( a_j < a_{de} \), the member [4] is provided from the bottom of the web at the edge of a segmental joint to the loading point. The member [3] is defined from the deviator to the loading point. The tension member [7] was modeled by the internal longitudinal steel bars as in Model I. The tension member [9], located with a segmental joint, was cut by a segmental joint.

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**Fig. 17** Schematic diagram of the extended modified model for segmental beams with draped tendons and \( \alpha > 2 \) deg. (a half of beam).

**Fig. 18** Schematic diagram of simplified truss model for segmental beams with draped tendons with \( \alpha \leq 2 \) deg. (a half of beam).

**Fig. 19** Horizontal distance from the loading point to the end node of member [4] with \( a_j > a_{de} \).

**Fig. 20** Horizontal distance from the loading point to the end node of member [3] with \( a_j < a_{de} \).
mainly resisted by the external tendon. The member [9], thus, can be modeled by the external tendon.

For an inclined angle less than 2 deg., the original modified model as shown in Fig. 18 can be applied to predict the shear carrying capacity of segmental beams with draped tendons. The effect of draped tendons, however, is also necessary to consider in the original modified model. In case of tendons, however, is also necessary to consider in the beams with draped tendons. The effect of draped tendons to the loading point, i.e. provided from the bottom of the web at the edge of a segmental joint to the loading point multiplied by the web thickness, while the cross sectional area of member [6] is assumed to be the distance from the segmental joint to the loading point multiplied by the web thickness. Transverse reinforcement is taken into account in the cross sectional area of member [5] and [6] by multiplying the ratio of elastic modulus of reinforcement to elastic modulus of concrete, $E_i/E_c$, and the ratio of area of transverse reinforcement to its spacing and its length.

(2) Cross section of members

The name of members in the modified model for segmental beams with draped tendons is identical in Figs. 17 and 18. The cross sectional area of the transverse tension member [5] is assumed to be the distance from the segmental joint to the loading point multiplied by the web thickness, while the cross sectional area of member [6] is assumed to be the distance from the segmental joint to the support multiplied by the web thickness. Transverse reinforcement is taken into account in the cross sectional area of members [5] and [6] by multiplying the ratio of elastic modulus of reinforcement to elastic modulus of concrete, $E_i/E_c$, and the ratio of area of transverse reinforcement to its spacing and its length. The cross sectional area of the flexural compression members is set as $t_f b_f$, where $t_f$ and $b_f$ are the thickness and the width of top flange, respectively. The cross sectional area of external tendons, $A_{ps}$, is used as the cross sectional area of members [11] and [14] in Model I and members [9], [11] and [14] in Model II. The cross sectional area of longitudinal tensile bars, $A_{ts}$, is provided for member [7]. Member [9] of Model I can be defined by the web area from the lowest longitudinal bars to the bottom.

The thickness of compression struts of the extended modified model is also formulated based on nonlinear FEM results as shown in Fig. 21. The compressive stress in the vertical direction at each Gauss’s point, $\sigma_{ji}$, at 90% of shear carrying capacity is considered. The element size has affected on the post-cracking behavior. In addition, in the compression failure mode the effect of element size on the stress and strain is insignificant in the prepeak behavior. Therefore, the thickness of compression struts has

Fig. 21 Evaluation of horizontal thickness of diagonal compression member.

Fig. 22 Effect of inclined angle $\alpha$ on thickness of compression struts where $a_i > a_{de}$.
less effect by the element size when it is determined at 90% of the shear carrying capacity. The stress distribution ratio of $\sigma_i$ to the maximum value $\sigma_{\gamma_{\text{max}}}$ ($\Delta \sigma_i = \sigma_i / \sigma_{\gamma_{\text{max}}}$) in horizontal direction is used to calculate the thickness of a strut. The horizontal width, $t_i$, of the distribution where $\Delta$ was equal to 0.1 is utilized to determine the thickness, because the disturbance is not observed in the width at this level as shown in Fig. 21(b). Three levels of Gauss’s point in the web below the top flange are considered to calculate the effect of a support plate. The average value of horizontal widths at the vicinity of a loading point is utilized to determine the thickness, because the disturbance is not observed in the width at this level.

In the extended modified model for draped tendons, the members [3] and [4] are considered to be affected by the width of a loading plate. Meanwhile members [1], [2], [12] and [13] are affected by the width of a support plate. The cross section area of each strut member is obtained by multiplying $t_{f,i}$, $t_{s,i}$ or $t_{f,si}$, $t_{s,si}$ with the web width of respective member and its inclination angle with respect to the beam axis.

(3) Evaluation of shear carrying capacity

In order to calculate the shear carrying capacity of slender segmental concrete beams with external tendons, equivalent elastic analysis is utilized. The member force, $F_i$, of each member is calculated based on Castigliano’s second theorem, i.e. the theorem of minimum strain energy, as summarized in Eqs. (6) and (7).

$$\frac{\partial U}{\partial X_i} = 0$$

(6)

$$\sum_{i=1}^{14} F_i \frac{\partial F_i}{\partial X_i} \frac{L_i}{E_i A_i} = 0$$

(7)

where $U$ is the strain energy, $X_i$ is the redundant member force, $F_i$ is the force of the $i$th member, $L_i$ and $A_i$ are length and area of the $i$th member, $E_i$ is stiffness of the $i$th member.

Figures 23 and 24 show the calculated results from the extended modified model for segmental concrete beams with draped external tendons with the variation of inclined angle of draped tendons. It confirms that dividing inclined angle $\alpha$ at 2 deg. is acceptable. However, a few test results conducted with draped tendons are out of the range of this study. Therefore, the applicability of the model needs to be confirmed in the future studies.

Table 5 tabulates the experimental results in this study and calculated values from the extended modified model for segmental beams with draped external tendons, JSCE Standard Specifications, AASHTO Specifications and the recommendation for shear design provisions for segmental box girders of AASHTO Specifications where the stress variable, $K$ was not limited. Although the experimental results in this study are restricted, the extended modified model for segmental beams with

$$t_{1-I} = t_{1-I} (1.03 \alpha^{0.11})$$

(4)

$$t_{1-II} = t_{1-II} (1.03 \alpha^{0.11})$$

(5)
draped tendons could be capable of predicting the shear carrying capacity of segmental beams with draped external tendons. Comparison of the mean value, standard deviation and coefficient of variation shows that the shear carrying capacity obtained from the extended modified model for segmental beams with draped tendons exhibits higher accuracy than that from existing equations in the current codes such as JSCE Standard Specifications and AASHTO Specifications for segmental beams. The recommendation for shear design provisions for segmental box girders of AASHTO Specifications also improved the accuracy to compare to AASHTO Specifications. The recommendation, however, has not considered the effect of the location of deviator and anchorage and local behavior near an opened segmental joint on the shear transfer mechanism of segmental concrete beams with draped external tendons.

### 6. CONCLUSIONS

From the study carried out on simply supported segmental concrete beams with draped external tendons, the following conclusions can be drawn:

1. From the experimental results, the shear carrying capacity of segmental concrete beams with draped external tendons was higher than that of segmental beams with straight external tendons. The shear failure of segmental concrete beams with draped external tendons was the shear compression failure mode and affected by the deviator force and the local behavior of the opened segmental joint.

2. The FEM analysis results showed that the shear failure mechanism of segmental concrete beams was affected by not only the deviator force and the local behavior of the segmental joint but also the transfer mechanism of prestressing force from the anchorage. The deviator force and transfer mechanism of prestressing force from an anchorage affected significantly the shear failure mechanism of segmental beams.
beams with draped external tendons as an incline angle of draped external tendons was larger than 2 deg.

3) The extended modified model for segmental beams with draped external tendons has been proposed based on the modified model. The effects of a deviator force, an inclined angle of draped tendons, a transfer mechanism of prestressing force from an anchorage and the local behavior of an opened segmental joint have been considered to set member forces and its thickness in the extended modified model. The shear carrying capacity assessed from the extended modified model for draped tendons has provided the good accuracy in comparison with the existing prediction models for segmental concrete beams with draped external tendons.

REFERENCES
5) Japan Society of Civil Engineers (JSCE) : Standard Specifications for Concrete Structures, Structure Performance Verification, 2002.