EFFECT OF ADDING CONSTANT TORQUE RESISTANCE ON THE DYNAMICS OF THE FLOAT COUNTERWEIGHT WAVE ENERGY DEVICE

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日本近海, 特に初期ステージの海上実験に適した内海では波高が小さく, 装置の機械ロスの評価が重要となることが多い. 著者らは釣瓶式波力発電装置の力学モデルにおいて回転体の運動方程式において, これまでは機械ロスを生むトルク抵抗を駆動プーリの回転速度に比例する項だけで表現していた.

本研究ではこれを見直して駆動プーリの回転速度に比例する項と固定トルクの合計で表現する形に変更した. 新たに追加した固定トルク抵抗は, 発電機の回転のための閾値のトルクである. この追加により得られた式と従来式の計算結果の比較から, 今後の実験検証でトルク抵抗を本研究で示した2パラメータで表現すべきである旨の示唆を得た.

Key Words : Torque resistance, float, counterweight, suspension cable, cable tension

1. INTRODUCTION

Development of the wave energy technology has become imperative in the context of today's global environment which faces the problem of a massive increase in energy demand coupled with the rising global warming^{1),2)}. The authors have been developing an innovative wave energy conversion device that can extract the energy of the waves and convert it into electric power. As shown in **Fig. 1**, the device mainly consists of a float, counterweight, suspension cable, driving pulley, ratchet, gearbox and generator^{3),4)}. Electric power is generated by the conversion of the motion of the float mass caused by wave into a rotational motion of the shaft connected to the electric generator. The ratchet mechanism converts the bi-directional rotation of the driving pulley into a unidirectional rotation of the shaft which is then accelerated by the gearbox.

Fig.1 Schematic diagram of the wave energy converter

Here, the application of a constant torque resistance is proposed. The effect of this resistance

in addition to that proportional to the angular velocity of the rotation of the driving pulley on various physical quantities of the float counterweight wave energy converter has been examined. The main objective of this research is to construct the dynamical model that can more effectively deal with the small-amplitude waves frequently present in the real sea condition. Calculation results showed a general reduction in the values of all the physical quantities reflecting the effect of adding this resistance.

2. DYNAMICS OF THE ENERGY CONVERTER

For a better understanding, the dynamical equations of the wave energy converter without considering the newly introduced constant torque resistance are derived first. Then, these equations are modified to include the newly proposed torque resistance.

(1) Equations for the generator

If θ is the angle of rotation of the driving pulley in anticlockwise direction, the torque that the driving pulley receives from the generator in anticlockwise direction, τ , and the potential difference or the voltage produced by the generator, e , are given by Eq.(1) and Eq.(2) respectively

$$\tau = -Gk_{\tau} i \cdot \text{sgn}(\dot{\theta}) \quad (1)$$

$$e = Gk_e \dot{\theta} \cdot \text{sgn}(\dot{\theta}) \quad (2)$$

where $\dot{\theta}$ is the angular velocity of the driving pulley, i is the current flowing in the coil of the generator, $\text{sgn}(x)=1$ for $x>0$ and -1 for $x<0$, G is the total gear ratio from the driving pulley to the generator, k_{τ} is the torque constant and k_e is the induced voltage constant. Here the minus sign in Eq.(1) indicates that the driving pulley receives an anticlockwise torque from the generator when the float is falling and vice versa. When G and k_e are kept constant, e is directly proportional to the angular velocity of the shaft's rotation.

(2) Force balance at stationary free state

The left part of Fig. 2 shows the position of the float and the water surface at stationary condition without work, and the right part shows their position at an arbitrary time when the system is at work. The force balance for the circular cylindrical float chosen for this work at stationary condition is given by Eq.(3).

Fig. 2 Sketch of the float. The left and the right parts indicate float position at stationary and working conditions respectively

$$M_c g + \frac{1}{4} \pi d_f^2 \rho_w h g = M_f g \quad (3)$$

where M_f is the mass of the float, M_c is the mass of the counterweight, d_f is the diameter of the float, h is the submerged height of the float and ρ_w is the density of water.

(3) Equations for the Heaving Motion of the Float

During its heaving motion, the float can exist in three different states of submergence depending on the wave condition. The corresponding equations of motion are given below.

a) Float partially submerged

$$(0 \leq h + x_w - x_f \leq H_f)$$

$$M_f \frac{d^2 x_f}{dt^2} = f_f + \frac{1}{4} \pi d_f^2 \rho_w (h + x_w - x_f) g - M_f g + \frac{1}{8} C_D \rho_w \left| \frac{dx_w}{dt} - \frac{dx_f}{dt} \right| \left(\frac{dx_w}{dt} - \frac{dx_f}{dt} \right) \pi d_f^2 - \frac{1}{4} C_M \pi d_f^2 \rho_w (h + x_w - x_f) \frac{d^2 x_f}{dt^2} \quad (4)$$

b) Float wholly submerged

$$(h + x_w - x_f > H_f)$$

$$M_f \frac{d^2 x_f}{dt^2} = f_f + \frac{1}{4} \pi d_f^2 \rho_w H_f g - M_f g + \frac{1}{8} C_D \rho_w \left| \frac{dx_w}{dt} - \frac{dx_f}{dt} \right| \left(\frac{dx_w}{dt} - \frac{dx_f}{dt} \right) \pi d_f^2 - \frac{1}{4} C_M \pi d_f^2 \rho_w H_f \frac{d^2 x_f}{dt^2} \quad (5)$$

c) Float hung in the air

$$(h + x_w - x_f < 0)$$

$$M_f \frac{d^2 x_f}{dt^2} = f_f - M_f g \quad (6)$$

In Eqs.(4)~(6), H_f is the height of the float, C_D is the drag coefficient, C_M is the coefficient of added mass, f_f is the tensile force in the cable on the float side, x_f and x_w are the displacements of the float and

water level respectively measured upward from the stationary free state as shown in **Fig. 2**.

(4) Equation for the driving pulley motion

The equation of the rotation of the driving pulley is given by

$$I \frac{d^2 \theta}{dt^2} + C \frac{d\theta}{dt} = \tau + (f_c - f_f) R_m \quad (7)$$

where I is the mass moment of inertia of all the rotating bodies, C is the viscous damping coefficient, R_m is the radius of the driving pulley and f_c is the tensile force of the cable on the counterweight side evaluated from Eq.(8).

$$f_c = M_c \left(g + \frac{d^2 x_c}{dt^2} \right) \quad (8)$$

(5) Combination of the equations

The wire tensile force, f_f , in Eqs.(4)~(6) can be eliminated using Eq.(7) leading to the following equations.

a) Float is partially submerged

$$(0 \leq h + x_w - x_f \leq H_f)$$

$$\begin{aligned} I \frac{d^2 \theta}{dt^2} + C \frac{d\theta}{dt} = & \tau + f_c R_m \\ & + \left(\frac{\rho_w g \pi}{4} d_f^2 (h + x_w - x_f) - M_f g - M_f \frac{d^2 x_f}{dt^2} \right) R_m \\ & + \left(\frac{1}{8} C_D \rho_w \cdot \left| \frac{dx_w}{dt} - \frac{dx_f}{dt} \right| \left(\frac{dx_w}{dt} - \frac{dx_f}{dt} \right) \pi d_f^2 \right. \\ & \left. - \frac{1}{4} C_M d_f^2 \rho_w (h + x_w - x_f) \frac{d^2 x_f}{dt^2} \right) R_m \end{aligned} \quad (9)$$

b) Float is wholly submerged

$$(h + x_w - x_f > H_f)$$

$$\begin{aligned} I \frac{d^2 \theta}{dt^2} + C \frac{d\theta}{dt} = & \tau + f_c R_m \\ & + \left(\frac{\rho_w g \pi}{4} d_f^2 H_f - M_f g - M_f \frac{d^2 x_f}{dt^2} \right) R_m \\ & + \left(\frac{1}{8} C_D \rho_w \cdot \left| \frac{dx_w}{dt} - \frac{dx_f}{dt} \right| \left(\frac{dx_w}{dt} - \frac{dx_f}{dt} \right) \pi d_f^2 \right. \\ & \left. - \frac{1}{4} C_M d_f^2 \rho_w H_f \frac{d^2 x_f}{dt^2} \right) R_m \end{aligned} \quad (10)$$

c) Float is hung in the air

$$(h + x_w - x_f < 0)$$

$$I \frac{d^2 \theta}{dt^2} + C \frac{d\theta}{dt} = \tau + f_c R_m - \left(M_f g + M_f \frac{d^2 x_f}{dt^2} \right) R_m \quad (11)$$

Since the cable is fixed at a point on the driving pulley and wound around it, the displacement of the float and the angle of rotation of the pulley can be written as follows

$$x_f = R_m \theta \quad (12)$$

Using Ohm's law, $e = i.r$, and combining Eq.(1) and Eq.(2) the torque can be written as

$$\tau = - \frac{G^2}{r} k_\tau k_e \frac{d\theta}{dt} \quad (13)$$

where r is the load resistance of the generator.

Using Eqs.(11)~(13) in Eqs.(9)~(11) the following second order differential equations are obtained in terms of the angle of rotation of the driving pulley.

(6) Final equations without the torque resistance

a) Float is partially submerged

$$(0 \leq h + x_w - x_f \leq H_f)$$

$$\begin{aligned} & \left\{ \frac{I}{R_m} + (M_f + M_c) R_m + \frac{\rho_w \pi C_M d_f^2}{4} (h + x_w - x_f) R_m \right\} \frac{d^2 \theta}{dt^2} \\ & + \frac{1}{R_m} \left(C + \frac{G^2 k_\tau k_e}{r} \right) \frac{d\theta}{dt} + \frac{\rho_w g \pi}{4} d_f^2 R_m \theta = \frac{\rho_w g \pi}{4} d_f^2 x_w \\ & + \frac{1}{8} \rho_w \pi C_D d_f^2 \left| \frac{dx_w}{dt} - R_m \frac{d\theta}{dt} \right| \cdot \left(\frac{dx_w}{dt} - R_m \frac{d\theta}{dt} \right) \end{aligned} \quad (14)$$

b) Float is wholly submerged

$$(h + x_w - x_f > H_f)$$

$$\begin{aligned} & \left\{ \frac{I}{R_m} + (M_f + M_c) R_m + \frac{\rho_w \pi C_M d_f^2 H_f}{4} R_m \right\} \frac{d^2 \theta}{dt^2} \\ & + \frac{1}{R_m} \left(C + \frac{G^2 k_\tau k_e}{r} \right) \frac{d\theta}{dt} = \frac{\rho_w g \pi}{4} d_f^2 H_f \\ & + \frac{1}{8} \rho_w \pi C_D d_f^2 \left| \frac{dx_w}{dt} - R_m \frac{d\theta}{dt} \right| \cdot \left(\frac{dx_w}{dt} - R_m \frac{d\theta}{dt} \right) \end{aligned} \quad (15)$$

c) Float is hung in the air

$$(h + x_w - x_f < 0)$$

$$\left\{ \frac{I}{R_m} + (M_f + M_c)R_m \right\} \frac{d^2\theta}{dt^2} + \frac{1}{R_m} \left(C + \frac{G^2 k_r k_e}{r} \right) \frac{d\theta}{dt} = (M_c - M_f)g \quad (16)$$

(7) Introduction of the constant torque resistance

As discussed earlier, a constant torque resistance is added to the system in addition to the velocity dependent damping. The mechanism of how this resistance acts is described in the following.

If $\frac{d\theta}{dt} < 0$, then torque resistance = R_τ

If $\frac{d\theta}{dt} = 0$, then torque resistance = 0

If $\frac{d\theta}{dt} > 0$, then torque resistance = $-R_\tau$

From Eq.(7), the equation of motion of the driving pulley considering the constant torque resistance, can be written as

$$I \frac{d^2\theta}{dt^2} + C \frac{d\theta}{dt} = \tau + (f_c - f_f)R_m - \text{sgn}(\dot{\theta})R_\tau \quad (17)$$

Rearranging Eq.(17), we obtain

$$\tau + (f_c - f_f)R_m - I \frac{d^2\theta}{dt^2} = C \frac{d\theta}{dt} + \text{sgn}(\dot{\theta})R_\tau \quad (18)$$

which has a form similar to the expression for the shear stress for Bingham fluids as given below

$$\tau^* = \eta \frac{du}{dy} + \tau_B \quad (19)$$

in which τ^* , $\frac{du}{dy}$, η and τ_B are respectively the shear stress, shear rate, coefficient of rigidity and the Bingham yield stress. Similar to the fact that Bingham fluid remains stationary under the action of a shear less than τ_B , in the case of the wave energy converter the motion of the driving pulley does not occur unless the torque exerted surpasses R_τ . Since the exerted torque directly corresponds to the external force acting on the float, the device remains stationary until the impinging waves are strong enough to surpass that threshold force.

Using Eq.(17) in Eqs.(4)~(6), Eqs.(9)~(11) can be rewritten as

a) Float is partially submerged

$$(0 \leq h + x_w - x_f \leq H_f)$$

$$\begin{aligned} I \frac{d^2\theta}{dt^2} + C \frac{d\theta}{dt} &= \tau + f_c R_m \\ &+ \left(\frac{\rho g \pi}{4} d_f^2 (h + x_w - x_f) - M_f g - M_f \frac{d^2 x_f}{dt^2} \right) R_m \\ &+ \left(\frac{1}{8} C_D \rho_w \cdot \left| \frac{dx_w}{dt} - \frac{dx_f}{dt} \right| \left(\frac{dx_w}{dt} - \frac{dx_f}{dt} \right) \pi d_f^2 \right. \\ &\left. - \frac{1}{4} C_M d_f^2 \rho_w (h + x_w - x_f) \frac{d^2 x_f}{dt^2} \right) R_m - \text{sgn}(\dot{\theta}) R_\tau \end{aligned} \quad (20)$$

b) Float is wholly submerged

$$(h + x_w - x_f > H_f)$$

$$\begin{aligned} I \frac{d^2\theta}{dt^2} + C \frac{d\theta}{dt} &= \tau \\ &+ f_c R_m + \left(\frac{\rho g \pi}{4} d_f^2 H_f - M_f g - M_f \frac{d^2 x_f}{dt^2} \right) R_m \\ &+ \left(\frac{1}{8} C_D \rho_w \cdot \left| \frac{dx_w}{dt} - \frac{dx_f}{dt} \right| \left(\frac{dx_w}{dt} - \frac{dx_f}{dt} \right) \pi d_f^2 \right. \\ &\left. - \frac{1}{4} C_M d_f^2 \rho_w H_f \frac{d^2 x_f}{dt^2} \right) R_m - \text{sgn}(\dot{\theta}) R_\tau \end{aligned} \quad (21)$$

c) Float is hung in the air

$$(h + x_w - x_f < 0)$$

$$\begin{aligned} I \frac{d^2\theta}{dt^2} + C \frac{d\theta}{dt} &= \tau + f_c R_m \\ &- \left(M_f g + M_f \frac{d^2 x_f}{dt^2} \right) R_m - \text{sgn}(\dot{\theta}) R_\tau \end{aligned} \quad (22)$$

(8) Final dynamical equations considering the constant torque resistance

a) Float is partially submerged

$$(0 \leq h + x_w - x_f \leq H_f)$$

$$\begin{aligned} &\left\{ \frac{I}{R_m} + (M_f + M_c)R_m + \frac{\rho_w \pi C_M d_f^2}{4} (h + x_w - x_f) R_m \right\} \frac{d^2\theta}{dt^2} \\ &+ \frac{1}{R_m} \left(C + \frac{G^2 k_r k_e}{r} \right) \frac{d\theta}{dt} + \frac{\rho g \pi}{4} d_f^2 R_m \theta = \frac{\rho g \pi}{4} d_f^2 x_w \\ &+ \frac{1}{8} \rho_w \pi C_D d_f^2 \left| \frac{dx_w}{dt} - \frac{dx_f}{dt} \right| \left(\frac{dx_w}{dt} - \frac{dx_f}{dt} \right) R_m - \text{sgn}(\dot{\theta}) \frac{R_\tau}{R_m} \end{aligned} \quad (23)$$

b) Float is wholly submerged

$$(h + x_w - x_f > H_f)$$

$$\begin{aligned}
& \left\{ \frac{I}{R_m} + (M_f + M_c)R_m + \frac{\rho_w \pi C_M d_f^2 H_f}{4} R_m \right\} \frac{d^2 \theta}{dt^2} \\
& + \frac{1}{R_m} \left(C + \frac{G^2 k_\tau k_e}{r} \right) \frac{d\theta}{dt} \\
& = \frac{\rho_w g \pi}{4} d_f^2 H_f + \frac{1}{8} \rho_w \pi C_D d_f^2 \left| \frac{dx_w}{dt} - R_m \frac{d\theta}{dt} \right| \\
& \cdot \left(\frac{dx_w}{dt} - R_m \frac{d\theta}{dt} \right) - \text{sgn} \frac{R_\tau}{R_m}
\end{aligned} \quad (24)$$

c) Float is hung in the air
($h + x_w - x_f < 0$)

$$\begin{aligned}
& \left\{ \frac{I}{R_m} + (M_f + M_c)R_m \right\} \frac{d^2 \theta}{dt^2} \\
& + \frac{1}{R_m} \left(C + \frac{G^2 k_\tau k_e}{r} \right) \frac{d\theta}{dt} = (M_c - M_f)g - \text{sgn} \frac{R_\tau}{R_m}
\end{aligned} \quad (25)$$

(9) Equations for the electrical power and the mechanical work rate

The instantaneous electrical power output, P_G , is calculated using the following equation.

$$P_G = r \cdot i^2 = r \left(\frac{-Gk_e}{r} \cdot \frac{d\theta}{dt} \right)^2 \quad (26)$$

The mechanical work rate is given by the following equation.

$$WR = f_f \cdot \frac{dx_f}{dt} \quad (27)$$

(10) Initial conditions

As represented by Eq.(28), x_w is a regular sinusoidal wave. Initial conditions represented by Eq.(29) describe the situation where the float is partially submerged at the crest of the wave form at $t=0$ seconds.

$$x_w = \frac{H}{2} \cos \left(\frac{2\pi}{T} t \right) \quad (28)$$

$$\theta(0) = \frac{H}{2R_m}, \quad \dot{\theta}(0) = 0 \quad (29)$$

Table 1 Dimensions for experiment

Float	Density (kg/m ³)	745.7
	Height (m)	0.7
	Diameter (m)	2
	Submergence ratio	0.5714
	Mass (kg)	1680
Counterweight	Mass (kg)	150
Driving Pulley	Radius (m)	0.18
Gearbox	Gear ratio	41.36

Table 2 Experimental conditions for significant energy conversion

Experiment No.	Wave Height	Time Period	Average Energy conversion rate
	H(m)	T(s)	(Σ wire tension*float speed*dt)/time(W)
1	0.27	4	80
2	0.24	3.5	60

Fig. 3 Effect of the torque resistance on the displacement of the float ($H=0.27\text{m}$, $T=4\text{s}$)

Fig. 4 Effect of the torque resistance on the cable tension ($H=0.27\text{m}$, $T=4\text{s}$)

Fig. 5 Effect of the torque resistance on the mechanical work rate ($H=0.27\text{m}$, $T=4\text{s}$)

3. COMPARISON WITH EXPERIMENTAL DATA

(1) Experimental Setup

Experiments were conducted in a wave tank at the Research and Development Center of Mitsubishi Heavy Industries LTD, Nagasaki, Japan^{3),4),5)}. The wave tank used was 3.2 m deep, 30m wide and had an effective length of 160 m. The experimental apparatus had the dimensions specified in **Table 1**. Only two out of several wave conditions used in the experiment produced a significant work rate. These wave conditions and the corresponding mechanical work rates are indicated in **Table 2**.

(2) Results and discussions

Comparisons of the model results with the experimental data are presented in **Figs. 3-6** for various physical quantities of the wave energy converter. **Fig. 3** indicates that both the models, i.e. with and without the torque resistance, underestimate the float's heave displacement when compared to the experimental value. Also, it can be observed that the addition of the torque resistance reduces the amplitude of the float displacement. The figure also shows that the displacement amplitude decreases with the increase in the torque resistance. **Fig. 4** shows the time series of the cable tension force. Here, both the models overestimate the cable tension when compared to the experimental value. When compared with each other, the model in which the torque resistance has been considered shows a reduction in the cable tension amplitude when compared to the one without the torque resistance and this reduction increases with the increase in the value of the torque resistance, a trend similar to that observed in **Fig. 3**. The time series of the mechanical work rate is shown in **Fig. 5**. It is observed from the figure that this quantity also shows a trend similar to that of the float displacement and the cable tension. In all the aforementioned figures a phase lag between the calculation and experiment data is observed.

4. CONCLUSIONS

A new dynamics model is examined which takes into account the newly proposed constant torque resistance. From the calculation results, it is found that there is a general reduction in the magnitude of all the examined physical quantities due to the added torque resistance.

Until now, the mechanical loss proportional to the velocity was considered. If so, when the change in the water level or the vertical motion of the float is slow the float motion would have followed the water level change more than the motion in the real situation, because of ignoring the constant torque which survives at slow rotational motion of the pulley. Therefore, the mechanical loss should be considered to occur due to both viscosity and friction in mechanical elements. This fact coincides with the characteristics of the dynamics of the mechanical element. Many papers consider only the viscous effect. The newly introduced frictional loss explains the situation realized when the external force is relatively small. This constant torque is appropriate to the threshold torque for rotation of the generator.

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