CLOSED CONDUIT SYSTEM FOR THE GENERATING BOUNDARY LAYER INDUCED BY SOLITARY WAVE

Bambang WINARTA¹ and Hitoshi TANAKA²

¹Student Member of JSCE, M. Eng., Graduate School of Eng., Dept. of Civil Engineering, Tohoku University (6-6-06 Aoba, Sendai 980-8579, Japan)
²Fellow Member of JSCE, Dr. of Eng., Professor, Dept. of Civil Engineering, Tohoku University (6-6-06 Aoba, Sendai 980-8579, Japan)

1. INTRODUCTION

Comprehension on sea bottom boundary layer characteristics is prirnacy in near-shore sediment transport modeling. A tsunami or seismic sea wave has behavior resembling to solitary waves. Beside that, as an oscillatory wave moves into shoaling water, its amplitude becomes progressively higher, the crests become shorter and the trough becomes longer and flatter and it is similar to solitary wave. Because of these reasons, laboratory experiment and numerical experiment studies concerning solitary wave boundary layer will be necessary to hold up in its application for practical purposes such as sediment transport.

Solitary wave boundary layer characteristics on laminar flow have been studied in both laboratory experiments and also in theoretical study. Wave flume with free surface was commonly used on previous studies. One of them is Liu et al., their experimental system is assisted by Particle Image Velocimetry (PIV) to measure velocity in the thin boundary layer. It is concluded that the experiments fall in laminar regime and still cannot investigate in transitional and turbulent regimes. Indeed, wave flume experiment facilities have difficulty to attain high Shield’s number. Another problem is hard to reproduce near-bed characteristics at practical scale. Due to these inconveniences, a proper generation set up to shore up an experiment on sediment transport induced by solitary wave is highly required.

To figure out some difficulties found in the previous sediment transport experiment, closed conduit and oscillating water tunnel are used. The purpose is to reproduce near-bed hydrodynamic and sediment transport phenomena at a realistic scale. Recently, Sumer et al. carried out laboratory experiment using an oscillating water tunnel on investigation turbulent solitary wave boundary layer. However, it will be practically difficult to generate boundary layer flow exactly corresponds to solitary wave motion because of restorative force in a tunnel, which may induce oscillating motion with flow reversal. Beside that U-shape oscillating water tunnel has difficulty to do periodical measurement. As we know that one wave cycle is not sufficient to make an adequate amount of sediment movement and consequently, it will be less of accuracy.

In the present study, a new generation system for bottom boundary layer under solitary wave which enable to do measurement under single and periodical oscillatory motion is proposed. Because of its applicability to assist continuous measurement, so this generation system can perform effectively in sediment transport experiment with movable bed.
2. GENERATION SYSTEM AND EXPERIMENT CONDITIONS

A closed conduit system for the generating solitary wave boundary layer applies in the present experiment shown in Fig. 1.

A new generation system proposed here has applicability to assist single and periodical or continuous measurement. Flow motion generated under continuous measurement is periodical oscillatory motion consists of solitary-wave-like with positive peaks and tranquil period in between two peaks. Therefore, it is necessary to evaluate sufficiency of tranquil period by means single measurement as purely solitary wave. And based on velocity analysis, single and periodical/continuous measurements give a good agreement among them. It has meaning that tranquil period is sufficient. This
is also indicating that continuous measurement can satisfy inherent requirement of solitary wave as single wave generation.

A series of experiments with different value of Reynolds number has been carried out using new generation system. LDV measured instantaneous velocity at 17-20 points in the vertical direction at 10 ms intervals. The mean velocity was obtained by averaging over 50 wave cycles, while the $\alpha$ values were found out by fitting Eq. (1) to the measured free stream velocity. Experimental Reynolds number ($R_e$) written in Table 1 were estimated by following equation\(^3\).

$$R_e = \frac{U_c^2}{\nu \alpha}$$ \hspace{1cm} (5)

where $\nu$: the kinematic viscosity of the fluid.

The various conditions in present experiment are summarized in Table 1.

### Table 1 Experimental conditions

<table>
<thead>
<tr>
<th></th>
<th>$T$ (s)</th>
<th>$\nu$ (cm²/s)</th>
<th>$U_c$ (cm/s)</th>
<th>$\alpha$ (s⁻¹)</th>
<th>$R_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>16.90</td>
<td>0.0116</td>
<td>78.7</td>
<td>0.95</td>
<td>$5.64 \times 10^5$</td>
</tr>
<tr>
<td>Case 2</td>
<td>15.36</td>
<td>0.0116</td>
<td>78.5</td>
<td>0.88</td>
<td>$6.06 \times 10^5$</td>
</tr>
<tr>
<td>Case 3</td>
<td>16.99</td>
<td>0.0114</td>
<td>81.3</td>
<td>0.81</td>
<td>$7.34 \times 10^5$</td>
</tr>
</tbody>
</table>

### 3. RESULTS AND DISCUSSIONS

**1) Shear stress estimation**

Bed shear stress quantity controls the sediment transport process. In oceans the total bed shear stress is summation of bed shear stress caused by tidal currents as unidirectional flows and also oscillatory wave.

Local bottom shear stress can be estimated from linear fitting measured velocity in the near bottom or in the viscous layer ($\delta_v$). It can be expressed as:

$$\frac{u}{u^*} = \frac{u^* z}{\nu}$$ \hspace{1cm} (6)

$$\delta_v = \frac{11.6 \nu}{u^*}$$ \hspace{1cm} (7)

where $u$ is the vertical velocity in the boundary layer, $u^*$ is the shear velocity, $z$ is the vertical coordinate. And bottom shear stress ($\tau_0$) is expressed by following equation,

$$\frac{\tau_0}{\rho} = u^* |u^*|$$ \hspace{1cm} (8)

where $\rho$ is the fluids density.

The other formulation to calculate bottom shear stress ($\tau_0$) is using Manning equation. This expression is frequently used to assess this quantity.

$$\tau_0 = \frac{g n^2 h}{R_h^{1/3}}$$ \hspace{1cm} (9)

where $g$ is gravity, $n$ is Manning coefficient, $R_h$ is hydraulics radius ($R_h = A/P$, where $A$ is wetted area and $P$ is wetted perimeter) and $U$ is depth averaged velocity.

Shear velocity ($u^*$) is calculated by linear fitting to measured velocity in the near wall using Eq. (6) and cross-checking the suitable viscous layer thickness with Eq. (7) simultaneously. And bottom shear stress is obtained by employing Eq. (8), it is square of shear velocity. Figure 2 illustrates the free stream velocity and Fig. 3 shows the viscous layer thickness in some variation of times of Case 2. In the present study, bottom shear stress is also estimated by Manning equation. Coefficient $n$ used in bottom shear stress calculation is 0.01 (for glass), this $n$ value reflects closed conduit material of generation system applied in the present experiment.

---

**Fig. 2** Free stream velocity (Case 2; $R_e = 6.06 \times 10^5$)

- Experiment
- Viscous layer thickness ($\delta_v$)

**Fig. 3** Estimation of shear velocity (Case 2; $R_e = 6.06 \times 10^5$)
Keulegan$^4$ have derived the expressions inside boundary layer for velocity and bottom shear stress in the spatial variation, considering linearized boundary layer equation in the laminar flow regime. And then Tanaka et al.$^5$ converted both expressions in temporal variation and written as follows,

\[
\frac{u}{U_c} = \sec h^2 (\alpha t) - \frac{2}{\sqrt{\pi}} \int_0^\infty \sec h^2 \left( \alpha t - \left( \frac{\beta}{\phi} \right)^2 \right) e^{-\phi^2} d\phi \tag{10}
\]

\[
\frac{\tau_0}{\rho} = \frac{4U_c^2}{\sqrt{\pi} R_c} \int_0^\infty \sec h^2 (\alpha t + \phi^2) \tan h (\alpha t + \phi^2) d\phi \tag{11}
\]

in which,

\[
\beta = \frac{I_2}{2 \sqrt{\nu}} \tag{12}
\]

Figure 4 illustrated the bottom shear in a variation of Reynolds number with different methods of calculation. During accelerating phase bottom shear stress has close agreement with analytical laminar solution, it means during this phase velocity distribution in viscous layer is in laminar distribution. At \( R_c = 5.64 \times 10^5 \), during accelerating phase up to end of decelerating phase, bottom shear stress has a good agreement with analytical laminar solution, it has meaning velocity distribution is almost laminar. However, slight deviation from analytical laminar solution occurs after end of decelerating phase, it is indicating that velocity distribution near wall has already moved to higher flow regime. When \( R_c = 6.06 \times 10^5 \) deviation from analytical laminar solution come earlier than previous case and bottom shear stress during flow reversal is almost zero. Quite different behavior when \( R_c = 7.34 \times 10^5 \), deviation from analytical laminar solution is getting higher and come earlier than 2 previous cases. From Fig. 3 we also can observe the magnitude of bottom shear stress using Manning equation is giving 3 times different with 2 other methods. The Manning equation estimates the bottom shear stress as a function of square of velocity per depth as shown in Eq.(10). In case of unsteady flow condition, this relation is not always correct and as consequence bottom shear stress is also not accurate.

### (3) Boundary layer thickness

Consistency in critical Reynolds number in terms of various quantities such as boundary layer thickness, friction factor and phase difference have been obtained in case of sinusoidal wave$^6$7). Based on stability diagram, critical Reynolds number of solitary wave case occur at \( R_c = 5 \times 10^5 \). In this present study boundary layer thickness and wave friction factor of solitary wave case will be estimated based on the present laboratory experiment and evaluated in connection with critical Reynolds number.

Boundary layer thickness is highest elevation above bottom where velocity of flow differs 1% of maximum velocity. Other threshold values 5% have been proposed$^8$. The reason of using threshold 5% instead of threshold 1% as commonly used is to avoid inevitable fluctuation in the experiments.

Analytical laminar solution for solitary wave boundary layer thickness can be derived from Eq. (10) and the expression in term of Reynolds number written as follows,

\[
\frac{\delta}{a_m} = \frac{2.9}{\sqrt{R_c}} \tag{13}
\]

Fig. 4 Bottom shear stress
In the present study, experimental boundary layer thickness ($\delta$) was estimated at point where $u = 0.95U_c$ as shown in Fig. 5 and parameter $a_m$ is half of total displacement of water particle ($d$),

$$d = \frac{u}{U_c} \sec h^2 \alpha t = \frac{2U_c}{\alpha}$$

and

$$a_m = \frac{U_c}{\alpha}$$

Boundary layer thickness from previous experiment study is plotted in same figure with present experiment as shown in Fig. 6 and both of experiment results show an excellent agreement with analytical laminar solution Eq. (13), although Reynolds number achieve $Re_1 = 1.8 \times 10^6$. It is much higher than critical Reynolds number at $Re_c = 5 \times 10^5$. The $Re_1$ numbers of present experiment are also above of critical Reynolds number but the boundary layer thickness follow well in the line of analytical laminar solution as shown in Fig. 6. It is caused by transition to turbulence phenomenon occurred in decelerating period, while during accelerating phase up to wave crest flows recover to laminar solution. It can be concluded that in-consistency of critical Reynolds number was found for solitary wave in case of boundary layer thickness. This characteristic is different with sinusoidal wave which has consistency in critical Reynolds number.

### Wave friction factor

Wave friction factor is dimensionless parameter used to estimate bed shear stress induced by wave. This parameter is related to the Shields parameter $\beta$ for unidirectional current. The experimental wave friction factor can be computed by following equation from measured bottom shear stress,

$$f_w = \frac{2\tau_{0\,\text{max}}}{\rho U_c^2}$$

where $\tau_{0\,\text{max}}$ is the maximum bottom shear stress, this value can be obtained from Fig. 4, it is maximum value of bottom shear stress obtained from linear fitting of measured velocity ($\tau_{0\,\text{max}}/\rho$)

Analytical laminar solution for boundary layer thickness under solitary wave can be obtained from the Eq. (11) and then expressed in term of Reynolds number as following equation,

$$f_w = \frac{1.71}{\sqrt{Re_c}}$$

The wave friction factor from the previous experiment study and DNS are plotted in similar figure with present laboratory experiment as shown in Fig. 7 and among them show a good agreement with Eq. (17), although Reynolds number attain at $Re_c = 2.7 \times 10^6$, it has to be 5 times of $Re_c = 5 \times 10^5$ as critical Reynolds number. The 3 cases of present experiments with $Re = 5.64 \times 10^5$, $Re = 6.06 \times 10^5$ and $Re = 7.34 \times 10^5$ are fall in analytical laminar solution (Eq. (17)). It is caused by transition to turbulence phenomenon happened in decelerating period, while during accelerating phase flows recover to laminar where the maximum bottom shear stress occurred. As a conclusion, wave friction factor under solitary wave also show in-consistency of critical Reynolds number, it is similar to boundary layer thickness.
4. CONCLUSIONS
A series of experiments with different value of Reynolds number has been carried out using a new generation system. Estimation and evaluation of some parameters have been done: shear stress, boundary layer thickness and wave friction factor. A new learning can be found from this present study is in-consistency of critical Reynolds number in terms of boundary layer thickness and wave friction factor for solitary wave. This observable fact is distinct difference with sinusoidal wave case which has consistency in critical Reynolds number of boundary layer thickness, phase difference and wave friction factor. A new generation system proposed here has purposes to overcome some difficulties in the previous experimental facilities. The weaknesses and the strengths of each experimental device in solitary wave boundary layer research are explained below,

(a) A wave flume with free surface
- It has difficulty to reproduce near-bed hydrodynamics and sediment transport phenomena at realistic scale. And it has problem to achieve high Shield’s number.

(b) U-shaped oscillating water tunnel
- It is practically difficult to generate boundary layer flow exactly corresponds to solitary wave motion because of restorative force in a tunnel, which may induce oscillating motion with flow reversal. The portion of the data with negative velocities at the trailing of the fluid motion was disregarded in their analysis.
- U-shaped oscillating water tunnel has difficulty to generate continuous motion of solitary wave. As a consequence, it will be primary problem for sediment transport experiment. As known, one wave cycle is not sufficient to make an adequate amount of sediment movement and consequently, it will be less of accuracy.

(c) A new generation system
- This experimental facility utilizes closed conduit water tunnel, therefore it can solve inconveniences or difficulties in previous experiment conducted by wave flume with free surface.

- A new generation set up has capability to do measurement under single and periodical or continuous measurements. Because of its applicability to conduct periodical motion measurement with sufficiency of tranquil period between two peaks of solitary motion, it will be appropriate for sediment transport.

As a main conclusion that a new generation system proposed in the present study will be able and applicable to shore up an experiment on sediment transport induced by solitary wave.

ACKNOWLEDGMENT: This research was partially supported by Open Fund for Scientific Research from State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University. The support from Hitachi Scholarship Foundation (HSF) for research activity of BW is also gratefully acknowledged.

REFERENCES