EFFICIENT CONFIGURATIONS FOR COMPUTING NONLINEAR ENERGY TRANSFER IN GRAVITY WAVE SPECTRUM

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Numerous efforts have been made for the improvement of nonlinear energy transfer computation. Until now, the DIA method is generally used for evaluating nonlinear energy transfer $S_{nl}$ in the practical wave model because of its low computational cost. Tamura et al. (2008), suggested SRIAM developed by Komatsu (1996) to be incorporated into the operational wave model. The performances of the wave model were significantly improved by using SRIAM. However, the computational cost is still 20 times larger than the existing operational wave model using DIA. Therefore, we modified SRIAM by reducing the number of the resonant configurations. It was found that 9 numbers of configurations produce the nonlinear energy transfer approximately well as compared to the original SRIAM which has 20 numbers of configurations. This method is then called as the Reduced SRIAM (R-SRIAM). The configurations of R-SRIAM show that the quasi-singular quadruplets contribute the most to the nonlinear energy transfer than the regular quadruplets. Hence, we proposed the efficient number of resonance configurations by selecting only from the quasi-singular quadruplets to be included in the calculation, which is called as the Alternative Multiple DIA (AM-DIA) method. The reduced number of the resonant configurations makes the computational method of the nonlinear energy transfer more economically acceptable.

Key Words: Resonant configurations, nonlinear energy transfer, wave modeling

1. INTRODUCTION

Among the source terms of the third-generation wave models, the nonlinear energy transfer is known as the most important component in the evolution of wave spectra. To predict the evolution of wave spectra with high accuracy, the nonlinear energy transfer source functions must be estimated accurately. Precise evaluation of nonlinear energy transfer requires a large number of the resonant configurations. Nevertheless, such calculations need huge computational costs which are not suitable for operation of wave model. Numerous efforts have been made to improve nonlinear energy transfer computation. Nowadays, the DIA (Hasselmann et. al., 1985) method is the most generally used method for evaluating nonlinear energy transfer $S_{nl}$ in the practical wave model because of its low computational cost. The DIA method substitutes a single configuration of resonant four-wave for an infinite number of configurations. Hence, the DIA method is considered to have limitation in accuracy.

Hashimoto and Kawaguchi (2001) developed the MDIA method by increasing the numbers of configurations in DIA as 2,3,4,5, etc. Each of the configuration uses different parameters $\lambda$ and $C$, where $\lambda$ is the parameter to determine the combination of the component waves. The optimum parameters $C$ is estimated for each $\lambda$ by applying the least square method to the exact value computed by RIAM (Komatsu, et. al., 1993). However, the applicability of the MDIA method has not been...
examined in wave model. In this paper, as a precursor of knowing the efficient number of resonance configuration, this study initially examine the validity and the effectiveness of the MDIA by integrating it in wave model under the duration-limited condition. On the other hand, Tamura et al. (2008), suggested SRIAM developed by Komatsu (1996), which able to reduce computational costs of nonlinear energy transfer by utilizing 20 numbers of configurations to be incorporated into the operational wave model. As the outcome, the wave model performance was significantly improved. However, even though improvement is achieved, the computational cost reduction still 20 times larger than the existing operational wave model using DIA.

It is necessary to develop a method, which can provide sufficient accuracy and efficiency, to be implemented into the wave model. Therefore, in order to decrease computation time, this study modified SRIAM by reducing the number of the resonant configurations. The method is then called as the Reduced SRIAM (R-SRIAM). Moreover, the efficient number of resonant configurations is also proposed by selecting only from the quasi-singular quadruplets to be included in the calculation, called as the Alternative Multiple DIA (AM-DIA) method. The numerical computation results of R-SRIAM and AM-DIA are compared with RIAM which has been proven to have the same degree of accuracy with the exact method developed by Masuda (1980) (Komatsu and Masuda, 1996).

2. INITIAL CONDITIONS

This study is limited to the deep water waves which assumes the directional spectrum $S(f,\theta)$ as a product of the frequency spectrum $S(f)$ and the Mitsuyatsu-type directional function $G(\theta|f)$ as explained as follows:

$$S(f,\theta) = S(f)G(\theta|f)$$

where,

$$S(f) = \sum_{m} \alpha_{m} H_{m/3}^{2} T_{m} (f/f_{p})^{m_{1}} \exp \left\{- \left( m_{1}/n_{1} \right) (f/f_{p})^{-n_{1}} \right\} \tilde{\gamma}_{i} \exp \left\{ - \left( f/f_{\alpha} - f^{2}/2\sigma^{2} \right) \right\}$$

where, $f_{p} = [1 - 0.132(\gamma + 0.2)^{-0.599}]/T_{1/3}$ and

$$\sigma = \begin{cases} 0.07 & (0.7 f_{p} < f < f_{p}) \\ 0.09 & (f_{p} \leq f < f_{p}) \end{cases}$$

and

$$G(\theta|f) = \sum_{i} \beta_{i} \cos \left( \left( \theta - \bar{\theta}_{i} \right)/2 \right)$$

where,

$$S = \begin{cases} S_{\text{max}} (f/f_{p})^{5} & (f < f_{p}) \\ S_{\text{max}} (f/f_{p})^{-2.5} & (f \geq f_{p}) \end{cases}$$

3. NUMERICAL EXAMINATIONS OF MDIA

Based on the previous numerical computation method by Hashimoto and Kawaguchi (2001), firstly, the validity and the effectiveness of the MDIA are examined. The Pierson-Moskowitz spectrum and the JONSWAP spectrum with $S_{\text{max}} = 10$ are used as the initial conditions. The optimum parameters of the MDIA are shown for each configuration in Hashimoto and Kawaguchi (2001). Hashimoto and Kawaguchi showed that the accuracy of the computations is improved as the number of configurations increase for some spectrum conditions. However, the negative values of $C$ are included, although $C$ should be positive in the original Boltzmann Integral which describes the nonlinear energy transfer. This is because the parameters $C$ are estimated by a simple least square method computation.

In order to confirm the behavior of MDIA in wave model, the optimum configurations are incorporated into the WAM model. It was found that MDIA applicable for only a short time in time integration under duration limited condition. The negative value of parameter $C$ seems to make the computation unstable as time increases.
Although the MDIA has the higher accuracy than those of the original DIA for a test wave spectrum, yet it is still impractical to be implemented in wave model.

4. EFFICIENT CONFIGURATIONS FOR COMPUTING NONLINEAR ENERGY TRANSFER

(1) The Reduced SRIAM (R-SRIAM)

In order to reduce the computational costs, in this study, SRIAM was modified by reducing the number of the resonant configurations. This method is called as the Reduced SRIAM (R-SRIAM).

The optimum parameters in the R-SRIAM method are shown in Table 1, which consists of 7 numbers of quasi-singular quadruplets and 2 numbers of regular quadruplets. The far left column is the number of resonant configurations, $\theta_1$ and $\theta_2$ are the angles of wave number vectors $k_1$ and $k_2$, respectively, $\tilde{\omega}_3$ is the normalized angular frequency divided by $(\tilde{\omega}_4 = 1)$, and $K$ is the kernel function for a specific quadruplet.

The R-SRIAM is implemented into the WAM model in order to perform the numerical simulations under duration-limited condition for 12 hours. Figures 1(a) and (b) show the initial frequency spectrum and the frequency spectra after 6 hours and 12 hours of simulations, respectively, using R-

<table>
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<th>$i_{rep}$</th>
<th>$\tilde{\theta}_1$</th>
<th>$\tilde{\theta}_2$</th>
<th>$\tilde{\theta}_3$</th>
<th>$\gamma$</th>
<th>$C_{REP}$</th>
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<tr>
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<td>-0.004</td>
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Table 1. The optimum parameters in the R-SRIAM method: singular quadruplets ($k_{rep}$: 1–7) and regular quadruplets ($i_{rep}$: 8–9).

Fig 1. The initial frequency spectrum and the frequency spectra after 6 and 12 hours of simulations using R-SRIAM, DIA, RIAM, and the original SRIAM (only $S_{so}$ is applied) at $t=6$ hours (a) and $t=12$ hours (b).

Fig 2. The frequency distribution of nonlinear energy transfer corresponding to the frequency spectrum in figure 1 (a) and (b) (at $t=0$ (a), $t=6$ hours (b), and $t=12$ hours (c) in Fig.2).
SRIAM, DIA, RIAM, and the original SRIAM by taking account only the nonlinear energy transfer \( S_{nl} \), without wind input \( S_{wi}=0 \) and the dissipation \( S_{di}=0 \). The energy concentration parameters are assumed as \( \gamma=3.3 \) and \( S_{max}=15 \).

As seen in Figs. 1 (a) and (b), the peak frequency which initially located at 0.12 Hz moves toward the lower frequency after 6 and 12 hours of simulations in all the cases of R-SRIAM, DIA, RIAM, and the original SRIAM. The energy transfer seems to be different in each method. After 6 and 12 hours of simulation, the peak value of the spectra in RIAM tends to overshoot. However, it should be noted that the R-SRIAM shows a smooth and continuous frequency downshift as the original SRIAM, whereas an original WAM with DIA shows relatively distorted frequency downshift in frequency spectra.

Figures 2 (a), (b) and (c) show the frequency distributions of the nonlinear energy transfer at \( t=0 \), 6 and 12 hours, correspond to the frequency spectrum in Fig. 1 (a) and (b). The intensity of the nonlinear energy transfer decreases as the time increases. Although the frequency distribution of nonlinear energy transfer of each method seems to be different, however, the nonlinear energy transfers by R-SRIAM are in agreement with those by the original SRIAM. Compared to DIA, the R-SRIAM produce better accuracy, yet has longer computational time.

### Table 2. The optimum parameters in the AM-DIA method for various numbers of configurations.

<table>
<thead>
<tr>
<th>No. of Conf</th>
<th>( \hat{\theta}_1 )</th>
<th>( \hat{\theta}_2 )</th>
<th>( \hat{\omega}_3 )</th>
<th>( \hat{K} )</th>
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<td>0.49</td>
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(2) **Alternative Multiple DIA (AM-DIA)**

It is preferred to process singular configurations since it improves the accuracy and requires less additional time for computation. Therefore, the effective configurations for computing the nonlinear energy transfer is proposed by using the quasi-singular quadruplets only. This method is called as the Alternative Multiple DIA (AM-DIA). The independent variables taken in this method are the frequency and the directions \( (\theta_1, \omega_2, \theta_2) \) same as those of RIAM and SRIAM.

The optimized quadruplets are identified for 1, 2, 3, 4, and 5 number of configurations to represent the infinite number of the resonant configurations. The coefficients of those parameters were defined similarly with SRIAM, which used RIAM for eight test spectra of the JONSWAP-type spectrum. The optimum parameters of the AM-DIA for various numbers of configurations are shown in Table 2. The far left column is the number of the quasi-singular quadruplets included in the calculation. The optimization of those coefficient parameters is executed by the non-negatives least square method.

Each number of configurations shown in Table 2 are implemented in the WAM model. The numerical simulations are performed under duration-limited condition for 12 hours in the cases of the PM and JONSWAP spectra, without taking account the wind input and the dissipation.

Figure 3 and Figure 4 shows the comparison of the frequency distributions of the nonlinear energy transfer by the AM-DIA, DIA, RIAM, and SRIAM which implemented in WAM model for PM and JONSWAP spectra. The comparisons of the nonlinear energy transfer are shown in the figures, from (a) to (e), the AM-DIAs for 1, 2, 3, 4, and 5 numbers of configurations, respectively. The AM-DIA is shown by solid red line in all the figures. When the number of configurations of the AM-DIA is only one, accuracy of the frequency distributions of the nonlinear energy transfer of the PM spectrum case is insufficient compared to those by the RIAM and SRIAM method, while the magnitude of the nonlinear energy transfer by the DIA are larger than those by RIAM and SRIAM. However, for the case of JONSWAP spectrum, the AM-DIA with one configuration shows much closer value to the nonlinear energy transfer by RIAM and SRIAM, which in contrast with that by DIA.

As shown in Fig. 3 and Fig. 4, the accuracy of the nonlinear energy transfer in PM spectrum by the
AM-DIA generally improves as the number of configurations increases, meanwhile in the case of the JONSWAP spectrum, the AM-DIA with more than 2 numbers of configurations turns out to give an unrealistic pattern of nonlinear energy transfer.

Incidentally, the AM-DIA with the two configurations is incorporated into the WAM model and carried out the integration under the duration limited condition. Stable computation is found during the long time integration, different from the case of MDIA.

(3) The applicability of the Alternative Multiple DIA (AM-DIA) method in complex situations
The superiority of the AM-DIA in the WAM model can be demonstrated in double-peaked spectra. Two different wave groups are simulated with different sets of parameters, $H_{1/3}$, $T_{1/3}$, $\gamma$, $S$, $\theta_i$.
(i=2), which superposed for double-peak spectrum. The examined wave spectra were a double peaked spectrum with \(f_{p1}=0.1\text{Hz}, f_{p2}=0.13\text{Hz}, \) and \(f_{p1}=0.1\text{Hz}, f_{p2}=0.2\text{Hz},\) where the energy concentration parameters are assumed as \(\gamma=3.3\) and \(S_{\text{max}}=15\) in the higher peak frequency spectrum, and \(\gamma=7.0\) and \(S_{\text{max}}=75\) in the lower frequency.

**Figure 5** shows the comparison of the frequency distributions of the nonlinear energy transfer for double peaked spectra (a) \(f_{p1}=0.1\) and \(f_{p2}=0.13\) and (b) \(f_{p1}=0.1\) and \(f_{p2}=0.2,\) by the AM-DIA (2), DIA, RIAM, and SRIAM which are implemented in WAM model with Δθ=0°. As seen in those figures, the three-lobe pattern of the nonlinear energy transfer are shown both in the higher and the lower peak frequency, transferring energy from intermediate frequencies to higher and lower frequencies. The AM-DIA results in agreement to the estimation by RIAM in both cases. In contrast, DIA results show only one three-lobe pattern of the nonlinear energy transfer at the high-frequency peak in the case where the ratio between the two peak frequencies is small (Fig. 5 (a)).** Figure 5 (b) shows the double-peaked spectra where the ratio between the two peak frequencies is large. The nonlinear energy transfer by AM-DIA sufficiently similar to those estimated by SRIAM and RIAM, whereas differently with those by DIA. It is clearly shown that the AM-DIA is better in accuracy than the DIA for double-peaked spectrum.

**5. CONCLUSIONS**

In this study, it can be concluded that reducing the number of configurations of the nonlinear energy transfer is possible to achieve less computational costs without losing the accuracy. In R-SRIAM, 9 configurations are able to show almost the same accuracy as the original SRIAM. As the outcome, R-SRIAM is less time consuming than the original SRIAM, yet it is still longer than DIA.

The AM-DIA is proposed with various efficient configurations which selected only from the quasi-singular quadruplets. The AM-DIA (2) is suggested as the superior method among other configurations of AM-DIA. Although it is not as accurate as those of RIAM, SRIAM and R-SRIAM, however, it is obviously better in accuracy than DIA, even for double peak spectra cases. This result suggests that the reduced number of the resonant configurations is efficient in practical application, even in complex situations. In terms of efficiency, the advantage of the AM-DIA (2) is more economically acceptable compared with RIAM, SRIAM, and R-SRIAM.

**ACKNOWLEDGMENT:** This work was supported by JSPS KAKENHI Grant Number 26289166.

**REFERENCES**


(Received February 4, 2016)