A method to remove self-intersections from dual cycles of a quadrilateral surface mesh for the generation of a hexahedral mesh

Yasumi KAWAMURA\(^1\) and Md. Shahidul ISLAM\(^2\)

1) Department of Systems Design for Ocean-Space, Faculty of Engineering, Yokohama National University
2) Department of Ocean and Space Engineering, Graduate School of Engineering, Yokohama National University

Due to the accuracy and the efficiency of hexahedral elements, the all-hexahedral element auto meshing has a growing demand for the finite element analysis. The authors have been studying whisker-weaving algorithm to generate a hexahedral mesh (HM) automatically\(^3\). In this method the prerequisite for generating a HM is a quadrilateral surface mesh (SM). From the given SM, combinatorial dual cycles (whisker sheet loops) for the whisker-weaving algorithm are generated to produce a HM. Generating a good quality HM does not depend only on the quality of quadrilaterals of the SM but also on the quality of the dual cycles generated from it. If the dual cycles have self-intersections, it could cause the formation of degenerated hexahedrons called knife elements, which are not usable in the finite element analysis. The presence of self-intersections in dual cycles depends on the SM. In this paper a detailed method is proposed to generate a HM free of knife elements, which includes modification of the SM to remove self-intersections from its dual loops. The SM modification procedure of the proposed method has three basic steps. These steps are (a) face collapsing, (b) new face generation and (c) template application.

Key Words: Surface mesh, Hexahedral mesh, Self-intersection, Dual cycle, Whisker-weaving

1. Introduction

To perform the finite element analysis, the problem domain must be discretized into simple cells. This discretization is called meshing. For three-dimensional models, hexahedral meshing is preferred. The advantages of a hexahedral mesh are: 1) it needs fewer elements to fill the domain, so needs less analysis time, 2) hexahedrons fit man-made objects better and 3) better numerical behavior of these elements in some problems e.g. stress analysis.

1.1 Review of previous work

A number of methods are studied to generate a hexahedral mesh (HM). These methods are reviewed here in brief.

Mapping\(^2\) is widely used in commercial software to produce a HM. In this method, the domain is divided into simple shapes (topologically cube) such that each opposing face of these shapes have the same surface mesh. These shapes are then meshed (HM) separately. Achieving such conditions can often be impossible for arbitrary geometry or can involve considerable user interaction to decompose the whole domain into map-meshable regions. Sweeping\(^2\) is another type of mapping method to produce a HM. A quadrilateral surface mesh can be swept through space along a curve if the start and end surfaces have a same mesh.

The medial surface method\(^5\) involves an initial decomposition of the structure domain by a set of medial surfaces. The decomposed volumes generated by medial surfaces are map-meshable regions. A series of templates for the expected topology of the regions formed by the medial surfaces are utilized to fill the volume with hexahedrons. This method, while proving useful for some geometry, has been less than reliable for general geometry.

The grid-based approach, proposed by Schneiders\(^5\) involves generating a perfect three-dimensional grid of hexahedrons on the interior of the volume. Hexahedrons are then added at the boundaries to fill the gaps where the regular grid of hexahedrons does not meet the surface. This method, while robust, tends to generate poor quality elements at the boundary of the volume.

Generating a SM first and then constructing a HM inward from the SM has several benefits. In many finite element analyses, a high quality mesh is needed near the boundary of a solid than deep inside of the volume. So if a surface mesh can be used, it is possible to generate good quality hexahedrons near the boundary.

For a large and complicated solid, it may sometimes be needed to decompose the whole region into sub domains (not necessarily map-meshable) to make the meshing procedure easier and these sub domains must have a compatible or same mesh in the common boundary of the adjacent regions. This also necessitates having a SM to generate a HM. The following two methods generate a HM from a SM.

With plastering\(^6\) method, elements are first placed starting with the boundaries and then advances towards the center of the volume. As the algorithm advances, complex interior voids may result, which in some cases are impossible to fill with hexahedrons. The remaining unplastered regions are then filled with tetrahedrons.

Whisker-weaving algorithm\(^7\) also starts generating a HM from a quadrilateral SM inward. To generate a HM, this algorithm produces a set of loops from the SM as described in section 1.2. These loops (dual of the SM) represent the outer boundary of a set of two-dimensional surfaces called whisker sheets. Then the algorithm seeks to complete the sheet diagrams by a set of rules. Each completed whisker sheet represents a layer of hexahedrons. Although the knife element generation problem exists in the whisker-weaving algorithm (see section 1.3), it can be considered that modified whisker-weaving\(^7\) associated with the post processing works is able to reliably generate all hexahedral mesh for large and complex geometries.

The above discussion justifies the selection of the whisker-weaving algorithm for generating a HM for the study. The present paper is intended to fix the knife element generation problem associated with this algorithm. As the finite element analysis cannot analyze these elements, it is very important to avoid forming knife elements.


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1.2 Procedure of constructing combinatorial duals (loops of whisker sheets) of the given quadrilateral surface mesh

The procedure is explained with a very simple example. Fig. 1 shows a cube with a quadrilateral surface mesh. The combinatorial dual loops of the surface mesh starts from an edge of a surface quadrilateral (face) and then continues moving to the opposite edge until it returns to the starting edge. In Fig. 2 the dotted lines on the surface quads represent the dual loops (L0-L5).

![Diagram](image1)

(a) Top, front and right sides  (b) left, back and bottom sides

Fig. 1 A simple block structure with surface mesh

![Diagram](image2)

(a) Top, front and right sides  (b) left, back and bottom sides

Fig. 2 A simple block with surface mesh and dual loops

1.3 Self-intersection of dual loops and knife element

Fig. 3(a) shows a simple block with a SM. In Fig. 3(b), the solid line other than the mesh represents a dual loop. The loop starts from the edge marked by the star and it crosses itself on its way. This crossing is called a self-intersection (SI). From Fig. 3(a) it is seen that the opposite geometric edges of some surfaces of the block don’t have same number of nodes and this is the reason for the presence of the self-intersection in this block. In general, due to the irregular surface meshes, self-intersections might take place.

![Diagram](image3)

(a) A block with surface mesh  (b) SI of the dual loop

Fig. 3 Self-intersection of the dual loop

Presence of self-intersections on these loops may cause the formation of degenerated hexahedrons called ‘knife elements’ while whisker-weaving. When the same face (having a self-intersection) is connected twice to make a hexahedron, a face (not of surface mesh) of that hexahedron is collapsed and a knife element is formed. In Fig. 4(a), if the node a of the hexahedron is merged to the node b, the face containing both node a and b, a knife element is formed (Fig. 4(b)). As all the faces of a knife element are quadrilaterals, the formation of these elements does not cause the whisker-weaving algorithm to fail.

![Diagram](image4)

(a) Hexahedron  (b) Knife element

Fig. 4 Definition of knife element

Fig. 5 shows a model, which has two self-intersecting loops. The self-intersecting faces are shown as shaded. It is obvious that if a column of hexahedrons can be arranged between these faces, the formation of a knife element can be avoided. Since whisker-weaving cannot perform such an operation, a column has to be arranged manually before meshing. However, meshing the rest of the domain will be like meshing a domain with a hole. Fig. 5 (b) and (c) shows that each column of hexahedrons causing a hole in the domain. We know that whisker-weaving cannot mesh a domain containing a hole or a void. This will necessitate decomposing the domain. As whisker-weaving is intended to mesh large structures (containing no hole) without decomposition, it is intended in this study to remove all self-intersections of the surface mesh.

![Diagram](image5)

Fig. 5 Column of hexahedrons to avoid knife elements

2. Proposal of the whole procedure of HM generation

The present study proposes a strategy to generate a hexahedral mesh (HM) from a quadrilateral surface mesh (SM) such that no knife element forms inside the domain. Fig. 6 shows the flow chart of the whole hexahedral mesh generation procedure. Firstly, the surface mesh is generated using an automatic quadrilateral surface mesh generator such as the paving method. Because the quadrilateral surface...
mesh should be of high quality without any self-intersections and contain even number of quadrilaterals, the surface mesh is modified in the next step. In this step, combinatorial duals are formed from the given SM. These duals are then checked for the presence of any self-intersections. If self-intersections are present then the SM is modified to remove them. In the following sections we will discuss the surface mesh modification procedure in details. After the modification of the SM, the dual loops are then created again which have no self-intersection. After the surface mesh modification procedure, the HM is generated which has no knife element from these loops, using the improved whisker-weaving algorithm (1). As our whisker-weaving method with the nodal placement technique is fully automatic, sometimes invalid elements like doublets (two hexahedrons sharing two faces between them), triplets (two hexahedrons sharing three faces between them) and quadruplets (two hexahedrons sharing four faces between them) are generated (Fig. 7) and also some distorted elements may be produced. For this reason a post-processing program is developed which includes 1) modifying doublets, triplets and quadruplets, 2) applying three-dimensional Laplacian smoothing (11) and 3) applying three-dimensional optimization based smoothing (12). The deviation of each interior angle of a quadrilateral, \( \delta \theta \), is defined as

\[
\delta \theta = \frac{\pi}{2} - \theta_i, \quad i = 1, 2, 3, 4.
\]

The distortion factor for quadrilateral \( F_q \) is defined as

\[
F_q = \left( \frac{1}{4} \sum_{i=1}^{4} (\delta \theta)^2 \right).
\]

It can be seen that the maximum value of \( F_q \) would be attained for a perfect square and the acceptable range of \( 90^\circ \pm 52.5^\circ \) defined by Lo and Lee (14) would correspond to \( F_q \leq 105^\circ \).

### 3. Some introductory concepts

In this section two important topics are discussed in brief.

#### 3.1 Checking the quality of a quadrilateral surface mesh using internal angles of the quadrilaterals

The quality of the surface mesh should be good to generate a good quality HM. To judge if a mesh is of sufficient good quality, it is needed to define a standard.

Zhu et al. (13) deemed a quadrilateral element satisfactory if all its internal angles \( \theta \) fall within \( 90^\circ \pm 60^\circ \) and was considered as unsatisfactory if \( \theta \) exceeds the limit \( 90^\circ \pm 60^\circ \). Lo and Lee (14) found that the first condition appeared to be too strict, so a more flexible range of \( 90^\circ \pm 52.5^\circ \) was used for quadrilateral interior angles. In the present study Lo and Lee’s range is chosen for acceptable quality of a quadrilateral element. Any element exceeding this range is considered unacceptable. The optimum shape for a quadrilateral is a square with interior angles \( 90^\circ \). The following equations were used to measure the distortion factor of quadrilaterals.

The deviation of each interior angle of a quadrilateral, \( \delta \theta_i \), is defined as

\[
\delta \theta_i = \left| \frac{\pi}{2} - \theta_i \right|, \quad i = 1, 2, 3, 4.
\]

The distortion factor for quadrilateral \( F_q \) is defined as

\[
F_q = \left( \frac{1}{4} \sum_{i=1}^{4} (\delta \theta)^2 \right).
\]

It can be seen that \( F_q \) would attain a minimum value of zero for a perfect square and the acceptable range of \( 90^\circ \pm 52.5^\circ \) defined by Lo and Lee (14) would correspond to \( F_q \leq 105^\circ \).

#### 3.2 Edge valence of a node

The edge valence of a node is defined as the number of nodes or edges connected to that particular node. In Fig. 8, the edge valence of a node \( i \) is 5. The concept of the edge valence is used in face collapsing operation (section 4.1).

### 4. Surface mesh modification techniques

The goal of the present study is to modify the surface mesh in such a way that not only self-intersections are removed but also the final surface mesh has requirement quality as well. To overcome the difficulties associated with other researchers methods (8, 13-17) (detailed discussion in section 6.3), we propose three steps to remove self-intersections of the dual loops. These steps are (1) face collapsing, (2) new face generation and (3) template application. The first and last steps make changes on the faces containing self-intersections, whereas the step 2 makes changes in surface mesh to remove self-intersections from any particular faces.

#### 4.1 Face collapsing

The most desirable technique to eliminate self-intersections is face collapsing operation proposed by Folwell and Mitchell (13). Here we introduce a face collapsing technique which performs positive collapsing before negative collapsing and is guided by our proposed technique of quality checking.

By collapsing a face (quadrilateral of a surface mesh) the redirection of the loop occurs which ultimately removes the self-intersection. The detailed description of the process is given below.

Face collapsing is done by merging a pair of nodes of any face to a new node \( n \). If a case like Fig. 9 (a) appears, the shaded face (containing a self-intersection) can be collapsed either by merging nodes 0 and 2 (Fig. 9 (b)), or by merging nodes 1 and 3 (Fig. 9 (c)). When nodes 0 and 2 of that face are merged, it is seen that two loops are formed from the original one. Such type of collapsing which breaks the original loop into two is called positive collapsing. If nodes 1 and 3 of the face are merged instead, the original loop remains intact. This type of collapsing is called negative collapsing. The neighboring quadrilaterals of the collapsed face, which shared the merging nodes, will then be provided with the new node \( n \) (which has the average of the two merging nodes coordinates) in exchange of the merging nodes.
loops are created and in this way two additional self-intersections are removed as these are no longer self-intersections. Therefore in the proposed technique, positive face collapsing is performed before negative collapsing in order to get the advantage of removing self-intersections with minimum number of collapsing.

Face collapsing certainly eliminates self-intersections but in some cases it may cause the formation of unacceptable quadrilateral elements (Fig. 11). For this reasons we developed two reliable checking procedures. If these checking procedures detect the formation of bad quality quadrilaterals then collapsing of that particular quadrilateral must be postponed. The checking procedures developed in this study are quadrilateral quality checking and edge valence checking. Both of the procedures are discussed next.

1) Face collapsing with quadrilateral quality checking

We developed this procedure, because it is very simple to understand and implement and also very effective. After a face is collapsed, all the corresponding quadrilaterals which (had the merging nodes) are checked if any one has quality less than desirable (see section 3.1). If a face of such quality is found then that particular collapsing is postponed and the next quadrilateral having a self-intersection is tested. After each collapsing, Laplacian smoothing on the local surface mesh of the collapse is performed to keep the mesh as smooth as possible to perform the next collapsing.

2) Face collapsing with edge valence checking

This is another way to evaluate whether a particular face collapsing would be allowed or not. In Fig. 12(a), a situation is depicted where the shaded quadrilateral has a self-intersection. For positive collapsing, node 1 and 3 should be merged to a new node. If the positive collapsing is performed then the surface mesh will be like Fig. 12(b).

4.2 New face generation

The users could regard some faces in stress concentration areas (shaded faces of Fig. 13) as important because a regular hexahedral mesh is desired. When the template is applied on a face (section 4.3), the surface mesh becomes unfavorable for generating good quality hexahedrons. Collapsing can also change the arrangement of nodes. For this reason, some faces may not be permitted by the user to be collapsed and templated. In this study, a way of avoiding self-intersections from any such constrained faces (constrained for both templating and collapsing) by generating new faces is proposed.

For any particular self-intersection (SI), the dual loop can be considered as a combination of two sub-loops (Fig. 14).

If the mesh can be modified in such a way that both of these sub-loops are broken, then the self-intersection on the original face might be avoided. In Fig. 15(a), one sub-loop and the face with a self-intersection is shown. Fig. 15(b) shows the modification of the SM, which breaks the loop. The modification is made in the region of the face (on the loop), which has a node with the edge valence 3. Such modification on the other sub-loop has also to be done. If the broken links do not connect each other again then the self-intersection on the face can be avoided.
4.3 Application of template

The dotted lines in Fig. 17 shows the shape of the template, which is applied on a face to remove a self-intersection. The original face has the shape represented by the solid lines. The dotted lines show that the original face is divided into 12 new faces. Hannemann (16) first proposed this particular type of the template to apply on all the faces having self-intersections.

When face collapsing is not possible, only then our proposed method applies the template on all the faces having self-intersections, and other faces (without self-intersections) of the mesh are divided into four. These two operations perform four important tasks. These are a) removal of all the remaining self-intersections, b) resolving connectivity problems due to template application, c) keeping the sizes of the faces as even as possible and d) also keeping the final number of quadrilaterals in the SM even. In Figure 18(a), the face labeled as SI has a self-intersection but cannot be collapsed as it is on the geometric edge. The dotted lines in Fig. 18(b) show the changes to be done to the faces of the mesh. One face is templated and the others are divided into four for the reasons discussed above. The advantages of our technique over Hannemann’s method are discussed in section 6.3.

5. Procedure of application of SM modification steps

In this study, we propose a strategy of application of the surface mesh modification techniques described above. Fig. 19 shows the flow chart of the surface mesh modification procedure.

The faces of the SM on geometric edges are never allowed to collapse. Depending on the geometry of the model and the region of interest for the finite element analysis, some faces can be constrained by the user for both face collapsing and templating. In this paper the constrained faces means faces constrained for both collapsing and template application.

The face collapsing technique is applied on the unconstrained faces. The positive collapsing is applied first. In this way, with a few numbers of collapsing, lots of self-intersections (SI) (including self-intersections on constrained faces) are possible to be removed.

If still any self-intersections are present on constrained faces, the new face generation step is applied. Face collapsing is applied next without collapsing the newly generated faces as well as the constrained faces. This loop of new face generation and face collapsing is continued until all the self-intersections in the constrained faces are removed.

After removing self-intersections by using the face collapsing and new face generation step, templates are applied on all faces having self-intersections if any self-intersection still exists. And all the remaining faces (not having self-intersections) are divided into four. If there is no self-intersection then only subdivision of faces is performed. If template application is not needed and if the mesh has even number of faces then subdivision of faces could be avoided if needed.

6. Results and discussion

A number of models of the different shape and the surface mesh are tested with the proposed method of SM modification.

6.1 Results of tested models

The models with the original and the modified surface mesh are presented here for comparison. The reasons why self-intersections exist in these models and the steps needed to
remove them are also discussed in detail.

(1) Some simple examples

These models are tested without constraining any faces.

Model 1: At the center of this model’s three surfaces, circular mesh is made. For this asymmetrical surface mesh, 4 self-intersections occur. Fig. 20 (a) shows meshes on top, front and left surfaces of the model, and Fig. 20 (b) shows the meshes on the other three surfaces. The original mesh has 240 faces.

![Mesh on model-1](image1)

(a) Top, front and left sides  
(b) Right, back and bottom sides  

Fig. 20  Mesh on model-1

Face collapsing with the face quality checking procedure collapses 2 faces each by positive and negative collapsing. At the end of this step, the total number of faces left is 236. Face collapsing with the edge valence checking produces the same result. As no self-intersection is left to remove with templating and the number of faces is also even, subdivision of the mesh is optional. Fig. 21 shows the model with modified surface mesh when subdivision is not applied, while Fig. 22 shows the model when subdivision is applied. After the subdivision, the number of faces in the SM is 944.

![Modified mesh on model-1](image2)

(a) Top, front and left sides  
(b) Right, back and bottom sides  

Fig. 21  Modified mesh on model-1 (no subdivision)

![Modified mesh on model-1 (after subdivision)](image3)

(a) Top, front and left sides  
(b) Right, back and bottom sides  

Fig. 22  Modified mesh on model-1 (after subdivision)

Model 2: The second model is a block having the surface mesh consisting of 156 quadrilateral elements. This mesh produces 30 self-intersections in its dual cycles. The reason for self-intersections is the different number of nodes applied on the opposite edges of the surfaces of the block to generate the surface mesh. Fig. 23 shows the original SM.

![Mesh on model-2](image4)

(a) Top, front and left sides  
(b) Right, back and bottom sides  

Fig. 23  Mesh on model-2

Using face collapsing with the face quality checking method, positive collapsing collapses 6 faces, which removes all 30 self-intersections. Fig. 24 shows the table where the effect of each face collapsing on the number of total self-intersections of the SM is presented. The data shows each positive collapsing causing the removal of a number of other self-intersections automatically.

At the end of the process, 150 faces remain. The modified mesh is shown in Fig. 25 where subdivision is applied. This mesh has a total of 600 faces. Face collapsing with the edge valence checking procedure produces the same result.

<table>
<thead>
<tr>
<th>Type of collapsing</th>
<th>Number of faces in SM</th>
<th>Number of SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>156 (Original)</td>
<td>30</td>
</tr>
<tr>
<td>Positive</td>
<td>155</td>
<td>23</td>
</tr>
<tr>
<td>Positive</td>
<td>154</td>
<td>14</td>
</tr>
<tr>
<td>Positive</td>
<td>153</td>
<td>11</td>
</tr>
<tr>
<td>Positive</td>
<td>152</td>
<td>6</td>
</tr>
<tr>
<td>Positive</td>
<td>151</td>
<td>3</td>
</tr>
<tr>
<td>Positive</td>
<td>150</td>
<td>0</td>
</tr>
</tbody>
</table>

![Effect of positive face collapsing on total SI](image5)

Fig. 24  Effect of positive face collapsing on total SI

![Modified mesh on model-2](image6)

(a) Top, front and left sides  
(b) Right, back and bottom sides  

Fig. 25  Modified mesh on model-2

Model 3: This model has 574 faces and 4 self-intersections. Fig. 26 shows the model with the original SM.

![Original mesh on model-3](image7)

Fig. 26 Original mesh on model-3

Face collapsing with the face quality checking, causes one positive collapsing, which removes three self-intersections. The remaining self-intersections are removed by applying templates. The final mesh is shown in Fig. 27 and it has 2300 faces. Face collapsing with the edge valence checking produces the same result.

![Modified mesh on model-3](image8)

Fig. 27 Modified mesh on model-3

(2) Models with constrained faces

Model 4: This model is a part of a crankshaft. The original surface mesh has 1008 quadrilateral elements and 84 of them have self-intersections. The reason of forming self-intersections is the presence of cylindrical extrusions in the lower parts of the model where the SM is made asymmetric. Constraints are applied on the faces near the neck of the model to avoid face collapsing and templating there. The original mesh is shown in Fig. 28(a). By collapsing 13 unconstrained faces (with the face quality checking), it is possible to remove 71 self-intersections. The remaining 13 self-intersections are removed by templating. It is noted that none of the constrained faces are templated by the proposed
method. The modified mesh has 4084 faces (Fig. 28(b)). Face collapsing with the edge valence checking produces the similar result.

Model 5: This model has 387 faces in its SM and its dual loops have 11 self-intersections. 3 of these self-intersections are on faces on geometric edge. The faces near that edge are constrained to avoid templating there. Fig. 29 shows the original SM with constrained faces (shaded ones).

By collapsing only 5 unconstrained faces (with the face quality checking), it is possible to remove all self-intersections. This example (also model-4) shows the benefit of constraining faces to avoid templating in the mesh area of interest (the self-intersections are automatically removed by positive collapsing). The modified final mesh pattern is shown in Fig. 30. This mesh has 1528 faces. Face collapsing with the edge valence checking produces the same result.

Model 6: The next model has 2584 quadrilateral elements and 88 of them have self-intersections. Some of these self-intersections take place near the inner edge of the model. Fig. 31 shows the model with the original SM. Some faces in the region shown in Fig. 31 are constrained (shaded faces) to avoid distortion of the final SM.

It is possible to remove 87 self-intersections by collapsing only 18 unconstrained faces in the first step of the face collapsing (with the edge valence checking). The remaining self-intersection exists in a constrained face. If the new face generation step is not applied, the final mesh pattern in the region of interest is shown in Fig. 32(a).

If the new face generation step is applied after the first face collapsing, then it is possible to avoid the self-intersection in the constrained face and for that, templating is needed on 6 unconstrained faces. Fig.32(b) shows the region of interest with the modified SM when the new face generation step is applied.

This example shows that our proposed new face generation method can effectively remove the self-intersection from a constrained face. Face collapsing with the face quality checking produces the similar result.

6.2 Discussion on results

Face collapsing operation can use the edge valence checking or the face quality checking. Sometimes these two methods generate different results. The particular face collapsing, which is not allowed by one method but allowed by the other, is the reason for this difference. The advantage of getting two results instead of one is that the better one can be chosen. By changing the value of the acceptable maximum and minimum angles in the face quality checking, it is possible to get the same results as the edge valence checking. So our proposed technique of the face quality checking is maneuverable and can be considered as more effective method.

If a self-intersection is not possible to be removed by face collapsing, application of the template is the only available solution. Generating fewer templates may be desirable for better quality mesh. The new face generation step cannot always provide a solution, as appropriate condition (described in section 4.2) must be reached to apply this step. Constraining a large number of faces compared to the total mesh size should be avoided to get the best result.

6.3 Comparative study

The surface mesh modification technique proposed in this study is effective and reliable. Here we describe the advantage of our method over the conventional methods of surface mesh modification.

Hannemann\(^\text{(16)}\) proposed a method to remove self-intersections, which is similar to the template application and the subdivision proposed in this paper (section 4.3). It is seen that to remove self-intersections by only this technique, the template has to be applied on all the faces having self-intersections, which degrades the quality of the surface mesh considerably and constrained condition cannot be applied also. Fig.33 shows the result of the model 2 (Fig.23) by applying only the Hannemann’s method (templates are applied on all 30 self-intersections). Our proposed method produces far better result (the average distortion factor computed by equation (2) is 22.1) and is shown in Fig. 25.

![Fig. 33 The result by Hannemann’s method](image-url)
Folwell and Mitchell\(^{(15)}\) proposed a method of removing self-intersections, but the present method provides a better and more detailed study. The present study proposes the checking procedures (the face quality and the edge valence) to allow face collapsing only when it results acceptable mesh quality. Moreover, our technique proposes a new idea about constraining faces, which avoids collapsing and templating in user-defined areas. Another advantage of the present method is that it can provide an output surface mesh which has even sized elements, and also can guarantee even number of quadrilateral elements which is a basic requirement for hexahedral mesh generation (model 5).

Egorova\(^{(17)}\) et al. proposed a method to produce surface mesh without self-intersections. From given geometry of the model, this method decomposes the surfaces of the model into a number of triangular or quadrilateral polygons. Then the polygons are surface meshed by pre-defined templates of quadrilaterals. After that by a heuristic method, the number of nodes on each edge is modified to produce dual cycles without self-intersections. At present, very reliable and good quality commercial quadrilateral surface mesh generators are available and in use for practical/industrial application. Our mesh modification technique can effectively work with these mesh generators and needs a very little change in the original mesh produced by these tools to remove self-intersections. Thus a good quality surface mesh might be found in many cases by our proposed techniques.

Wenjie\(^{(8)}\) et al. presented a study, which states that if any dual sheets contains even number of self-intersections, it is possible to generate hexahedral mesh by making particular connectivity among the self-intersected quadrilaterals. The disadvantage of this procedure is discussed in section 1.3 with Fig. 5. Wenjie et al. also mentioned that removing self-intersections by face collapsing causes distortion of geometry. In the present study, as face collapsing is totally avoided on the quadrilaterals on geometric edges (detailed discussion on section 4), it is possible to successfully avoid such kind of distortion in most cases.

![Fig. 34  Templates of Wenjie et al.](image)

Fig. 34 (a) and (b) shows two templates proposed by Wenjie et al. Removing self-intersections only by using templates needs all the faces having self-intersections to be templated which degrades the quality of the surface mesh. The situation worsens when some of these faces (with self-intersections) are located nearby. Fig. 35 (a) shows such an example model, which has self-intersections on four neighbor quads (shaded quads of Fig. 35 (a)). The first template proposed by Wenjie et al. is unusable. If the second template is applied, the mesh becomes like Fig. 35 (b), which has the average distortion factor 37.62. It is also seen that elements with high aspect ratio are generated (shaded) as this template causes the other elements (not having self-intersections) on the same dual loop to be divided into two in only one direction. Our method is applied on the same model of Fig. 35(a) and the mesh result is shown in Fig. 35 (c). The average distortion factor of this surface mesh is only 19.1, which is far better from the result found by using the template proposed by Wenjie et al.

Wenjie et al. also proposed structure insertion to remove a self-intersection. Fig.36(a) shows, if a face has self-intersection and if no edge of the face is on geometric edge, then node A and B can be opened to edges (A to A\(_1\) and A\(_2\) to B, B to B\(_1\) and B\(_2\) as shown in Fig.36(b)(c)). The effect of this change on the neighbor faces is examined and seen that this method also generates elements with high aspect ratio and also a lot of bad quality elements are generated when applied to models like Fig. 35 (a) (when some self-intersecting faces are located nearby).

![Fig. 36 Structure insertion method proposed by Wenjie et al.](image)

6.4 Advantages of the proposed methods

From the above discussion, the advantages of our proposed method can be concluded as follows.

1. Our method of modifying surface mesh for removing self-intersections is a combined method of the face collapsing and the template application. As a result it is possible to get advantages of both of these techniques so that the resulting surface quadrilateral mesh has better quality compared with the conventional methods.

2. We propose the idea of constraining faces to avoid collapsing and templating in a particular face. This idea is especially important for the stress concentration zone and also for a common surface shared by two decomposed domains where the same surface mesh is the requirement for merging the domains. The positive collapsing and the new face generation techniques are proposed in this paper, to remove self-intersections from these constrained faces.

3. After removing all self-intersections, our method subdivides all faces of the surface mesh as discussed in section 4.3. This makes the quality of final surface mesh even sized (better aspect ratio), which is not possible by using the conventional methods. This action also reduces the distortion factor.

4. A fully automatic program is developed on the basis of our proposed method. The program is applied to a number of complicated and practical problems. The results achieved are quite satisfactory and shows the validity of our proposed method.

7. Conclusion

1. In this paper, a total HM generation procedure is proposed
firstly. The procedure is composed of four steps, (1) the surface quadrilateral mesh generation, (2) the surface mesh modification, (3) the whisker-weaving operation, and (4) the post-processing of the mesh generation. By using this strategy, it becomes possible to avoid the formation of knife elements during the whisker-weaving operation.

2. Secondly, a surface mesh modification method with the concept of constrained faces is developed in this paper. This method has three steps. The first step collapses the surface quadrilaterals to remove self-intersections. This collapsing is carried out such that no unacceptable elements are formed. For this purpose, two checking procedures are proposed. The second step breaks the dual loops to avoid self-intersections on constrained faces by generating new faces. In the third step, templates are applied on all the faces having self-intersections (if present), while all other faces are divided into four. This step not only removes the self-intersections but also keeps the number of faces even, which is an essential condition for hexahedral mesh generation.

3. A detailed comparative study of the surface modification methods is made with other conventional methods, which proves that our proposed technique is effective in many cases. The output surface mesh is ready to be used in HM generation by the improved whisker-weaving algorithm with the nodal placement technique1).

References