Practical Monte Carlo Simulation Method Highlighting on Tail Probability with Application to Biomechanics Analysis of Pressure Ulcer*

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The Monte Carlo method is a well-known method for calculating uncertainty, but it has the disadvantage of high computational cost because it usually requires 10,000 sampling points. In this paper, a practical sampling algorithm named Stepwise Limited Sampling (SLS) is proposed to obtain both accurate expected values and very accurate tail probability values in the Monte Carlo simulation. This method was then applied to a biomechanics problem concerning the risk prediction of pressure ulcers. It is known that the initial damage that leads to a fatal stage of pressure ulcer occurs in deep muscle, but its location has not been clarified. By modeling assumed damage at the bone-muscle interface in the human buttock as a cutout, and by judging whether the damage propagates or not, sets of material properties of muscle and fat in the tail probability that result in high interface shear strain were obtained as a function of body positioning during nursing.

Key Words: FEM, Monte Carlo Simulation, Pressure Ulcer, Interface Strain, Tail Probability

1. Introduction

For the quality assurance of engineering simulations, the uncertainty modeling and practical computational methods for uncertainty quantification have been a matter for concern ever since the guidelines for verification and validation (V&V) were published by the American Society of Mechanical Engineers(1)~(3). Their disadvantages lie in the computational cost, especially in industry, because 10,000 analyses are usually required to obtain reliable probability density for the quantity of interest. To shorten the computational time in 3D and/or non-linear problems, the efficient sampling scheme such as Latin Hypercube has been proposed.

Another approach is the use of the stochastic finite element method (FEM)(7),(8). This method provides the expected value and standard deviation of the quantity of interest with much smaller computational cost. Therefore, one of the authors has applied the stochastic multi-scale method to the analysis of human bone, which has a very complicated micro-architecture(9).

With FEM, when a perturbation method is used with respect to a random variable, arbitrary uncertainty with relatively large scattering cannot be solved. In such a case, the Monte Carlo method has the advantage over the stochastic FEM.

Depending on the problem, the purpose of the stochastic/probabilistic FEM with uncertainty modeling
is not always to obtain both expected value and standard deviation, in other words, the probability density function. If the final goal is to predict the possibility of fracture, it is often important to predict if there are a special set of parameters set that may lead to fracture even when the probability of such a set is low. In such a case, what is important is to predict the expected value with moderate accuracy and the tail probability with great accuracy.

The accuracy of a Monte Carlo simulation depends on the algorithm for generating random numbers along with the number of computational cases. Even if the Mersenne Twister method is used for the generation of 10,000 random numbers, the reliability of the tail probability value is not good.\(^\text{(10)}\)

Hence, in this study, a practical sampling algorithm named stepwise limited sampling (SLS) is proposed to obtain both accurate enough expected values and the tail probabilities as accurate as those derived from Monte Carlo simulations. There has been no similar approach to this two-step sampling method in finding the tail probability. This method was then applied to a biomechanics problem concerning the risk prediction of the pressure ulcers. The SLS algorithm is described in Chapter 2 and application to the pressure ulcer problem is described in Chapters 3-5.

Pressure ulcers occurs through sustained pressure and cutoff of blood supply. It has been found that internal damage in deep muscle layers covering bony prominences can result in fatal pressure ulcers.\(^\text{(11)(12)}\). However, the initial location of that damage has not yet been found. Since it is important to know the local internal strain/stress regions under external pressure, some numerical studies using CT/MRI image-based FEM have been reported. However, Bouten et al.\(^\text{(12)}\) claimed that FEM is not a familiar method to clinical and nursing staff. Determining the correct nursing care is important in order to reduce the chance of pressure ulcers occurrences. Therefore, this study aims at developing a practical simulation methodology. Each person has different material properties that were influenced by age, gender, nutrition intake, and wet or dry skin. These differences will affect on how the nursing method should be performed and what positioning will be best for each individual patient. The final goal of the developed biomechanical simulation is to obtain the set of dangerous material parameters for muscle and fat that can lead to high strain at the bone-muscle interface depending on the load conditions for different body positions.

In other studies, 3D FEM analyses have been reported based on MRI images. Makhsous et al.\(^\text{(15)}\) used the Mooney-Rivlin model, and a uniform contact pressure of 20.34 kPa was applied to the 3D model. The differences between FEM and measurement from MRI images were compared. The measured displacement at a certain point was 16.8±16.5 mm, while the numerical prediction was 10.7±8.0 mm. In another region, the measured value was 36.6±9.0 mm, while the predicted one was 18.1±5.8 mm. The accuracy was not very good probably because of the Neumann condition. On the other hand, the prediction by Linder-Ganz et al.\(^\text{(16)}\) was very accurate. The Neo-Hookean model, Prony series expansion type viscoelastic model and Dirichlet condition using the measured deformation by MRI were adopted. The measured pressure was 17±4 kPa, and the predicted value was 18±3 kPa. Yamamoto et al.\(^\text{(17)}\) used a 2D model and Ogden model, but a multi-scale analysis was carried out to predict not only the strain distribution but also capillary deformation and cutaneous blood flow. The correlation between blood flow and contact pressure was compared qualitatively with experimental measurement. It should be noted that Yamamoto et al.’s model was 2D, but novel advanced simulation was conducted. Our study also employed 2D modeling as a first step in a new simulation considering uncertainty factors.

One interesting result by Linder-Ganz et al.\(^\text{(16)}\) was that large inter-individual differences were seen among 6 subjects. The maximum von Mises stress ranged from 20 to 53 kPa in gluteus muscle and 14 to 25 kPa in enveloping fat. Makhsous et al.\(^\text{(15)}\) also noted that the stresses reported by many others showed great variation, which may be due to differences of the configurations, material parameters, loading and boundary conditions. We postulate that the consideration of uncertainty factors in the simulation is a critical issue, but we can find no literature on this point.

Another interesting result of the work by Linder–Ganz et al.\(^\text{(16)}\) were the values for maximum shear stress was reported. They ranged from 12 to 30 kPa in gluteus muscle among 6 subjects and 7 to 13 kPa in fat, where inter-individual differences were again seen. We have previously reported that interfacial shear stress may lead to the breakage of fibril tissue at the bone-muscle interface.\(^\text{(13)}\)

Chapter 3 describes the finite element model and Chapter 4 shows the computational procedure using the SLS method in the analysis of pressure ulcer. Chapter 5 shows numerical results and discussion.

2. Stepwise Limited Sampling (SLS) algorithm

By giving the random distribution with relatively large scattering, the Monte Carlo method provides us with
the probability density of the quantity of interest, its expected value and standard deviation. It is known that analyses of 10,000 cases are usually required to reach the convergence of both expected value and standard deviation. The accuracy is dependent on the generation scheme of random numbers. The Mersenne Twister, known to give high-quality random numbers, is used in this study.

The convergence of the expected value is, in general, more easily obtained than the standard deviation. One of the reasons is that the quality of random numbers generated in the tail probability is not good enough using 10,000 random numbers, even if the Mersenne Twister method is adopted. In other words, if one wants to put highlight on the reliability of the tail probability, 10,000 cases are not enough. There would be such a case, for instance, when the prediction of fracture/failure is required, even if probability is very low. Considering that the disadvantage of Monte Carlo simulation is the high computational cost, a new and cost-effective sampling scheme that emphasizes accuracy in the tail probability would be valuable in many industries.

Therefore, an SLS algorithm is proposed in this paper, which stops the iteration when the expected value is converges and spends the rest of the computational time on analysis of the tail probability. SLS consists of the following two main steps:

(1) Step 1: Convergence check and post-processing

SLS only assures the moderate accuracy for the expected value of the quantity of interest. The expected value itself is distributed, and therefore, the convergence is estimated based on the central limited theorem. Let \( E_i \) be the expected value and \( \sigma_i \) be the standard deviation after \( i \) sets of analyses in standard Monte Carlo simulation. The convergence is judged by Eq. (1).

\[
E_i - E_{(i-1)} \leq \frac{\sigma_i}{E_i \sqrt{n_{\text{max}}}} \tag{1}
\]

where the right-hand-side of the equation is normalized by the expected value after the first set of analyses. We recommend using \( n_{\text{max}} = 10,000 \). Considering the analysis of 100 cases, Eq. (1) then yields:

\[
E_{(100j)} - E_{(100(j-1))} \leq \frac{\sigma_{(100)}}{E_{(100)} \sqrt{n_{\text{max}}}} \tag{2}
\]

If Eq. (2) holds three times consecutively, then the expected value is judged to have converged. This is because the expected value may oscillate in the Monte Carlo simulation. When the convergence is obtained, the Monte Carlo simulation is suspended.

After this first step, the correlation between the input parameters sets and the quantity of interest must be investigated. Assuming that the number of parameters is large; then sets considering just two parameters among all the parameters are chosen. Then the quantity of interest can be plotted in the 2D space of the parameters. To this end, it is easy to determine the limited sampling zone for the specific value of the quantity of interest by the following equation:

\[
p_i x_i + p_j x_j + q \geq 0 \quad (i \neq j) \tag{3}
\]

Here, \( x_i \) and \( x_j \) are the chosen two parameters, and \( p_i, p_j, \) and \( q \) are scalar factors. By the combination of multiple linear equations using Eq. (3) for the multi-dimensional space of all parameters, the limited sampling zone for the quantity of interest can be defined. This procedure is easy to be automated, because simple linear equations are used.

In this study, as a demonstration, we determined the threshold of \( \mu + 3\sigma \) for the quantity of interest, where \( \mu \) and \( \sigma \) are the average and standard deviation, respectively. This threshold was decided after looking into the results generated from the analysis. Different limitation can be used depending on the problem solved.

(2) Step 2: Prediction of tail probability

After determination of this limited sampling zone the Monte Carlo simulation resumes by using only the random numbers generated in the limited sampling zone determined from step 1. This step is called the rejection step. The rejection ratio is equivalent to the efficiency ratio of SLS. The number of analyses in this step is user-determined. In the later analysis for pressure ulcers, 1,000 cases were analyzed in the limited sampling zone.

Note: if the reject ratio is high, i.e., the acceptance ratio is low, the rejection method can be replaced by the Metropolis-Hasting algorithm.

3. Computational model of the human buttock for risk prediction of pressure ulcers

3.1 Assumption of bio-mechanism of pressure ulcer occurrence

As was discussed in the introduction, the initial damage that finally results in pressure ulcers occurs in deep muscle layers, but the initial location has not yet been found. For the purposes of this study, the authors have assumed that tiny damage of fibril tissues at the interface between bone and muscle in the human buttock is the trigger for muscle damage. The exact size of the tiny damage of fibril tissues is unknown at this moment as it was never measured.

Figure 1 shows a typical CT image of a healthy human buttock. Bone, muscle, fat and skin are the main tissues, and the center part is the target region where muscle covering bony prominences is seen.
By using a simplified numerical modeling and assuming that the initial damage occurs at the interface between bone and muscle by the loose fibril tissue damage\(^{(13)}\), the authors successfully evaluated where there would be risk for reoccurrence of pressure ulcers after surgery was successfully evaluated. Also, we pointed out that severe shear loading to the patient’s buttock may become a trigger for pressure ulcers. Bansal et al.\(^{(14)}\) listed up do’s and don’ts to prevent pressure ulcers and wrote that keeping the patient incline at no higher than 30° is important to prevent sliding and friction on the buttock, which supports our previous discussion\(^{(13)}\). Therefore, the same bio-mechanism assumption is employed in this paper, and the interface damage is modeled by a cutout in the finite element model.

### 3.2 Finite element model

In this paper, 2D plane strain linear finite element analysis based on the CT image in Fig. 1 was carried out in the same way as was done in the authors’ previous paper\(^{(13)}\) and the fibril tissue damage was modeled by a cutout as shown in Fig. 2. The location of 4 mm cutout is fixed as a demonstrative example in this paper. The size of the cutout was large enough to assume the tiny damage of fibril tissue. The number of four-noded elements is 2,396.

The skin was neglected, because Makhsous et al.\(^{(15)}\) have reported that the deformation of skin is much

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus, E (MPa)</th>
<th>Shear modulus, G (MPa)</th>
<th>Coefficient of correlation between E and G</th>
<th>Poisson’s ratio, ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>8.0×10^2</td>
<td>2.857×10^2</td>
<td>0.995</td>
<td>0.4</td>
</tr>
<tr>
<td>Muscle</td>
<td>7.5×10^2</td>
<td>2.517×10^2</td>
<td>-</td>
<td>0.49</td>
</tr>
<tr>
<td>Bone</td>
<td>2.0×10^4</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Fig.3  Boundary conditions for different body positions.
smaller than that for muscle and fat. Table 1 shows the material properties based on a linear isotropic model\(^{(17)\sim(20)}\). Note that the Young’s moduli and/or shear moduli for muscle and fat are scattered. These are the parameters that are going to be varied randomly in the following analysis. A normal distribution is assumed for simplicity. The coefficient of correlation between Young’s modulus and the shear modulus for fat was determined so that the Poisson’s ratio would not exceed 0.5. For muscle, only the variation of Young’s modulus was considered because its Poisson’s ratio is close to 0.5. For the cutout element, a value \(10^{-5}\) times smaller value than the mean Young’s modulus of muscle was used.

For the boundary conditions, three typical body positions—the supine position and two lateral positions—were analyzed as shown in Fig. 3. Since a linear problem setting was used in this paper, the contact area is fixed by experimental measurement, and 8 kPa pressure was used for the supine position, whilst 18 kPa pressure was used for lateral positions. Note the pressure value used is close to that used by Mahksous et al.\(^{(15)}\)

Since the boundary condition in this model was a simplified one, and only one node was fixed in the x-direction in Fig. 3, a regularization technique was used as shown in Eq. (4) in order to escape the singularity of the model\(^{(21),(22)}\). \(K\) and \(N\) are the stiffness matrix and shape function and \(u\) and \(f\) are the displacement and force vectors. The regularization was applied after diagonal scaling in the scaled conjugate gradient (SCG) solver. The coefficient factor \(\lambda = 10^{-10}\) was determined by sensitivity analysis.

\[
(K + \lambda \int_V N^T N dV) u = f
\]  

(4)

4. Computational procedure

4.1 Risk prediction method

In the biomechanical analysis of a pressure ulcer, where the fibril tissue damage at the interface between bone and muscle is modeled by a cutout, the strains at the right and left cutout tips are the quantity of interest. These strains were transformed into a normal (\(n\)) and tangential (\(t\)) component along the interface as shown in Fig. 4. The cutout tip strain was extrapolated from

Fig.5 Flowchart of risk prediction.
Fig. 6 Convergence of expected strain value under supine position.

(a) $\varepsilon_n$ at left cutout tip

(b) $|\gamma_{tn}|$ at left cutout tip

(c) $\varepsilon_n$ at right cutout tip

(d) $|\gamma_{tn}|$ at right cutout tip

Fig. 7 Convergence of expected strain value under lateral position A.

(a) $\varepsilon_n$ at left cutout tip

(b) $|\gamma_{tn}|$ at left cutout tip

(c) $\varepsilon_n$ at right cutout tip

(d) $|\gamma_{tn}|$ at right cutout tip
Fig. 8 Comparison of $|\gamma_n|$ at right cutout tip with and without cutout under supine position.

Fig. 9 Comparison of the strains with and without cutout under lateral position A.

The values at Gauss points in the neighboring element, as shown in Fig. 4, so that the extrapolation could be automated. We denote normal strain and shear strain by $\varepsilon_n$ and $\gamma_n$. Concerning normal strain $\varepsilon_n$, it is assumed that only tensile strain contributes to the breakage of fibril tissue and propagation of damage area. High shear strain is also supposed to be dangerous, especially when repeated shear strain is applied to the fibril tissue.

Table 2 Risk prediction for further analysis under supine position.

<table>
<thead>
<tr>
<th>Supine Position</th>
<th>Left cutout tip</th>
<th>Right cutout tip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_n$</td>
<td>$</td>
</tr>
<tr>
<td>Without cutout</td>
<td>-0.0130</td>
<td>0.0270</td>
</tr>
<tr>
<td>SD</td>
<td>0.0014</td>
<td>0.0023</td>
</tr>
<tr>
<td>With cutout</td>
<td>-0.0570</td>
<td>0.0092</td>
</tr>
<tr>
<td>SD</td>
<td>0.0061</td>
<td>0.0033</td>
</tr>
<tr>
<td>Risk of pressure ulcer’s occurrence</td>
<td>No Risk</td>
<td>Possible Risk</td>
</tr>
</tbody>
</table>

EV: Expected Value  
SD: Standard Deviation

Table 3 Risk prediction for further analysis under lateral position A.

<table>
<thead>
<tr>
<th>Lateral Position A</th>
<th>Left cutout tip</th>
<th>Right cutout tip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_n$</td>
<td>$</td>
</tr>
<tr>
<td>Without cutout</td>
<td>0.00550</td>
<td>0.03000</td>
</tr>
<tr>
<td>SD</td>
<td>0.00064</td>
<td>0.00320</td>
</tr>
<tr>
<td>With cutout</td>
<td>0.00560</td>
<td>0.03700</td>
</tr>
<tr>
<td>SD</td>
<td>0.00063</td>
<td>0.00330</td>
</tr>
<tr>
<td>Risk of pressure ulcer’s occurrence</td>
<td>Dangerous</td>
<td>Dangerous</td>
</tr>
</tbody>
</table>

Table 4 Risk prediction for further analysis under lateral position B.

<table>
<thead>
<tr>
<th>Lateral Position B</th>
<th>Left cutout tip</th>
<th>Right cutout tip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_n$</td>
<td>$</td>
</tr>
<tr>
<td>Without cutout</td>
<td>-0.000290</td>
<td>0.002900</td>
</tr>
<tr>
<td>SD</td>
<td>0.000066</td>
<td>0.000690</td>
</tr>
<tr>
<td>With cutout</td>
<td>-0.000160</td>
<td>0.003800</td>
</tr>
<tr>
<td>SD</td>
<td>0.000064</td>
<td>0.000750</td>
</tr>
<tr>
<td>Risk of pressure ulcer’s occurrence</td>
<td>No Risk</td>
<td>Dangerous</td>
</tr>
</tbody>
</table>

The interface strains at the cutout tips, $\varepsilon_n$ and $|\gamma_n|$, are compared to the case without cutout, $\varepsilon_{n,\text{nocutout}}$ and $|\gamma_{n,\text{nocutout}}|$, at the same cutout location. If $\varepsilon_n$ or $|\gamma_n|$ is higher than $\varepsilon_{n,\text{nocutout}}$ or $|\gamma_{n,\text{nocutout}}|$, then we can postulate that the fibril tissue damage may propagate. In that case, the second step of SLS can be carried out. The detail of the flowchart of risk prediction is shown in Fig. 5, where the subscript exp refers to the expected value and the subscript max denotes the maximum value in the tail probability.

4.2 Convergence check and post-processing in the first step of SLS

In the standard Monte Carlo simulation considering the normal distribution of material properties under the three boundary conditions, the expected values of strains at the right and left tips of the cutout were monitored after every 100 analyses. Figure 6 shows the history of the Monte Carlo simulation under supine position of every 100 cases shown with the blue line. The value of the right hand side of Eq. (2) is also plotted as shown with the red
line. The three red circles indicate the location where three continuous sets of analyses occurred indicating the convergence. All components reached the convergence after 2,000 analyses except for $|\gamma_{tn}|$ at the left cutout tip where convergence was reached at 1,500 analyses. Then, the highest number of analysis for convergence at 2,000 analyses was chosen, at which point the Monte Carlo simulation was suspended.

The behaviors after 2,000 analyses were also plotted in this paper to investigate the effectiveness of the proposed convergence rule. In Fig. 6, the expected value was oscillated after 2,000 analyses. This oscillation happened because very rare sampling points with low probability were generated, and we confirmed the convergence rule had no serious problems. We confirmed the oscillation by performing each analysis up to 10,000 data and compared it with 2,000 data.

The Monte Carlo histories for the case of lateral position A are shown in Fig. 7. In this case, faster convergence was obtained, and no oscillation was found after convergence. So, 2,000 was assumed to be a sufficient number of cases to ensure convergence for all boundary conditions.

After the modeling, the risk of damage propagation was judged under the three boundary conditions, as shown in Fig. 5. Figure 8 shows the distribution of $|\gamma_{tn}|$ at the right cutout tip under supine position. Obviously, this result implies the risk of pressure ulcer. The tail probability of the strains with cutout will always be higher than strains without cutout. Table 2 shows the detail of all strain values and the final results for the supine position. For $\varepsilon_n$ at the left and right cutout tip, $\varepsilon_{n,\text{exp}} < 0$ which means no risk of pressure ulcers. For $|\gamma_{tn}|$ at the left cutout tip, the $|\gamma_{tn}|_{\text{exp}} < |\gamma_{tn,\text{nocutout}}|_{\text{exp}}$ which means possible risk of pressure ulcers while at the right cutout tip, the $|\gamma_{tn}|_{\text{exp}} \geq |\gamma_{tn,\text{nocutout}}|_{\text{exp}}$ which means danger of pressure ulcers.

Figure 9 shows $\varepsilon_n$ and $|\gamma_{tn}|$ at the right cutout tip under lateral position A, and the final results are summarized in Table 3. Unlike the results for the supine position, the strain distribution is overlapped, which means that risk was predicted. The tail probability of the
limited sampling zone under lateral position B.

Fig. 12 Limited sampling zone under supine position.

strains with cutout may sometimes be the same value with strains without cutout. Similar results for lateral position B are shown in Table 4. Finally, further analyses were carried out for all boundary conditions in the second step of SLS. The results from Table 2 to Table 4 showed that position supine is the most dangerous as it has the highest value of shear strains compared to lateral position A and lateral position B.

4.3 Limited sampling in the second step of SLS

To define the limited sampling zone in the 3D space of random input parameters, $E_{fat}$, $G_{fat}$ and $E_{muscle}$, multiple linear equations using Eq. (3), were determined.

The parameters sets in this limited sampling zone can result in high strain values at either cutout tip. As an example, the correlation between $|\gamma_{\tau n}|$ under lateral position B and the parameters set is shown in Fig. 10. Each plot is colored by the strain value. As mentioned in Chapter 2, $\mu + 3\sigma$ was used as the threshold for the limited sampling zone. Concerning the $|\gamma_{\tau n}|$ at right and left tips, the limited sampling zone could be defined by the combinations of $E_{fat} - G_{fat}$ and $E_{fat} - E_{muscle}$, but could not be defined for $E_{muscle} - G_{fat}$. As shown in Fig. 10, the following four equations were obtained.

$$E_{fat} \geq 0.075$$

$$E_{muscle} - 0.253E_{fat} \leq 0.0365$$

$$E_{fat} \geq 0.065$$

$$E_{muscle} - 0.16E_{fat} \leq 0.045$$

By taking the union of these regions, the limited sampling zone was obtained as shown in Fig. 11. In the same manner, the limited sampling zone was determined under the supine position in Fig. 12, and under the lateral position A in Fig. 13.

Note that only 10 to 20 sampling points were generated among 2,000 analyses in the first step of SLS. Moreover, the distribution of those points was irregular. In the second step of SLS, however, 1,000 more sampling points were added in this limited sampling zone to assure the reliability of the analysis of the tail probability.

5. Numerical results and discussion

As mentioned in the introduction, the goal of the biomechanical simulation was to obtain the sets of material parameters that would lead to high interface strain between bone and muscle and propagation of fibril tissue damage area. This set must be defined by the limited sampling zones in Figs. 11-13 for the three positions. For the following discussion, only $|\gamma_{\tau n}|$
is considered to demonstrate the effectiveness of SLS.

Figure 14 shows the calculated $|\gamma_{tn}|$ for 1,000 more cases at the right cutout tip under lateral position B. Unlike the determination of the limited sampling zone in Fig. 11, the boundary values for strain values were set by $\mu + 7\sigma$ in Fig. 14.

The results are summarized in Fig. 15 for the set of parameters with strain higher than $\mu + 7\sigma$ for the three boundary conditions.

The following new findings were obtained that would be helpful in deciding nursing strategy for patients:

1. For the subject in Fig. 1, if Young’s modulus of muscle $E_{\text{muscle}}$ is low, and tiny bone-muscle interface damage occurs then very high shear strain will propagate the damaged area.

2. In the above case, keeping the supine position is more dangerous if Young’s modulus of fat $E_{\text{fat}}$ is high, while keeping the lateral position is more dangerous if $E_{\text{fat}}$ is low.

This new finding was obtained by considering the limited sampling zone in the multi-dimensional space of multiple random parameters.

6. Conclusions

To ensure quality assurance of engineering simulations of uncertainty—which is of great importance to many different industries—a sampling algorithm in conjunction with Monte Carlo simulation emphasizing the tail probability value, called SLS, was proposed. Its use was demonstrated by its application to a biomechanics problem to predict the risk of pressure ulcers. The correlation between the random material parameters and strain at the interface between bone and muscle was calculated by using multiple linear equations to define the dangerous zone in the tail probability. A new finding was obtained, which will be useful for nursing of patients to prevent pressure ulcer.

To ensure the value of this new method, the biomechanical simulations in the future should contain more random parameters, such as the location of the cutout, configuration of fat, muscle and bone and boundary conditions\(^{(15)}\). The proposed SLS algorithm can be applied to a problem with larger number of input parameters, which we hope to prove in the future simulation.

Only 3,000 analyses were needed to obtain reasonably reliable average values and very accurate tail probabilities. Compared to the conventional Monte Carlo simulation of 10,000 cases, the proposed method is cost-effective and practical. Although 2D linear analyses were carried out as a demonstration in this paper, 3D nonlinear analyses should be employed considering large deformation, contact between body and bed and viscoelasticity.

Acknowledgment

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References


(3) The American Society of Mechanical Engineers, An Illustration of the Concepts of Verification


(20) Shimizu, Y., Okamoto, K., Sugiki, T., Takano, N. and Nagasao, T., Development of Robust Design System for Medical Microneedle Array - Monte Carlo Simulation using Measured Results of Inter-individual Differences of Skin Characteristics, Proc. of the Conf. on Computational Engineering and Science, Kyoto: JSCES, May 2012, CD-ROM.
