IMPACT FORCE IDENTIFICATION OF ALUMINUM PLATES USING STRAIN SENSORS

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This paper proposes a reliable method identifying both a location and a history of an impact force acting on a plate using multiple sensor responses. First, a fundamental relationship between an impact force and the strain responses is formulated based on the finite element method. The impact force history is precisely identified by the least-squares method containing a penalty term on derivative of force history. The location of impact force is determined by minimizing an error vector between measured strain responses and analytically evaluated ones. An application of proposed method to aluminum plates equipped with sensors is developed to discuss the validity of the present method.

Key Words: *impact force identification, aluminum plate, strain sensors, PZT sensors, strain gauges, finite element method*

1. INTRODUCTION

For a long-term operation of structures, detecting internal structural damages is a significant concern. Especially, some damages induced by external impact events are apparently invisible and might degrade structural strength extremely. Additionally, the impact-induced damages spread into structures rapidly and dynamically unlike fatigue damages. Thus, it is inevitable to evaluate structural integrity with nondestructive inspection to avert a sudden structural breakdown or collapse and to carry out long-term operation of structures. As nondestructive inspection techniques such as coin tapping, X-ray and ultrasonic inspections have been conventionally and widely accepted. However, these inspections not only need to suspend the operation but also may lead to some human oversights. Therefore, they are unable to guarantee the structural integrity with high reliability. Accordingly, in recent years a new inspection approach, i.e., structural health monitoring^{1),2)}, has been focused by lots of researchers. In this inspection technique, sensing- actuating net-

works and algorithm for damage detection are incorporated into structures, so that the structural integrity can be automatically monitored with sensor information. To establish such a damage monitoring system, external impact forces should be effectively identified first. It is well known that the maximum impact force or energy is tightly connected with the occurrence of impact-induced damages. If impact events can be identified in real time during operation, the safety and the reliability of structures can be greatly enhanced.

In some previous studies, a technique for identifying impact forces has been proposed using a transfer matrix relating the impact force with induced strain responses. For instance, Inoue et al.³⁾ have experimentally obtained the transfer matrices between an impact force and strain responses, and obtained impact force histories through the deconvolution approach. Doyle⁴⁾⁻⁶⁾ identified impact force histories both in time and frequency domains by using the transfer matrices obtained numerically. The validity of the method was verified through experiments for isotropic beams⁴⁾ and plates^{5),6)} with strain

gauges. In addition, Martin and Doyle⁷⁾ applied the same method to frame structures. Wu et al.^{8),9)} have constructed Green's functions with the eigenmode expansion method and succeeded in identifying impact force histories by minimizing a residual error between measured strain responses and numerically evaluated ones. This method has also been extended to identification of multiple impact forces⁹⁾. However in their work, it was assumed that the force locations were known in advance.

As no information of impact locations is given usually, some methods to identify both locations and histories of impact forces have been proposed. Yen and Wu^{10),11)} have identified impact force locations by minimizing an objective function derived from multiple strain responses and transfer matrices. In addition, the force histories have been identified using the same approach in ref. 8) and ref. 9). Choi and Chang¹²⁾ have minimized the error vector between measured strain responses from PZT piezoelectric sensors and numerically evaluated responses, and obtained both impact force locations and histories. Although a lot of approaches have been reported as mentioned above, an effective impact force identification method has not been established yet.

In this paper, we propose a method to identify the force location and the force history at low-velocity impact. First, a relationship between an impact force and the corresponding strain responses for a plate structure is formulated based on the finite element method as a forward problem. Then, the impact force history is given as a solution of an inverse problem derived from the above forward problem and nominally solved by the least-squares method. However, in actual systems, an identified force history oscillates remarkably due to measurement noises in sensor responses. Therefore, a reliable method where a regularization term, i.e., a penalty term of the derivative of force history is brought into the conventional least-squares method is put forward to obtain the stable history. In the force location identification, first, some locations are chosen as candidate force locations based on the reciprocal relationships of strain responses. Among the candidates, the true force location is given as a location where a residual error vector between experimentally measured strain responses and numerically evaluated strain responses using the finite element model is minimized. Those numerical responses are estimated from the forward problem using the temporally identified force history at a candidate force location. Once the force location can be identified, the true force history can also be determined through the solution of the modified

Fig.1 Rectangular plate under an impact force

least-squares problem.

The present identification method is verified experimentally with an impact force identification system composed of a measurement part and an identification part. The measurement part consists of a sensor network, that is, PZT piezoelectric sensors or strain gauges, and various signal conditioners. This practical system is applied to aluminum plates under a cantilever support condition. It is reported in this paper that the validity of the present method is verified through the comparison with experimental identification results.

2. FORMULATION

(1) Impact force and strain response

A sufficiently thin rectangular plate under an impact force is shown in Fig.1. The motion equation of plate based on the finite element method under the undamped forced vibration is expressed as follows.

$$[M]\{\ddot{u}\} + [K]\{u\} = f(t)\{\delta\} \quad (1)$$

where $[M]$, $[K]$, $\{u\}$ and $f(t)$ are the mass matrix, the stiffness matrix, the nodal displacement vector and the external force history respectively. In addition, $\{\ddot{\cdot}\}$ denotes the second derivative with respect to time and $\{\delta\}$ is a vector which means a force location. In this paper, we assume that one concentrated force acts on a node in the finite element model. Thus, $\{\delta\}$ can be expressed as

$$\{\delta\} = [0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0]^T \quad (2)$$

where the index $[\]^T$ represents transpositional operation of the associated vectors or matrices. The nodal displacement $\{u\}$ is described by the linear combination based on a limited set of n natural modes as

$$\{u\} = \sum_{i=1}^n \{\phi_i\} \xi_i = [\Phi]\{\xi\} \quad (3)$$

where $\{\phi_i\}$ and ξ_i are the i th natural vibration

mode and modal displacement. In addition, $[\Phi]$ and $\{\xi\}$ are respectively the modal matrix and the modal displacement vector, which are expressed as $[\Phi] = [\phi_1 \ \phi_2 \ \dots \ \phi_n]$ and $\{\xi\} = [\xi_1 \ \xi_2 \ \dots \ \xi_n]^T$. Thus, the motion equation in the modal coordinates is obtained in the following Eq.(4) by substituting Eq.(3) into Eq.(1).

$$\ddot{\xi}_i(t) + \omega_i^2 \xi_i(t) = f(t) \{\phi_i\}^T \{\delta\} \quad (4)$$

where ω_i is the i th natural angular frequency. This second-order ordinary differential equation can be easily solved by applying Duhamel's integral. The i th modal displacement is obtained as

$$\xi_i(t) = \frac{1}{\omega_i} \int_0^t f(\tau) \{\phi_i\}^T \{\delta\} \sin \omega_i(t - \tau) d\tau \quad (5)$$

By applying the conception of discrete time domain $t = k\Delta t$ ($k=1,2,\dots,m$) and a linear interpolation of impact force $f(t)$ in each time interval Δt , the modal displacement at $t = t_m (= m\Delta t)$ is expressed as

$$\xi_i(t_m) = \frac{\{\phi_i\}^T \{\delta\}}{\omega_i} \left[\sum_{s=1}^{m-1} \Omega(i, m-s) f_s + \Psi(i) f_m \right] \quad (6)$$

where

$$\Omega(i, j) \equiv \frac{1}{\omega_i^2 \Delta t} \{-\sin \omega_i(j-1)\Delta t + 2\sin \omega_i j\Delta t - \sin \omega_i(j+1)\Delta t\} \quad (7)$$

$$\Psi(i) \equiv \frac{1}{\omega_i} \left(1 - \frac{1}{\omega_i \Delta t} \sin \omega_i \Delta t \right) \quad (8)$$

and f_s denotes the value of impact force at $t = t_s$. In terms of the impact force history vector $\{\tilde{f}\}$, which consists of scalar values of impact force at each discrete time point, $f_1, f_2, \dots, f_m$, Eq.(6) can be rewritten into

$$\{\xi\} = [R][T]\{\tilde{f}\} \quad (9)$$

where

$$[R] = \text{diag} \left[\frac{\{\phi_1\}^T \{\delta\}}{\omega_1}, \frac{\{\phi_2\}^T \{\delta\}}{\omega_2}, \dots, \frac{\{\phi_n\}^T \{\delta\}}{\omega_n} \right] \quad (10)$$

$$[T] = \begin{bmatrix} \Omega_{m-1}^1 & \Omega_{m-2}^1 & \dots & \Omega_1^1 & \Psi_1 \\ \Omega_{m-1}^2 & \Omega_{m-2}^2 & \dots & \Omega_1^2 & \Psi_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Omega_{m-1}^n & \Omega_{m-2}^n & \dots & \Omega_1^n & \Psi_n \end{bmatrix} \quad (11)$$

In addition, $\Omega_j^i \equiv \Omega(i, j)$, $\Psi_i \equiv \Psi(i)$ and an index *diag* denotes a diagonal matrix. Substituting Eq.(9) into Eq.(3) leads to an expression of the nodal displacement $\{u_m\}$ at $t = t_m$

$$\{u_m\} = [\Phi][R][T]\{\tilde{f}\} \quad (12)$$

A strain response $\{\varepsilon\} = [\varepsilon_x \ \varepsilon_y \ \gamma_{xy}]^T$ at (x, y, z) on the plate is given as

$$\{\varepsilon\} = \left[-z \frac{\partial^2 u}{\partial x^2} \quad -z \frac{\partial^2 u}{\partial y^2} \quad -2z \frac{\partial^2 u}{\partial x \partial y} \right]^T \quad (13)$$

Therefore, by employing the strain-displacement matrix $[B]$ and Eq.(12), strain response $\{\varepsilon_m\}$ at $t = t_m$ can be expressed as

$$\begin{aligned} \{\varepsilon_m\} &= z[B][\Phi][R][T]\{\tilde{f}\} \\ &\equiv [\bar{g}]\{\tilde{f}\} \end{aligned} \quad (14)$$

In Eq.(14), the matrix $[\bar{g}]$ is strongly affected by the characteristic of the convolution integral in Duhamel's integral. Therefore, focusing on the x-axis strain ε_x , one can easily transcribe Eq.(14) at each sampling time into the following equations

$$\begin{aligned} \varepsilon_x^1 &= g_x^1 f_1 \\ \varepsilon_x^2 &= g_x^2 f_1 + g_x^1 f_2 \\ &\vdots \\ \varepsilon_x^k &= g_x^k f_1 + g_x^{k-1} f_2 + \dots + g_x^1 f_k \\ &\vdots \\ \varepsilon_x^m &= g_x^m f_1 + g_x^{m-1} f_2 + \dots + g_x^k f_{m-k+1} + \dots + g_x^1 f_m \end{aligned} \quad (15)$$

Defining a strain history vector $\{\tilde{\varepsilon}_x\}$ which consists of $\varepsilon_x^1, \varepsilon_x^2, \dots, \varepsilon_x^m$, the relationship between the strain history $\{\tilde{\varepsilon}_x\}$ and the force history vector is cast into the following equation.

$$\{\tilde{\varepsilon}_x\} = [g_x]\{\tilde{f}\} \quad (16)$$

where the transfer matrix $[g_x]$ is a lower triangular matrix composed of m independent components as follows

$$[g_x] = \begin{bmatrix} g_x^1 & & & \\ g_x^2 & g_x^1 & & 0 \\ g_x^3 & g_x^2 & g_x^1 & \\ \vdots & \vdots & \vdots & \ddots \\ g_x^m & g_x^{m-1} & g_x^{m-2} & \dots & g_x^1 \end{bmatrix} \quad (17)$$

For y-axis strain ε_y or shear strain γ_{xy} , Eq.(16) can be also formulated. Therefore, the relationship Eq.(16) can be generalized as the next equation.

$$\{\tilde{\varepsilon}\} = [g]\{\tilde{f}\} \quad (18)$$

Once the finite element model of structures is constructed, the transfer matrix $[g]$ is determined by both impact force location (x_f, y_f) and location of strain sensor (x_s, y_s, z_s) , and each component of the matrix denotes the strain response corresponding

Fig.2 PZT piezoelectric sensor model

to an impulse force.

(2) Modeling of PZT sensor

Piezoelectric materials typified by lead zirconate titanate (PZT) or polyvinylidene fluoride (PVDF) are sensitive to responses to structural deflection in high frequency range. These materials can be employed as not only sensors for passive sensing with their piezoelectricity but also actuator for active sensing with their converse effect. In recent years, new distributed sensing and actuating networks called SMART Layer¹³⁾ were developed in Stanford University. This network in which multiple PZT piezoelectric devices are wired on a thin polymer film enables both active and passive sensing. Moreover, its film shape has the advantage of integration with various structures. In this section, modeling of output from PZT devices as strain sensors is discussed.

In this study, a circular PZT device shown in **Fig.2** is employed. This device is sufficiently thin compared with its diameter and therefore can be assumed to be under plane stress. Additionally, its poling direction is along 3 axis and perpendicular to its surface. We also assume that the strain within the device is uniform, and elasticity and piezoelectricity are both isotropic in the 1-2 plane. The governing equations of the piezoelectric device can be written as follows.

$$\begin{aligned}\varepsilon &= s^E \sigma + dE \\ D &= d\sigma + \varepsilon^T E\end{aligned}\quad (19)$$

where ε and σ are mechanical strain and stress, D and E are the electric displacement and the electric field. d , s^E and ε^T are respectively the piezoelectric strain constant, compliance constant at a constant electric field and dielectric permittivity at constant stress. In this study, PZT device is used as sensors but not actuators, thus, the electric field E therefore is zero. If stresses $\{\sigma\} = [\sigma_1 \ \sigma_2 \ \tau_{12}]^T$ induce strains $\{\varepsilon\} = [\varepsilon_1 \ \varepsilon_2 \ \gamma_{12}]^T$ within the device, the equivalent electric displacement is also excited on account of the direct piezoelectric effect according to the second equation in Eq.(19). In consequence, the interaction between stresses and strains

as well as between the electric displacement and stresses are respectively expressed as follows.

$$\{\sigma\} = \frac{Y_{11}}{1-\nu_{12}^2} \begin{bmatrix} 1 & \nu_{12} & 0 \\ \nu_{12} & 1 & 0 \\ 0 & 0 & (1-\nu_{12})/2 \end{bmatrix} \quad (20)$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{31} & 0 \end{bmatrix} \{\sigma\} \quad (21)$$

where Y_{11} and ν_{12} respectively indicate Young's modulus and Poisson's ratio of the device. Thus the lateral electric displacement D_3 can be written by substituting Eq.(20) into Eq.(21) as follows.

$$D_3 = d_{31} \frac{Y_{11}}{1-\nu_{12}} (\varepsilon_1 + \varepsilon_2) \quad (22)$$

Accordingly, the electric charge Q_{PZT} stored within the PZT device is expressed in the following equation by using the cross section of the device S .

$$Q_{PZT} = S d_{31} \frac{Y_{11}}{1-\nu_{12}} (\varepsilon_1 + \varepsilon_2) \quad (23)$$

In this study, the stored electric charge Q_{PZT} is transformed into voltage through a charge amplifier. The measurable output voltage V_{PZT} is estimated by

$$V_{PZT} = g_{amp} Q_{PZT} = g_{PZT} (\varepsilon_1 + \varepsilon_2) \quad (24)$$

where

$$g_{PZT} \equiv g_{amp} S d_{31} \frac{Y_{11}}{1-\nu_{12}} \quad (25)$$

and g_{amp} denotes the gain of a charge amplifier. According to Eq.(24), it is found that the output voltage from the PZT device is proportional to the sum of in-plane strains so that it is possible to measure strain responses by measurement of the output voltage. In this paper, employing PZT devices as strain sensors, we call them PZT sensors. In Eq.(24), the value of g_{PZT} is affected by the installation of devices, and it is determined by a calibration test using biaxial strain gauges.

As shown in Eq.(24), the measured strain response from the PZT sensor is the sum of in-plane strains. By orienting the in-plane axes on the PZT to ones on the rectangular plate, the relationship between an impact force and the strain response in Eq.(18) can be rewritten as follows.

$$\begin{aligned}\{\tilde{\varepsilon}\} &= \{\tilde{\varepsilon}_x\} + \{\tilde{\varepsilon}_y\} = ([g_x] + [g_y]) \{\tilde{f}\} \\ &\equiv [G] \{\tilde{f}\}\end{aligned}\quad (26)$$

(3) Estimation of force location

When an impact force $\{\tilde{f}\}$ acts on (x_f, y_f) in

the plate structures and the induced strain responses can be at once measured through $N(\geq 3)$ strain sensors arranged in or on the plate, the strain responses, $\{\tilde{\varepsilon}_i\}$ and $\{\tilde{\varepsilon}_j\}$, which are respectively responses from i th or j th sensor are given by Eq.(26) as follows.

$$\{\tilde{\varepsilon}_i\} = [G_i]\{\tilde{f}\} \quad (27a)$$

$$\{\tilde{\varepsilon}_j\} = [G_j]\{\tilde{f}\} \quad (27b)$$

Note that matrices $[G_i]$ and $[G_j]$ are the transfer matrices corresponding to i th and j th sensor. By multiplying transfer matrices $[G_j]$ and $[G_i]$ on both sides in each equation respectively, both equations are written as

$$[G_j]\{\tilde{\varepsilon}_i\} = [G_j][G_i]\{\tilde{f}\} \quad (28a)$$

$$[G_i]\{\tilde{\varepsilon}_j\} = [G_i][G_j]\{\tilde{f}\} \quad (28b)$$

Recalling that the transfer matrix $[G]$ is derived from Duhamel's integral shown in Eq.(5) and the integral is commutative, the following commutative equation can be given.

$$[G_j][G_i] = [G_i][G_j] \quad (29)$$

According to Eq.(28) and Eq.(29), one reciprocal relationship between two different strain responses $\{\tilde{\varepsilon}_i\}$ and $\{\tilde{\varepsilon}_j\}$ can be expressed as^{(10),(11)}

$$[G_j]\{\tilde{\varepsilon}_i\} = [G_i]\{\tilde{\varepsilon}_j\} \quad (30)$$

Eq.(30) is valid only when the true force location (x_f, y_f) is given. Then, estimating a force location as (x_e, y_e) on the plate, a residual between both sides in Eq.(30) means a deviation between true force location and another estimated force location. Thus, true force location can be given as a solution of the following minimization problem.

$$\min_{x_e, y_e} E = \sum_{i=1}^N \sum_{j \neq i}^N \frac{\|[G_i]\{\tilde{\varepsilon}_j\} - [G_j]\{\tilde{\varepsilon}_i\}\|}{\|[G_j]\{\tilde{\varepsilon}_i\}\|} \quad (31)$$

where $\|\bullet\|$ gives the Euclidean norm of the associated vector.

If there are no errors, e.g., such as modeling errors affecting the matrix $[G]$ or measurement errors in strain responses in the above problem, the minimization procedure will precisely give a true force location where the objective function E has the minimum value. However, the actual system should be accompanied with those errors, which possibly poses obscurity to accurate force location identification. In this study, Eq.(31) is evaluated not for identifying the true location, but for estimation of possible force location, that is to say, some locations with

small value of objective function E are chosen as candidate force locations.

(4) Identification of impact force history

If it is assumed that the force location (x_f, y_f) is known and N sensor responses are measured, N relationships in Eq.(26) can be given as a forward problem. Thereby, the identification problem of impact force history can be formulated as an inverse problem derived from the forward problems and force history is obtained as a solution of the following least-squares problem.

$$\min_{\{\tilde{f}\}} F = \sum_{i=1}^N \|\{\tilde{\varepsilon}_i\} - [G_i]\{\tilde{f}\}\|^2 \quad (32)$$

Generally, inverse problems tend to be ill-posed. In fact, the force history identification problem in Eq.(32) is so ill-posed that the identified force history $\{\tilde{f}\}$ has oscillation due to the infection of errors involved in sensor responses or the limitation of employed modal data in numerical analysis. Thus, in order to improve the inverse problem itself by suppressing the oscillation and smoothing the force history, we propose a reliable method where a regularization term, that is, a penalty term imposed on the force history, is introduced into Eq.(32) and expressed as

$$\begin{aligned} \min_{\{\tilde{f}\}} F &= \sum_{i=1}^N \|\{\tilde{\varepsilon}_i\} - [G_i]\{\tilde{f}\}\|^2 + \alpha \|[L]\{\tilde{f}\}\|^2 \\ \text{subject to } &\{\tilde{f}\} \geq 0 \end{aligned} \quad (33)$$

where $[L]$ gives the first order derivative of force history with respect to time calculated by the finite difference method. Moreover, α denotes a weight factor of the regularization term and is chosen through trial-and-error technique in advance in this paper. In Eq.(33), under a constraint that identified impact forces should be compressive, the quadratic programming method is employed to get impact force history.

(5) Identification of impact force location

Making use of an estimated force location (x_e, y_e) obtained by the minimization process in Eq.(31), a force history $\{\tilde{f}_e(x_e, y_e)\}$ can temporally be obtained by the modified least-squares method presented in Eq.(33). According to Eq.(26), an error vector $\{\tilde{e}_i\}$ can be defined as

$$\{\tilde{e}_i\} = \{\tilde{\varepsilon}_i\} - [G_i(x_e, y_e)]\{\tilde{f}_e(x_e, y_e)\} \quad (34)$$

This error vector indicates a deviation between a measured strain responses $\{\tilde{\varepsilon}_i\}$ and an evaluated strain response $[G_i]\{\tilde{f}\}$ using the numerical model.

Fig.3 Impact force identification system

Fig.4 Specimen and sensor arrangement

Table1 Material properties of aluminum plate

E [GPa]	G [GPa]	ν	ρ [kg/m ³]
70	26.3	0.33	2710

Therefore, the force location which minimizes the following objective function is determined as the true force location.

$$\min_{x_f, y_f} H = \sqrt{\frac{1}{mN} \sum_{i=1}^N \|\tilde{e}_i\|^2} \quad (35)$$

The minimization in Eq.(35) is parametrically solved among the nodes in the finite element model. Once the true force location (x_f, y_f) is determined, the corresponding force history can also be identified by solving Eq.(33).

Eq.(35) enables to identify the precise location, although it leads to the increase of identification time. In the experimental verification, we employ the above two different force location identification method presented in Eq.(31) and Eq.(35) in order to discuss the identification accuracy.

3. IMPACT FORCE IDENTIFICATION SYTEM

(1) Experimental setup

Setup to verify the present impact force identification method is shown in Fig.3. The identification system consists of an aluminum plate specimen, a jig for cantilever support, an amplifier, a digital oscilloscope (Yokogawa Electric, DL708E), a computer and an impulse hammer (ONO SOKKI, GK-3100).

The aluminum plate of 300mm × 300mm × 2mm is shown in Fig.4. The employed material properties are shown in Table 1 as well. The specimen is uniformly clamped by a few bolts on the jig and supported as a cantilever plate shown in Fig.3.

As strain sensors, biaxial strain gauges (NEC San-ei Instrument, N22-FA-5-120-11-VS1) or PZT piezoelectric sensors (diameter 6.35mm, thickness 0.25mm, $d_{31} = -175(\text{pC/N})$) are located on SMART Layer (Acellent Technology). The biaxial strain gauges are stuck on the top surface of the specimen, while SMART Layer is attached on the underneath of the specimen. The number of sensors is four and they are located at (85,90), (213,90), (86,216) and (213,218)mm from Sensor1 to Sensor4 as shown in Fig.4.

When the impact force is exerted on the specimen using the impulse hammer, the induced sensor responses are amplified through the amplifier and A/D conversion is conducted through the digital oscilloscope. Note that the output of gauges is the sum of strain responses along x-axis and y-axis and amplified through a direct amplifier (UNIPULSE, AM32-AZ), while electric charges from PZT are transformed to voltage and amplified through a charge amplifier (ONO SOKKI, GK-1100). Finally, responses are imported into the computer. The response of the impulse hammer, that is, the true impact response, measured coincidentally is also stored in the computer to compare with the

Fig.5 Finite element model

identified force history.

(2) Finite element model

The finite element model of 10×10 division consisting of 100 elements is shown in Fig.5. The 4-noded plate bending element which has 3 degrees of freedom at each node, i.e., the rotation about x-axis θ_x , the rotation about y-axis θ_y and the deflection w , is employed. The total number of degrees of freedom is 363. The boundary conditions are imposed on 22 nodes of 10 elements in the clamped area by the jig. At 11 nodes (illustrated as “■”) along the edge of the plate, all degrees of freedom are fixed. On the other hand, at the other 11 nodes (illustrated as “□”), the rotation θ_x is only set to be free. The effects of the thickness and the stiffness of SMART Layer on the finite element model are ignored and the direction of impact force is defined to be positive when impact acts on the top surface of the plate.

In order to determine the necessary number of natural vibration modes, i.e., n , used in Eq.(3), an impact experiment was performed. From the results, we found that very low spectrum energy was observed in the frequencies higher than 2kHz. Consequently, the first 30 natural vibration modes corresponding to the frequencies lower than 2kHz are utilized in the present numerical analysis.

(3) Flow chart of impact force identification

The flow chart of impact force identification is

Fig.6 Flow chart of impact force identification

presented in Fig.6. The finite element model is assumed to be constructed beforehand. Impact events are monitored using a trigger function in the oscilloscope. When one of induced strain responses passes over the defined threshold, the system detects the occurrence of impact event and starts sweeping and holding all the responses. After responses are imported into computer, first, the detected area in identification of force location has been zoomed to shorten the identification time according to sensor responses. This is performed under an assumption that the sensor closest to the force location should respond the most quickly. A threshold value $\varepsilon_{\text{threshold}}$ is imposed on the sensor responses as shown in Fig.7. A trigger is pulled on a sensor of which response crosses over the threshold first and the identified area is zoomed into a small area according to the triggered sensor. For instance in the case of Fig.7, one can find that the response of PZT sensor 2 crosses over the defined threshold, therefore, the estimation of force location, that is, the parametric minimization in Eq.(31), is carried out within the assigned area on the sensor 2 shown in Fig.5.

After determination of an identification area, calculation to identify the impact force is conducted and described as: First, some candidates of force location are estimated based on Eq.(31). Next, force history identification in Eq.(33) is performed at these candidates. Then, the force location is evaluated by using Eq.(35) with the temporally obtained force history and measured strains. If the optimal force location and the corresponding force history can be identified, all of the results are outputted to terminate identification process. Otherwise, the candidate force location is renewed

Fig.7 Threshold and trigger point on sensor responses

and the same process is continually executed.

In this study, the total measurement time t is $t=10\text{ms}$ and the sampling time Δt is $\Delta t=0.1\text{ms}$. To identify the location, the first 5ms strain responses are employed. When strain gauges are used as sensors, content of higher frequencies than 10kHz is cut off through a low pass filter in the direct amplifier. In addition, the weight parameter α in Eq.(33) and the threshold value for narrowing the identified region are set to be $\alpha=1.0\times 10^{-19}$ and $\varepsilon_{\text{threshold}}=3.0\times 10^{-6}(\mu\text{m/m})$.

4. IDENTIFICATION RESULTS

(1) Calibration of PZT sensors

In section 2.(2), it is found that the output voltage of the PZT sensors is proportional to the sum of the in-plane strains. The conversion factor g_{PZT} of PZT sensors in Eq.(25) is determined through the calibration test using biaxial strain gauges. The biaxial gauges are attached on the plate to adjust each axis with in-plane coordinates, i.e., x-axis and y-axis. The sum of the measured strains at PZT sensor location is used to calibrate the conversion factor g_{PZT} . The average factor is calculated by comparing the maximum peaks of both strain responses. As a result of calibration, average value of g_{PZT} is obtained as $g_{\text{PZT}}=-2.19\times 10^4$. For example, when the impact is exerted on No.48 node (see Fig.5, illustrated as “▽”), both strain responses at sensor 1 are shown in Fig.8. The calibrated response of PZT sensors coincides with that of strain gauge well and the validity of the present PZT sensor model is verified.

(2) Identification results of force location

The identification results of force location are shown in Fig.9 and Fig.10. They illustrate the

Fig.8 Calibration of PZT sensors

results using strain gauges and PZT sensors, respectively. In Fig.9(a), the successfully identified nodes are presented with gray circles “●”, while the failed-identified nodes are marked with crosses “×”. Additionally, the alternatively identified nodes to those unsuccessful nodes are illustrated with black circles “●” in Fig.9(b). When strain gauges being used, 87% nodes can be identified successfully as shown in Fig.9(a). On the other hand, 94% nodes can be more successfully and precisely identified with PZT sensors as shown in Fig.10. In Fig.9(b), one can also find that all of the unsuccessful force locations are near the clamped boundary and the corresponding wrongly identified nodes are just next to the true ones. In the case of PZT sensors, the similar results are confirmed as well. The reason is that the strain responses near the boundary are easily affected by the modeling error.

In this study, a true force location is determined based on the evaluation of Eq.(35) among some locations for candidates chosen by Eq.(31). Fig.11 shows the results based on Eq.(31) as the only criterion for determination of the force locations. Note that PZT piezoelectric sensors are employed. In Fig.11, the identification accuracy falls to 87% comparing with the result of 94% in Fig.10 where Eq.(35) is employed to determine the true force location. That is because the dimensionless objective function in Eq.(31) cannot always be minimized at the true location due to some errors involved in the sensor responses or the structural modeling. On the other hand, a criterion of Eq.(35) compares measured strain responses with evaluated ones and is competent for more accurate identification of the force location.

(3) Identification results of force history

The results of force history identification when the impact forces are exerted at No.40 node (see Fig.5 and illustrated as “▽”) are shown in Fig.12. Fig.12(a) and Fig.12(b) show respectively the

Fig.9 Identification results of force location using strain gauges (Eq.(35) is employed)

Fig. 10 Identification results of force location using PZT sensors (Eq.(35) is employed)

Fig. 11 Identification results of force location using Eq.(31) and PZT sensors

results with strain gauges and PZT sensors. One can find that the force location identification can successfully be achieved according to **Fig.9(a)** and **Fig.10**, so that both identified force histories have good agreements with the measured ones from the impulse hammer. To summarize these results, when the force location can correctly be identified, the force history can be determined with high precision as well.

Next, we discuss the importance of force location identification method in Eq.(35). As an example, we consider the impact case of No.86 (see **Fig.5** and illustrated as “▼”). In case of the criterion in Eq.(31), the force location is unsuccessfully identified and thereby the identified force history shown in **Fig.13(a)** has large disagreement with the measured one. On the other hand, the criterion in Eq.(35) can give the well-identified force history as shown in **Fig.13(b)** as well as the successful force location identification as shown in **Fig.10**. In consequence, not only force location but also force history can be precisely identified by using the criterion in Eq.(35) where the measured and the numerically evaluated responses are directly compared.

5. CONCLUSION

In this paper, a method for identifying both the location and the history of an impact force using multiple strain sensors is proposed. First, the fundamental relationship between an impact force and induced strain responses for a plate structure is formulated as a forward problem based on the finite element method. In impact force identification, first, some force locations are estimated as candidates through the optimization procedure with a reciprocal relationship between two different measured strain responses. Next, in the force history identification, a reliable method by introducing a penalty term on the first derivative of the force history is proposed to alleviate the numerical errors in the ill-conditioned inverse problem. The impact force location is determined by minimization of a deviation between measured strain responses and numerically evaluated ones. Finally, the proposed identification method is applied to an aluminum plate with strain gauges or a thin sensor network film. The experimental verification shows that both the location and the history of impact forces can be identified with high precision.

Fig.12 Identification results of force history

Fig.13 Influence of different criteria in force location identification to accuracy of identified force history (PZT piezoelectric sensors are employed)

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