EFFECTIVENESS OF SEISMIC DISPLACEMENT RESPONSE CONTROL FOR NONLINEAR ISOLATED BRIDGE

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The effectiveness of seismic displacement response control for isolated bridges, which behave nonlinearly at both the columns and the isolators, is studied using the LQR optimal active control. A typical viaduct is analyzed for evaluation. An extensive parametric study of weighting matrices is carried out to examine the effect on reducing the displacement response. The results indicate that the active control is effective in reducing the deck displacement at small control force levels. Further, the effectiveness of saturation of control force is investigated for preventing excessive control at large control force levels. Finally, the active control is compared with the passive control using viscous dampers.

Key Words: nonlinear isolated bridge, active control, passive control, saturation.

1. INTRODUCTION

The isolators in bridge structures are effective in mitigating the induced seismic force by a shift of natural period. However, the deck displacement becomes excessively large when subjected to a ground motion with large intensity or unexpected characteristics. Even in a standard-size bridge a deck displacement reaches 0.5 m under the ground motions developed in the 1995 Kobe earthquake and the 1999 Chi-Chi earthquake. Such a large displacement may result in the higher-than-expected seismic force due to the pounding effect of decks and $P - \delta$ effect¹, hence reducing deck displacement using seismic response control technology is emphasized in this study. In the past studies, structural controls were useful on reducing seismic responses of isolated bridges ²)-⁷). However, only the isolators (sliding isolators ²)-⁴),⁷) and LRB isolators ⁷)) were regarded as either nonlinear elements or hysteretic elements with all the columns being assumed to behave linearly or to be rigid. In reality, the columns may exhibit hysteretic behavior when they are subjected to extreme excitations. It is apparent that the existing studies cannot reveal the real seismic control performance of isolated bridges once the columns start to undergo inelastic range.

Active control systems for civil infrastructures have been studied to mitigate the seismic response effectively ⁸). The recent active control theories for nonlinear or inelastic systems were proposed by Yang et al. ⁹)-¹³). Considering practical applications, the linear quadratic regulator (LQR) optimal control, which is simple and reliable for on-line operations ¹²),¹³), will be used in this study. Thus, the objective is to investigate the control effectiveness of isolated bridges, which exhibit nonlinear behavior at not only the isolators but also the columns under extreme earthquake events, on reducing the deck displacement by using the LQR optimal active control. A typical five-span continuous highway elevated bridge is analyzed. With an actuator exerting the active control force, an extensive parametric study on the weighting matrices of the LQR performance index is carried out to evaluate the control efficiency. It is found that larger control force may cause larger column yielding and render less efficiency on reducing the deck displacement. Hence, the saturation of control force is adopted to prevent inefficient control. Finally, the effect of the active
control is compared with that of the passive control using viscous dampers.

2. ANALYSIS MODEL FOR ISOLATED BRIDGE WITH CONTROL DEVICE

Assuming the deck of a typical isolated bridge is rigid in the longitudinal direction, a column with the effective deck mass on the top can be taken apart as a unit for seismic analysis, as shown in Fig. 1. For study of control effectiveness, the column-isolator-deck system may be idealized as a two-degree-of-freedom lumped-mass system. A control device is set between the deck and the column.

The column and the isolator are assumed here to be perfect elastoplastic and bilinear elastoplastic, respectively, as shown in Fig. 2. The Bouc-Wen hysteretic model\(^\text{14}\) is used for the stiffness restoring force of both the column and the isolator as

\[
F_{si}(t) = \alpha_i k_i x_i(t) + (1 - \alpha_i) k_i x_{yi} v_i(t) \quad (i = c \text{ and } b)
\]

in which the subscripts \(c\) and \(b\) denote the column and the isolator, respectively; \(x_c\) = the column displacement relative to the ground; \(x_b\) = the isolator displacement being the same as the deck displacement relative to the column, \(k_i\) = initial stiffness; \(\alpha_i\) = ratio of the post-yielding to pre-yielding stiffness; \(x_{yi}\) = yield displacement; and \(v_i\) is a nondimensional variable introduced to describe the hysteretic component of the displacement with \(|v_i| \leq 1\), where

\[
\dot{v}_i = \dot{x}_{yi}^{-1} \left[ A_i \dot{x}_i - \beta_i \dot{x}_i |v_i|^n |v_i|^{-1} v_i - \gamma_i |v_i|^{n-1} |v_i|^{n-1} \right] \quad (2)
\]

in which parameters \(A_i\), \(\beta_i\) and \(\gamma_i\) govern the scale and general shape of hysteresis loops, whereas the smoothness of force-displacement curve is determined by the parameter \(n_i\). These parameters are considered time invariant herein.

The equations of motion of the isolated bridge system may be expressed as

\[
M \ddot{x}(t) + C \dot{x}(t) + K_c x(t) + K_f v(t) = \eta \ddot{x}_g(t) + H(t) U(t) \quad (3)
\]

in which \(x = [x_c \ x_b]^T\) is a vector with the relative displacements of the column and the isolator. The deck displacement relative to the ground can be obtained as \(x_c + x_b\). \(v = [v_c \ v_b]^T\) is a hysteretic vector; \(\ddot{x}_g(t)\) is an absolute ground acceleration; \(U(t)\) is the control force generated by the control device; \(M\), \(C\), \(K_c\) and \(K_f\) are mass, damping, elastic stiffness and hysteretic stiffness matrices, respectively; \(\eta\) and \(H\) are the location matrices of the excitation and the control force, respectively. These matrices are given by

\[
M = \begin{bmatrix} m_c & 0 \\ m_d & m_f \end{bmatrix}; \quad C = \begin{bmatrix} c_c & -c_b \\ 0 & c_b \end{bmatrix};
\]

\[
K_c = \begin{bmatrix} \alpha_i k_c & -\alpha_i k_b \\ 0 & \alpha_b k_b \end{bmatrix};
\]

![Fig.1](image1.png) Analytical idealization: (a) analytical unit of column, and (b) 2DOF system.

![Fig.2](image2.png) Material behavior: (a) column – perfect elastoplasticity, and (b) isolator – bilinear elastoplasticity.
\[ K_f = \begin{bmatrix} (1-\alpha_c)k_c x_c & -(1-\alpha_b)k_b x_y b \\ 0 & (1-\alpha_b)k_b x_y b \end{bmatrix}; \]

\[ \eta = \begin{bmatrix} -m_d \\ -m_c \end{bmatrix}; \quad H = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \] (4)

where \( m_c \) and \( m_d \) are the masses of the column and the deck, respectively; \( c_c \) and \( c_b \) are the damping coefficients of the column and the isolator, respectively.

### 3. OPTIMAL ACTIVE CONTROL

The equations of motion by Eq. (3) can be written using a state space formulation as follows

\[ \dot{Z}(t) = g[Z(t), v(t)] + BU(t) + W\ddot{x}_g(t) \] (5)

where the space-state vector \( Z(t) = [x(t) \ \dot{x}(t)]^T \);

\[ g[Z(t), v(t)] \] is a nonlinear function of \( Z(t) \) and \( v(t) \);

\( B \) and \( W \) are the matrices of the control location and the excitation location, respectively. \( g, \) \( B \) and \( W \) are defined as follows

\[ g[Z(t)] = \begin{bmatrix} \dot{x} \\ -M^{-1}[C\dot{x} + K_c x + K_f v] \end{bmatrix}; \]

\[ B = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}; \quad W = \begin{bmatrix} 0 \\ M^{-1} \eta \end{bmatrix} \] (6)

The LQR performance index is given by

\[ J = \int_0^t [Z^T(t)QZ(t) + RU^2(t)]dt \] (7)

in which \( Q \) is a \((4 \times 4)\) symmetric positive semidefinite weighting matrix and \( R \) is a positive weighting scalar. The weighting values should be determined depending on the design performance goals and the constraints on the controller.

Under the constraint of the state equations of motion by Eq. (5), the optimal solution that minimizes the performance index, as shown in Eq. (7), is obtained as

\[ U(t) = -0.5R^{-1}B^T PZ(t) \] (8)

in which \( P \) is the solution of Ricatti equation given by

\[ \Lambda_0^T P + PA_0 - 0.5PBBR^{-1}B^T P = -2Q \] (9)

where

\[ \Lambda_0 = \frac{\partial g(Z)}{\partial Z} \bigg|_{Z=0} \] (10)

Note that the constant Ricatti matrix \( P \) in Eq. (9) is obtained by linearizing the structure at \( Z = 0 \), where the structure is stable, as shown in Eq. (10) and by neglecting the earthquake excitation \( \ddot{x}_g \).

The control force obtained by Eq. (8) can be written as

\[ U(t) = -GZ(t) = -G_c \dot{x} - G_x x \] (11)

where the gain matrix \( G \) is time-invariant. It is noted that civil engineering structures are designed to be stable. The typical closed-loop controller in Eq. (8) for the LQR control maintains structural stability by modifying mainly the controlled system to have larger damping values than the uncontrolled system. For a nonlinear system, it may be more effective to
regulate the active control gain with variation of structural properties. However, in reality, there has been the difficulty to accurately identify the varying structural properties when the structure exhibits high hysteresis. With consideration of practical applications, the control gain remains constant at all times in this study. The stability and effectiveness of applying the same control gain to nonlinear structures will be shown for the case in this study in the latter section.

Additionally, passive control is applied to the same isolated bridge for comparison. A linear viscous damper with a constant damping coefficient, $c_D$, is set between the deck and the column. The passive control system is decentralized so the equations of motion by Eq. (3) is rewritten by

$$\mathbf{M}\ddot{\mathbf{x}}(t) + [\mathbf{C} + \mathbf{C}_D] \dot{\mathbf{x}}(t) + \mathbf{K}_c \mathbf{x}(t) + \mathbf{K}_I \mathbf{v}(t) = \mathbf{n}_D(t)$$

where

$$\mathbf{C}_D = \begin{bmatrix} 0 & -c_D \\ 0 & c_D \end{bmatrix}$$

$$\text{(12)}$$

4. TARGET ISOLATED BRIDGE AND SIMULATIONS

In this study, a typical isolated viaduct designed based on Japan Design Specifications for Highway Bridges, Part V Seismic Design is analyzed to investigate the effectiveness of structural control in terms of peak seismic displacement responses as shown in Fig. 3. The superstructure consists of a five-span continuous deck with a total length of 5@40 m = 200 m and a width of 12 m, which is supported by four reinforced concrete columns with a height of 12 m in each and two abutments. The longitudinal reinforcement ratio is 0.91% and the tie reinforcement ratio (volumetric ratio) is 0.53%. Five high-damping-rubber isolators with a size of 112mm×600mm×600mm (H×B×D) are used per column.

The bridge is idealized as a two-degree-of-freedom lumped-mass system. The effective masses of the deck and the column are perfect elastoplastic and bilinear elastoplastic, respectively. The parameters in Eq. (1) are $k_c = 112.7 \text{ MN/m}$, $\alpha_c = 0$, $x_{yc} = 0.0309 \text{ m}$, $A_c = 1$, $\beta_c = \gamma_c = 0.5$ and $n_c = 95$ for the column, and $k_b = 47.6 \text{ MN/m}$, $\alpha_b = 0.1912$, $x_{yb} = 0.016 \text{ m}$, $A_b = 1$, $\beta_b = \gamma_b = 0.5$ and $n_b = 95$ for five isolators. The first and second natural periods of the isolated bridge with the initial elastic stiffness are 0.86 sec and 0.24 sec, respectively. The damping ratios of the system are assumed to be 2% for both the modes.

Four near-field ground motions are selected as the input excitations. They were measured at JMA Kobe Observatory and JR-Takatori Station in the 1995 Kobe, Japan earthquake, Sylmar Parking Lots in the 1994 Northridge, USA earthquake, and Sun-Moon Lake in the 1999 Chi-Chi, Taiwan earthquake, as shown in Table 1 and Fig. 4. All the excitations are applied at the full intensity for the evaluation of the real seismic performance. Time histories of all the response quantities are computed with 30 seconds of the records except Chi-Chi earthquake with 40 seconds.

An actuator is used to apply control force to the isolated bridge. The weighting matrix $\mathbf{Q}$ in Eq. (7) is assumed as

$$\mathbf{Q} = \begin{bmatrix} Q_{cd} & 0 & 0 & 0 \\ 0 & Q_{bd} & 0 & 0 \\ 0 & 0 & Q_{cv} & 0 \\ 0 & 0 & 0 & Q_{bv} \end{bmatrix}$$

$$\text{(14)}$$

where $Q_{cd}$ and $Q_{bd}$ represent the weighting factors related to the potential energy of the column and the isolator, respectively, while $Q_{cv}$ and $Q_{bv}$ denote the kinematic energy of the column and the isolator, respectively. The performance index $J$ by Eq. (7) is referred to the summation of energy (unit: N-m) with respect to state variables and control force. The unit of $Q_{cd}$ and $Q_{bd}$ corresponding to displacement is N/m, which is the same as the unit of stiffness, while the unit of $Q_{cv}$ and $Q_{bv}$ corresponding to velocity is kg, which is the same as the unit of mass. Accordingly, the unit of $R$ corresponding to control force is m/N. The units of the weighting factors will be omitted for brief expression hereinafter.

As a result of numerical analyses, Fig. 5 shows the control force and the seismic responses of the isolated bridge subjected to Sun-Moon Lake record with and without control. The weighting parameters in Eq. (14) are assumed as $Q_{cd} = Q_{cv} = Q_{cv} = 1$ and $Q_{bd} = 10^2$, and the weighting $R$ in Eq. (7) is varied from $10^{-11}$ to $3 \times 10^{-12}$. The hysteretic loops of the isolator and the column with and without active control are shown in Fig. 6, which demonstrates that the column yields even in the controlled systems. As observed from Fig. 5, the control with $R = 10^{-11}$ demands smaller control force than the control with $R = 3 \times 10^{-12}$. The peak control force is 1.28 MN (22% of the deck weight) and 1.98 MN (34% of the deck weight) for $R = 10^{-11}$ and $R = 3 \times 10^{-12}$, respectively. In the uncontrolled system, the peak deck displacement reaches 0.55 m. It is noted that the column has a residual displacement of 0.1 m. This results in the same magnitude of residual displace-
ment in the deck. Because it is difficult to restore the
deck to the original alignment once a residual drift
occurs, it is required to control the residual drift as
small as possible. Under the control with $R = 10^{-11}$,
the peak deck displacement of the bridge reduces to
0.35 m, which is 64% of that under uncontrolled,
with the residual displacement of 0.1 m. It is obvious
that both isolator displacement and column dis-
placement have substantially reduced. However,
under the control with $R = 3 \times 10^{-12}$, which exerts
larger control force, the peak deck displacement is
0.37 m, 67% of that under uncontrolled, and the re-
sidual displacement is 0.2 m. The deck displacement
does not further decrease, even increases, as the
control force increases. Compared to the control with
$R = 10^{-11}$, the isolator displacement decreases as the
control force increases, but it costs the increase of the
column displacement. The reason will be interpreted
later. **Figure 7** presents the hysteretic loops of con-
trol force and corresponding stroke.

### Table 1 Near-field ground motions.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Station</th>
<th>Epicentral distance (km)</th>
<th>Magnitude</th>
<th>Peak acceleration (m/sec²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyogo-Ken Nanbu</td>
<td>JMA Kobe Observatory</td>
<td>20</td>
<td>7.3</td>
<td>8.18</td>
</tr>
<tr>
<td>January 17, 1995</td>
<td>NS component</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hyogo-Ken Nanbu</td>
<td>JR Takatori station</td>
<td>11</td>
<td>7.3</td>
<td>6.66</td>
</tr>
<tr>
<td>January 17, 1995</td>
<td>EW component</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northridge</td>
<td>Sylmar Parking Lot</td>
<td>16</td>
<td>6.8</td>
<td>5.93</td>
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<tr>
<td>January 17, 1994</td>
<td>NS component</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chi-Chi</td>
<td>Sun Moon Lake</td>
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<td>7.3</td>
<td>9.83</td>
</tr>
<tr>
<td>September 21, 1999</td>
<td>EW component</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**5. PARAMETRIC STUDY OF WEIGHTING MATRICES**

The active control force and the corresponding
structural seismic response depend on the gain matrix
from the Ricatti equation by Eq. (9). The selections
of $Q$ and $R$ will determine whether the control
performance is under the structural or device con-
straints and achieves the “optimum” control effect. A
parametric study on $Q$ and $R$ is conducted herein.

Referring to the seismic response simulation in the
previous section, it is noted that the demand for
control force increases by decreasing $R$. Regarding
the weighting matrix $Q$, the weighting factors $Q_{cv}$
and $Q_{bv}$ with respect to velocity are set equal to unity
and the weighting factors $Q_{cd}$ and $Q_{bd}$ with
respect to displacement will be varied because the
objective of this study is to reduce the peak deck
displacement. In general, the stiffness of the isolator
is much smaller than that of the column so that the
deck displacement results mostly from the isolator
displacement. It is thus more effective to control the
isolator displacement for reducing the deck displacement. Therefore, $Q_{bd}$ is varied as $3 \times 10^{-3}, 1, 3 \times 10^{-1}$ and $10^0$ with setting $Q_{cd}$ being unity to have different levels of control performance. However, because it is found that the response under $Q_{bd} = 3 \times 10^{-3}$ is very close to that under $Q_{bd} = 1$ through numerical simulation, the response under $Q_{bd} = 3 \times 10^{-3}$ will not be discussed herein.

The effectiveness of the controlled system is evaluated in terms of the peak normalized deck displacement $J_d$, the peak normalized isolator displacement $J_b$, the column ductility factor $\mu$ as well as the peak normalized control force $J_U$ as

$$J_d = \max_t |u_d(t)| / \max_t |\dot{u}_d(t)|$$  \hspace{1cm} (15)

$$J_b = \max_t |x_b(t)| / \max_t |\dot{x}_b(t)|$$  \hspace{1cm} (16)

$$J_U = \max_t |U(t)| / W_{deck}$$  \hspace{1cm} (17)

where $u_d(t)$ and $\dot{u}_d(t)$ are the deck displacements under controlled and uncontrolled systems, respectively; $x_b(t)$ and $\dot{x}_b(t)$ are the isolator displacements under controlled and uncontrolled systems, respectively; $U(t)$ is the control force, and $W_{deck}$ is the deck weight.

**Figure 8** shows the peak normalized control force $J_U$ versus the weighting $R$ relation under $Q_{bd} = 1$ and $10^0$. It is observed that the peak normalized control force $J_U$ increases as the weighting $R$ decreases. Although there is a certain scattering depending on ground motions, $J_U$ reaches $40\%$ at around $R = 10^{-13}$ under $Q_{bd} = 1$ and at around $R = 10^{-9}$ under $Q_{bd} = 10^0$, respectively. Because the control force over $50\%$ of the deck weight is not practically feasible, $R > 10^{-14}$ under $Q_{bd} = 1$ and $R > 10^{-10}$ under $Q_{bd} = 10^0$ may be the possible combinations for the design purpose.

**Figure 9** shows how the peak normalized deck displacement $J_d$, the peak normalized isolator displacement $J_b$, and the column ductility factor $\mu$ depend on the peak normalized control force $J_U$ under the control with $Q_{bd} = 1, 10^1$ and $10^0$. Since the control performance under Sylmar and JR Taka-tori records is similar to that under JMA Kobe and Sun-Moon Lake records, respectively, only the results under JMA Kobe and Sun-Moon Lake records are shown in **Fig. 9**.
As observed from the results, at smaller peak normalized control force $U_J$, say $U_J \leq 27\%$ under JMA Kobe record and $U_J \leq 20\%$ under Sun-Moon Lake record, the control effect on reducing the peak deck displacement under $Q_{bd} = 10^3$ and $10^6$ is close to each other and better than that under $Q_{bd} = 1$. Both the deck displacement and the isolator displacement decrease as the peak normalized control force $J_U$
increases in those ranges. It is noted that the column tends to develop extensive hysteretic behavior \((\mu = 8)\) under the uncontrolled system \((J_U = 0)\) subjected to Sun-Moon Lake record. The column ductility factor decreases as \(J_U\) increases under Sun-Moon Lake record, whereas the column ductility does not decrease monotonically as \(J_U\) increases under JMA Kobe record. The peak normalized deck displacement \(J_d\) reduces to 67% at \(J_U = 27\%\) under JMA Kobe record, while \(J_d\) reduces to 65% at \(J_U = 18\%\) under Sun-Moon Lake record.

It is noted however in Fig. 9 that at larger peak normalized control force \(J_U\), say \(J_U > 27\%\) under JMA Kobe record and \(J_U > 20\%\) under Sun-Moon Lake record, the peak deck displacement does not decrease monotonically, even increases, as \(J_U\) increases. Similar to the previous response simulation, although the peak normalized isolator displacement \(J_b\) monotonically decreases as \(J_U\) increases until 40%, large inelastic displacement is conversely induced in the column. Referring to the analytical study of Erkus et al.\(^6\) for an elevated highway isolated bridge of which columns are assumed to remain linear under active control, it has been shown that the displacement of elastomeric isolator monotonously decreases, whereas the column displacement increases moderately and monotonously as the peak control force increases under El Centro, Northridge and Kobe earthquakes. In this study, considering the nonlinearity of the column, once the control force is too large to cause the column to undergo inelasticity, the column displacement increases sharply with increase of the control force. Moreover, high hysteresis of the column may result in irregular control effect on the column displacement under some ground motions, such as Sun-Moon Lake record. Therefore, it is not

![Fig. 9](image-url)  

**Fig. 9** Peak normalized deck displacement, peak normalized isolator displacement and column ductility factor vs. peak normalized control force under: (a) JMA Kobe, and (b) Sun-Moon Lake.
beneficial to increase the control force over the range of large control force for avoiding large column post-yielding displacement. Although the column displacement decreases again with increase of the control force in the interval $33\% < U < 43\%$ under Sun-Moon Lake record, it is unreliable to adopt the control cases in this interval.

6. SATURATED CONTROL

Figure 10 shows an isolated bridge system in its first deflected mode. Positive control force $U$ decreases the isolator displacement while it increases the column displacement, and vice versa. The control force makes a trade-off between the isolator displacement and the column displacement. In other words, although the control force is effective in reducing the isolator displacement, it is also effective in transmitting force between the deck and the column so that it increases the column displacement. However, larger positive control force may hence cause larger column yielding, which in turn increases the column displacement in inelastic range largely. Once the increase of the column displacement surpasses the decrease of the isolator displacement, the larger control force inversely increases the deck displacement. It should be reminded that in the previous section the deck displacement does not decrease monotonically with increase of control force at larger control force levels.

To avoid such ineffective control, it is needed to mitigate large column post-yielding displacement by restricting the control force. However, it is not easy to restrict the control force based on the feedback state due to the complicated nonlinear interaction between the column and the deck. In this study, we try to utilize the saturation strategy, where the control force is clipped under a certain level at all times, to investigate its effect on avoiding large column post-yielding displacement. The concept of saturation started from the inherent limitation of capacity of control devices. Saturation has been taken into account in some control algorithms to consider device limitation. Chase et al. presented an explicit method to guarantee the stability of controlled systems in the presence of actuator saturation. Mongkol et al. developed a control algorithm combining bang-bang control and low-gain linear control to maximize the effect of control device and to avoid the chattering effect. Both the developed control methods have shown promising results. Saturation has also been employed to clip control force in the control algorithms, where the demanded control force generally or occasionally exceeds the feasible capacity of control devices, such as sliding mode control, nonlinear control and LQR control. Such a clipped method is employed herein by

$$U(t) = \begin{cases} U^*(t) & \text{if } |U^*(t)| < U_{\text{max}} \\ \text{sign}(U^*(t)) \cdot U_{\text{max}} & \text{if } |U^*(t)| \geq U_{\text{max}} \end{cases}$$

in which $U^*(t)$ is the optimal control force demanded by Eq. (8) and $U_{\text{max}}$ is a limitation of the control force. It is noted that the LQR control with saturation is not a linear control algorithm any more. Once the control force is truncated, the control force cannot be referred to optimal control force. However, it is important to ascertain that the controlled system under the clipped control force is stable. Chase et al. showed that the closed-loop controlled systems by LQR control still possess stability as the control force is clipped. However, different from the purpose of the previous studies, this study investigates the effect of saturation with various values on avoiding

![Fig.10](imageurl) The influence of the control force on the column displacement and the isolator displacement: (a) positive control force, and (b) negative control force.
large column post-yielding displacement.

**Figure 11** shows how the peak normalized deck displacement $J_d$, the peak normalized isolator displacement $J_b$ and the column ductility factor $\mu$ depend on the peak normalized demanded control force $J_U^*$ under various saturation values. $J_U^*$ is defined as

$$J_U^* = \max_t \left| U^* (t) \right| / W_{deck} \quad (19)$$

Note $J_U^*$ is the peak normalized demanded control force whereas $J_U$ by Eq. (17) is the peak normalized applied control force. The weighting matrix $Q$ in Eq. (14) is here assumed as $Q_{cd} = Q_{cv} = Q_{bd} = 1$ and $Q_{bd} = 10^3$. The weighting $R$ is varied in the range described previously. The saturation $U_{max}$ varies as 10%, 15%, 20% and 30% of the deck weight, corresponding to 16.8%-50.6% of the column flexural yielding strength. The responses of the bridge controlled without saturation ($J_U^* = J_U$) are also presented in **Fig. 11**. The saturation $U_{max}$ influences the control performance only when the peak demanded control force $\max \left| U^* (t) \right|$ is larger than the saturation $U_{max}$.

For larger peak normalized demanded control force $J_U^*$, namely $J_U^* \geq 20\%$, it is seen from **Fig. 11** that the column ductility factor effectively decreases as the saturation of control force $U_{max}$ decreases even though the trend is less straightforward under Sun-Moon Lake record. Inversely, the peak normalized isolator displacement increases with decrease of saturation $U_{max}$. It again shows the trade-off between the isolator displacement and the column dis-

![Figure 11](image-url)
placement. The resulting deck displacement reduces in some saturated cases compared to unsaturated cases. The peak deck displacement \( J_d \) achieves the smallest value under the saturation of \( U_{\text{max}} = 15\% W_{\text{deck}} \) under JMA Kobe record. The column ductility factor achieves the smallest value while \( J_d \) remains nearly constant as \( J_U \) increases under the saturation of \( U_{\text{max}} = 20\% W_{\text{deck}} \) under Sun-Moon Lake record. The saturated control performance under JR Takatori and Sylmar records is similar to that under Sun-Moon Lake and JMA Kobe records, respectively. The saturation of 15\%\( W_{\text{deck}} \) is recommended to prevent the ineffective control, which corresponds to 25\% of the column yielding strength. Under the saturation of 15\%\( W_{\text{deck}} \), the peak deck displacement reduces a maximum of 30\% under all ground motions. Comparing the saturated controllers having \( U_{\text{max}} = 15\% W_{\text{deck}} \) and larger gains (\( J_U > 15\% \)) to the unsaturated controller having \( J_U = 15\% \), the saturated controllers with gains ranging in 15\% < \( J_U < 40\% \) achieve smaller peak deck displacement but slightly larger column ductility factor under JMA Kobe record. The saturated controllers with gains ranging in 15\% < \( J_U < 35\% \) perform almost the identical or smaller peak deck displacement and smaller column ductility under Sun-Moon Lake record. In general, the saturated controller provides better control performance than the unsaturated controller with the same peak control force. It is understandable that the saturation controller applied the maximum control force for longer times. However, the saturated controllers with quite large gains (\( J_U > 40\% \)) are not recommended because the control performance does not further improve. It can be inferred that the bang-bang controller based on the LQR control (\( J_U \rightarrow \infty \)) cannot achieve better performance.

### 7. Comparison of Active Control and Passive Control

Consider the use of passive viscous dampers instead of active actuators. The seismic responses of the same bridge are computed under various damping coefficients \( c_d \).

Figure 12 shows the control force and the seismic responses of the isolated bridge subjected to Sun-Moon Lake record under no control, control with an active actuator, and control with a passive viscous damper. In the active control, the weighting matrix \( Q \) in Eq. (14) is assumed as \( Q_{cd} = Q_{cv} = Q_{bd} = 1 \) and \( Q_{bd} = 10^5 \) for the 
weighting \( R \) is assumed as \( 10^{-11} \), which have been referred previously. For the passive control, the viscous coefficient \( c_d \) is assumed as 758 kN/m/s. It is noted that both the active control and the passive control exert almost identical maximum control force, i.e. around 1.28 MN. As observed from the results, the passive control achieves worse control performance on reducing the peak deck and isolator displacements than the active control. The peak deck displacement decreases to 0.40 m and 0.35 m, under the passive control and the active control, respectively, while the peak isolator displacement decreases to 0.30 m and 0.26 m, respectively. Figure 13 presents the hysteretic loops of the isolator, column and viscous damper under the passive control. The hysteretic loops without control and with the active control have already been showed in Figs. 6 and 7.

Figure 14 shows how the peak normalized damping force changes depending on the damping coefficient \( c_d \). It can be seen that the peak normalized damping force approaches to a certain limit as the damping coefficient increases under all ground motions. This is different from the active control systems as presented earlier.

The peak normalized deck displacement \( J_d \) versus the peak normalized control force \( J_U \) is compared in Fig. 15 between the active control and the passive control, where the damping force is used instead of the active control force as \( U \) in Eq. (8). As observed from the results, at smaller peak normalized control force with \( J_U \leq 20\% \), viscous dampers provide close performance to the active control with \( Q_{bd} = 10^3 \) and \( 10^6 \), and slightly better performance than the active control with \( Q_{bd} = 1 \) under JMA Kobe record, while viscous dampers are less effective than the active control with \( Q_{bd} = 10^3 \) and \( 10^6 \), and more effective than the active control with \( Q_{bd} = 1 \) under Sun-Moon Lake record. It is concluded that viscous dampers can provide close control performance to some active controllers.

In LQR optimal active controller, the control gain \( G \) is time-invariant as shown in Eq. (11). Upon substituting Eq. (11) into Eq. (3), the equations of motion of a controlled structure are described by

\[
[M] \ddot{\mathbf{x}}(t) + [C + HG_x] \dot{\mathbf{x}}(t) + [K_{e} + HG_x] \mathbf{x}(t) + [K_f] \mathbf{v}(t) = \eta_{g}(t) \tag{20}
\]

This implies that the effect of control force leads the modification of structural damping and stiffness. Note that the structural stiffness at each instant is determined by elastic stiffness matrix \( K_e \), hysteretic stiffness matrices \( K_f \) and hysteretic vector \( \mathbf{v} \). For example, when the element \( i \) behaves in inelastic range, the corresponding hysteretic variable \( \nu_i \) is equal to unity so that the element stiffness is equal to \( a_{ii} \). The time-invariant gain vector \( G_x \) in this study yields the same modified magnitude but the
different ratio on pre-yielding stiffness and post-yielding stiffness. For the target isolated bridge, since the yielding strength of the isolator is much smaller than that of the column, the structural dynamic responses may be simply categorized to three conditions: (i) both column and isolator with pre-yielding stiffness, (ii) column with pre-yielding stiffness and isolator with post-yielding stiffness, and (iii) both column and isolator with post-yielding stiffness. Each dynamic response condition has different modified stiffness depending on the weighting matrices. Let us study how the weighting values modify the structural stiffness and damping along with the corresponding

![Fig. 12 Comparison of (a) control force, (b) deck displacement, (c) isolator displacement, and (d) column displacement under active control and passive control subjected to Sun-Moon Lake ground motion.](image)

![Fig. 13 Hysteretic loops under passive control subjected to Sun-Moon Lake record: (a) isolator, (b) column, and (c) viscous damper.](image)
control performance. Corresponding to each condition, the equations of motion by Eq. (20) can be written as

\[
0() \begin{bmatrix} \mathbf{x} \end{bmatrix} + [\mathbf{C} + \mathbf{H} \mathbf{G}] \dot{\mathbf{x}}(t) + [\mathbf{K}_0 + \mathbf{H} \mathbf{G}] \mathbf{x}(t) = \eta \ddot{\mathbf{y}}(t)
\]

(21)

where \( \mathbf{K}_0 \) is the stiffness matrix consisting of pre-yielding stiffness by assuming \( \alpha_1 = 1 \) or post-yielding stiffness by multiplying pre-yielding stiffness by \( \alpha_i \) of the isolator and the column for each corresponding condition. Transforming Eq. (21) into a state space formulation yields

\[
\mathbf{Z}(t) = \bar{\mathbf{A}} \mathbf{Z}(t) + \mathbf{W} \ddot{\mathbf{y}}(t)
\]

(22)

where \( \bar{\mathbf{A}} \) is the modified structural property matrix as

\[
\bar{\mathbf{A}} = \begin{bmatrix}
0 & \mathbf{I} \\
-M^{-1}[\mathbf{K}_0 + \mathbf{H} \mathbf{G}] & -M^{-1}[\mathbf{C} + \mathbf{H} \mathbf{G}]
\end{bmatrix}
\]

(23)

The eigenvalues of \( \bar{\mathbf{A}} \) may be given as

\[
\lambda_{2r-1,2r} = -\zeta_r i \omega_r, \quad (r = 1, 2)
\]

(24)

where \( \omega_r \) and \( \zeta_r \) are the \( r \)th modified modal undamped frequency and damped frequency, respectively; and \( \zeta_r \) is the \( r \)th modified modal damping ratio. It is noted that since the column post-yielding stiffness is null, there is a rigid mode in the condition (iii). Moreover, once an overdamped mode occurs in the controlled system, a conjugated complex pair reduces to two negative real numbers.

**Figure 16** shows the modified undamped period and the modified damping ratio for the condition (i) and the modified damping value for the condition (iii) in the first mode, which dominates the dynamic response, with respect to the peak normalized control force \( J_U \) for the active control and the passive control under Sun-Moon Lake record. The trend of the modified undamped period and the damping ratio for the condition (ii) is similar to that for the condition (i). Since the other records show almost similar features, only the results under Sun-Moon Lake record are presented here. Although the active control gains obtained by linearizing the structure are applied to all the conditions no matter how the response is elastic or inelastic, the modified properties of the controlled system remain stable because all the eigenvalues have negative real values. On the other hand, note that the resulting damping matrices in Eqs. (12) and (20) cannot be uncoupled by the mode-shape matrix.
of the undamped system. Simulation results reveal that the larger the active control gain or the passive damping coefficient, the larger the off-diagonal damping terms in the normal-coordinate equations. Only when the modified damping matrix can be uncoupled by the mode-shape matrix of the undamped system, the modified undamped periods obtained from Eq. (23) remain the same as the undamped system regardless of variation of damping matrix. It can be observed from Fig. 16 that larger viscous damping ($J_U > 30\%$) results in obvious decrease of the modified undamped period of the first mode. In fact, the modified undamped period of the second mode increases inversely, which is not shown herein.

At smaller peak normalized control force with $J_U \leq 30\%$, it is observed that there is less change in the modified undamped period, especially for the passive control and the active control with $Q_{bd} = 1$, but a substantial increase in the damping ratio as the peak normalized control force $J_U$ increases. In particular, the modified undamped periods and damping ratios under $Q_{bd} = 10^3$ and $10^6$ are virtually close. This verifies that the control effect of $Q_{bd} = 10^3$ and $10^6$ is similar to each other as shown in Fig. 9. Likewise, the modified undamped periods and damping ratios of viscous dampers are almost between those of the active control with $Q_{bd} = 1$ and $10^3$ so that the control effect of viscous dampers is also between those of the active control with $Q_{bd} = 1$ and $10^3$. It is noted that the larger modified damping ratio generally leads larger decrease in the peak deck displacement and isolator displacement at smaller control force levels $J_U \leq 30\%$.

8. CONCLUSIONS

The effectiveness of seismic displacement response control for nonlinear isolated bridges, which exhibit inelastic response at both the column and the isolator, was studied. The LQR optimal active control was adopted to study the control performance for a typical five-span viaduct under four near-field ground motions. An extensive parametric study of the weighting matrices as well as various saturation values of control force was carried out to evaluate the control effect on reducing the displacement response. Comparison was made between the active control and the passive control with a constant viscous coefficient. The following conclusions may be obtained from the results presented herein.

(1) The LQR optimal active control is effective in reducing the deck displacement and the isolator displacement at smaller control force levels. The effect is significant on mitigating the column ductility factor even under Sun Moon Lake and JR Takatori records, which likely develop extreme inelastic response in structures.

(2) At larger control force levels, although the isolator displacement further decreases, it potentially results in larger column yielding, which inversely increases the deck displacement. Such a situation may render an ineffective control on the deck displacement. It is preferable to apply the saturation of control force to prevent such an ineffective control unless the control objective is to reduce only the isolator displacement.

(3) Based on an extensive parametric study of the weighting matrices of the LQR active control, larger $Q_{bd}$ has more efficient reduction in the deck displacement, especially under Sun-Moon Lake and JR Takatori records.

(4) By comparison, the passive viscous dampers provide close control effect to some active control cases. Therefore, it is important to select carefully appropriate weighting matrices for the LQR active control to achieve better performance. Although the
passive control may perform as well as the active control under some ground motions, active controls however possess versatility and adaptability, which is capable of reacting to different ground excitations. In this study, the conventional LQR control algorithm was adopted to study the control effect of nonlinear bridges. In the future study, more effective control algorithms or strategies are expected to improve the active control performance for the bridges with nonlinearity on not only the isolators but also the columns.

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