STUDY ON TORSION PROPERTIES OF CARBON FIBER SHEET STRENGTHENED PC MEMBER WITH ZEBRA-SHAPED

Haiming HE\(^1\) and Osamu KIYOMIYA\(^2\)

\(^1\)Member of JSCE, Research Associate, Dept. of Civil Eng., Waseda University (Okubo 3-4-1, Shinjuku-ku, Tokyo 161-0032, Japan) E-mail: hehaiming0830@ybb.ne.jp
\(^2\)Fellow of JSCE, Professor, Dept. of Civil Eng., Waseda University (Okubo 3-4-1, Shinjuku-ku, Tokyo 161-0032, Japan) E-mail: k9036@waseda.jp

In this research, carbon fiber sheet (CFS) substitute to lateral tie was used for torsional strengthening of PC member. To investigate the mechanical properties of CFS strengthened PC member subjected to torsion and torsional reinforcement effects of CFS, monotonic and cyclic torsional loading tests were carried out with specimens which strengthened by lateral ties and unreinforced specimen for comparison. From both monotonic and cyclic loading test results, it was proved that the CFS can increase the ultimate torsional capacity and cracking torque. And it was also confirmed that the CFS had sufficient reinforcement effect, and for CFS strengthened specimen, the cracks of concrete were dispersed and widths were small. Mechanical properties such as torsional rigidity retention, residual twist, resilience and ductility were excellent. As for three-dimension finite element method analysis, analysis results matched the results of monotonic loading tests well and tracked the cyclic torsional loading tests when both nonlinear properties and hysteresis properties of each material were applied. A calculation concept including torsional effective cross-sectional area and thickness of shearing flow for the torsional capacity of CFS strengthened PC member was proposed.

**Key Words** : torsion, torsional reinforcement, CFS, loading test, FEM analysis, torsional rigidity

1. INTRODUCTION

The history of the research on mechanical behaviors of reinforced concrete (RC) member and prestressed concrete (PC) member subjected to torsion or cyclic torsion, especially for PC member, is not long and mechanical properties such as cracks, failure mode are not well known. It is necessary to grasp the mechanical properties and characteristics for RC, PC member subjected to cyclic torsion during earthquakes and so on.

On the other side, as a reinforcement method to the existing RC, PC structure, the application of carbon fiber sheet (CFS) became common. CFS has properties of high tensile strength, light weight and easy execution. CFS is easy to adjust the reinforcement volume whenever necessary and excellent in endurance because the rust will not occur. Actual achievements of research work were increased recently, design and execution specifications\(^3\) were made as well. However, the current design and execution specifications pointed out that [more research needs to be done]\(^1\) and [it is necessary to depend on testable, reliable analysis for design torsional capacity]\(^1\) towards torsional reinforcement used CFS.

Authors have carried out a series of research works\(^2, 3, 4\). And mechanical properties of various types of concrete members were fairly made clear. Based on these previous studies, in this paper, mechanical properties of PC members covered by CFS with zebra shap reinforcement subjected to torsion and cyclic torsion were focused on. Thus both static loading test of pure torsion and finite element method (FEM) analysis were carried out in order to investigate the basic properties of torsion and mechanical behaviors of PC member and PC member which covered by CFS. In the test, the loading method (monotonic or cyclic torsional loading) was used as a parameter to perform a torsional loading test for unreinforced PC member, PC member reinforced by CFS and reinforcing steel bars. Mechanical behav-
iors such as torque-angle of twist relationship, crack characteristics and reinforcement effects of CFS were investigated. Three dimension FEM analysis which considered anisotropy of CFS, nonlinear and hysteresis properties of each material were carried out. The test result was compared and discussed. Based on the test result and analysis result, a torsional capacity calculation method based on shearing flow theory was proposed for the member covered by CFS.

2. TEST SUMMARY

(1) Specimen
There were totally five target specimens in this research, which included one unreinforced specimen, two lateral ties strengthened specimens and two specimens reinforced by CFS with zebra-shaped. The reason of using CFS with zebra-shaped was as follows. In the previous test\(^3\), torsional loading tests of specimens reinforced by CFS with complete wrap or zebra-shaped using same CFS reinforcement volume were carried out. Test results showed that even though the ultimate torsional capacity of specimen reinforced by complete wrap was a little bit higher than the specimen reinforced by zebra-shaped because CFS with complete wrap brought more confined effect, but there was no difference of mechanical properties such as torsional rigidity and ductility. Further more, if CFS with complete wrap used, it was difficult to observe and investigate the damage of the concrete. We believed that zebra type was better than complete wrap type as to investigation and repair work when members had damage by earthquake, etc. CFS reinforcement volume of specimen reinforced at transverse direction had the same tensile rigidity \(A \times E\) (Area of cross section\(\times\)Young’s modulus) as the lateral tie of specimen reinforced by lateral tie and the ratio of tensile strength (Area of cross section\(\times\)tensile strength) was 5.92 in this case. The details of unreinforced specimen, specimen reinforced by CFS and specimen reinforced by reinforcing steel bars were shown in Fig.1, Fig.2 and Fig.3, respectively. A list of specimen including loading method, reinforcing method was shown in Table 1. Loading test was divided into 2 groups. Group 1 was monotonic torsional loading test for unreinforced specimen Co-1, specimen Re-1 which was reinforced by reinforcing steel bars, and specimen CFS-1 which was reinforced by CFS. Group 2 was cyclic torsional loading test for specimen CFS-2 which was reinforced by CFS and specimen Re-2 which was reinforced by reinforcing steel bars. For each specimen, cross section was square with 200mm\(\times\)200mm, the length was 1300mm.

![Fig.1 Unreinforced Specimen (unit mm)](image1)

![Fig.2 Specimen reinforced by CFS (unit mm)](image2)

![Fig.3 Specimen reinforced by lateral tie (unit mm)](image3)

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen No.</th>
<th>Loading Method</th>
<th>Reinforcement Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Co-1</td>
<td>Monotonic</td>
<td>Nothing</td>
</tr>
<tr>
<td></td>
<td>CFS-1</td>
<td></td>
<td>CFS</td>
</tr>
<tr>
<td></td>
<td>Re-1</td>
<td></td>
<td>Lateral tie</td>
</tr>
<tr>
<td>2</td>
<td>CFS-2</td>
<td>Cyclic</td>
<td>CFS</td>
</tr>
<tr>
<td></td>
<td>Re-2</td>
<td></td>
<td>Lateral tie</td>
</tr>
</tbody>
</table>

Table 1 List of test specimens
Introduced prestress was 200kN (5N/mm$^2$ for concrete), which was introduced right before the loading test. To prevent any local failure occurring on loading position, end boxes for each specimen was reinforced by outside steel plate. PC steel bar (SBPR Φ19mm B type No. 1) which introduced prestress was arranged in the middle of the specimen section. Normal concrete with design strength 30N/mm$^2$ was casted and strength tests were carried out. Average compressive strength was 39.4 N/mm$^2$ and average tensile strength was 3.24 N/mm$^2$. For specimen reinforced by reinforcing steel bars, longitudinal reinforcement were 4 D13 (SD295) steel bars and transverse reinforcement were 7 D10 (SD295) steel bars at intervals of 100mm. Table 2 showed the material properties of reinforcing steel bars. For specimen reinforced by CFS, longitudinal reinforcement were 4 D13 (SD295) steel bars and transverse reinforcement were CFS with zebra-shaped at intervals of 100mm. The edges of cross section was not chamfered. As shown in Fig.3, 4 layers of CFS with a width of 48 mm were used for each piece and fiber of CFS was arranged in one direction. The material properties were shown in Table 3.

(2) Loading procedure and measuring items

a) Loading procedure

Fig.4 showed the loading set-up used in the test. Supporting point of bottom end was fixed and the other end was free for torsion. A pure torsional moment was applied through the overhang beam from upper bearing pressure plate to the top end of a specimen. To produce the required torsion, two hydraulic jacks were installed to apply reversal tensile force and compressive force. The load was applied statically with taking care of the amount of stroke of two hydraulic jacks to be almost equal at loading steps. Torsional moment was applied to control in monotonic loading until crack occurred in the specimen. Loading step was around 0.14 kN-m. After that, angle of twist was controlled and loading step was 0.0004rad/m. For the first loop at cyclic loading, loading at both positive and negative directions was kept on until crack was observed in the specimen, and the angle of twist as $\theta^+$ and $\theta^-$ was recorded. From second loop onward, angle of twist was controlled during the loading test, loading was done at speed of 0.0004rad/m. For CFS-2 specimen, the cyclic loading for 4 loops was kept on until the angle of twist reached twice, four times and eight times at both positive and negative directions. After finishing the loading of fourth loop, a large deformation by increasing the angle of twist till 16 times more at both positive and negative directions was applied. However, it was very difficult to observe the whole cracks on the concrete section because of CFS, therefore the exposed concrete part was chosen to observe the crack. For Re-2 specimen, the angle of twist was increased till twice more at both positive and negative directions, and load was applied for second loop until a few new cracks were observed on the specimen. For third loop, a large deformation at both positive and negative directions was applied.

b) Measuring items

The main measuring items of the test were torsional moment, angle of twist and strains of reinforcing bars. Table 2 showed the material properties of reinforcing steel bars. Table 3 showed the material properties of CFS.

<table>
<thead>
<tr>
<th>Reinforcing bar Type</th>
<th>Yield strength (N/mm$^2$)</th>
<th>Tensile strength (N/mm$^2$)</th>
<th>Young’s Modulus (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D10</td>
<td>360</td>
<td>515</td>
<td>2.06×10$^5$</td>
</tr>
<tr>
<td>D13</td>
<td>356</td>
<td>505</td>
<td>1.98×10$^5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio g/m$^2$</th>
<th>thickness (mm)</th>
<th>Tensile strength (N/mm$^2$)</th>
<th>Young’s Modulus (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0.333</td>
<td>3400</td>
<td>2.30×10$^5$</td>
</tr>
</tbody>
</table>
steel bar, CFS, and concrete. The measure method for angle of twist was shown in Fig.5. Aluminum bars were arranged in the 350mm location from the bottom end and 450mm location from the upper end of specimen. Each rotation angle was measured from the difference between the displacement of V1 and V2, and the difference between the displacement of V3 and V4. The angle of twist was determined by the change of the rotation angle per unit length from the differences of these rotation angles.

3. TEST RESULT

(1) Test results of monotonic loading test (group 1 specimen)

a) Torque and angle of twist

Fig.6 showed the relationship between torque and angle of twist for group 1 specimen. For each specimen, the general relationship between torque and angle of twist were explained. Initially linear elastic behavior at a low loading stage was observed. The load was gradually increased up to $P$ point (Co-1 specimen was 11.80kN-m, 0.00574 rad/m; Re-1 specimen was 11.80kN-m, 0.00574 rad/m; CFS-1 specimen was 14.13kN-m, 0.00675 rad/m) where the torque-twist curve was greatly bent. After that, depending on the specimen, the relationship between torque and angle of twist became very different. For Co-1 specimen, it appeared to rapid collapse after reaching $P$ point with the extension of the crack. Then the torsional capacity was rapidly lost. As for Re-1 specimen, torsional moment dropped in a slow pace after reaching $P$ point. Even the twist was over 0.06, it still kept 70% of the maximum torsional moment. It was clear that it had certain residual torsional capacity after the maximum torsional moment. On the other side, for CFS-1 specimen, torsional moment was kept on increasing after reaching $P$ point, even the angle of twist reached 14 times or more than $P$ point. Torsion torsional capacity did not drop. The maximum torsional capacity for Co-1 specimen and Re-1 specimen were 11.80kN-m, 1.52 times and 1.29 times higher than designed torsional capacity, respectively. Compared to these, torsional capacity of CFS-1 specimen was about twice, which was 23.61 kN-m. Apparently CFS had strong reinforcement effects.

b) Cracks and fracture condition

Fig.7 showed the crack details for group 1 specimen. Cracks of Co-1 specimen and Re-1 specimen were observed when the load reached its maximum. The position of the initial crack was a little bit lower than the middle of the specimen. As for Co-1 specimen, with the increase of the load, the crack extended to the other sides of the specimen, eventually the crack connected to the four sides of specimen and the specimen was failure. As for Re-1 specimen, simultaneously with the occurrence of the first crack, many new cracks were occurred through the other sides of specimen with the increase of the load. The cracks extended and the crack connected to the four sides of specimen in ultimate state. As for CFS-1 specimen, it was very difficult to observe the crack on the concrete section because of CFS. For the exposed concrete part, the cracks were more dispersed than Re-1 specimen, and the crack widths were smaller. As for the fracture condition, by the time finishing the loading, torsional moment did not drop,
large damage was not observed on the specimen either. After the stroke of the jack reached its maximum output, load was eliminated and the jacks position was replaced and loading was restarted to destroy the specimen. Photo.1 showed the fracture condition of specimen. There were large damages at the exposed concrete section which located at 200mm-276mm from the bottom of the specimen. Since the width of exposed concrete (76mm) between reinforcing steel plate and CFS was wider than the width of the exposed concrete (52mm) between CFS and CFS, cracks of concrete became wider and was finally exfoliated by the torsional moment. As for CFS, there was no signal stripping or rupture of CFS observed during the whole loading tests.

Next, the cracking torsional moment of group 1 specimen is discussed. There was a possibility that the crack had already occurred on the concrete part covered by CFS at the time when the first crack on CFS-1 specimen was observed. With occurrence of crack on the specimen, torsional rigidity of specimen should be dropped and the slope of torque-angle of twist curve should become moderate. From the fact that cracking torsional moment of Co-1 specimen and Re-1 specimen were near the P point, torsional moment at P point could be treated as common cracking torsional moment of specimen. The torsional moment at P point for each specimen in group 1, Co-1 specimen and Re-1 specimen were 11.80kN-m, CFS-1 specimen was 14.13kN-m. It was clear that cracking torsional moment of CFS-1 specimen was larger. As for Re-1 specimen, when prestress was introduced, concrete was confined by tie hoop. There was restrain effect on core concrete part but this did not affect the covered concrete. Also, before crack occurred, the torsional resistance on the concrete section was dominated and torsional resistance shared by reinforcing bars could be ignored. As a result, the cracking torsional moment of Co-1 specimen and Re-1 specimen became same. As for CFS-1 specimen, when prestress was introduced, the whole section including concrete coverage part was confined by CFS. Also, since CFS was covered on the surface of the specimen, the arm length of torsional moment to specimen central axis became longer, therefore shared torsional resistance of CFS became larger accordingly. As a result, cracking torsional moment increased.

c) Strain of reinforcement material
(lateral tie or CFS)

Fig.8 showed the torque-strain relationship of Re-1 and CFS-1 specimen. Fig.9 showed the angle of twist-strain relationship of Re-1 and CFS-1 specimen. Discussion points was difference between reinforcement materials. For Re-1 specimen and CFS-1 specimen, the strain of middle reinforcement material and bottom reinforcement material were almost same till P point (Explained in section 3 (1) a)). After that (P point), the strain behaviors of reinforcement material for both specimen became different. For Re-1 specimen, even after passing P point (11.80kN-m, 0.00574 rad/m), strain of middle lateral tie dramatically increased, and strain of bottom lateral tie increased moderately. On the other side, for CFS-1 specimen, strain of middle and bottom CFS increased in a similar pace until it reached P point (17.04kN-m, 0.03089 rad/m). After that, strain of bottom CFS became increased dramatically faster than middle CFS. More details were discussed on the difference between these two in the later chapter of comparison of the test results.

(2) Test results of cyclic loading specimen
(group 2 specimen)

Table 4 showed the torsional moment and angle of twist at the unloading point for CFS-2 specimen in
each loop. Torsional moment at unloading point for each loop was almost the maximum torsional moment for each loop.

**a) Torque and angle of twist**

Fig.10 was the diagram which showed the relationship between torque and angle of twist for specimen in group 2. The relationship between torque and angle of twist for first loop (the curve of loading part) of CFS-2 specimen was almost as same as CFS-1, which had been explained previously. From second loop onward, torsional rigidity decreased while torsional moment increased. As for the torsional moment at the point when the crack was observed, the cracking torsional moment in the negative direction was lower than in the positive direction. On the other hand, for Re-2 specimen, torsional moment of third loop was lower than first loop and second loop. And from Fig.10, it was observed that the residual angle of twist for CFS-2 specimen was smaller than Re-2 specimen. It was considered as following two causes: One was that the damage of concrete in CFS-2 specimen was much smaller than in Re-2 specimen. Other was that CFS material had no residual strain subjected to unloading unlike reinforced bars.

**b) Crack and fracture condition**

For the cracks of CFS-2 specimen after finishing the loading, same as CFS-1 specimen, there were many lattice-shaped cracks on the four surfaces of specimen, but the width of cracks was very small and it did not reach a complete failure. For Re-2 specimen after finishing the loading, there were lattice-shaped cracks on four surfaces of the specimen, the distribution of cracks was more concentrated and the widths were wider. It was observed that some part of covering concrete was peeled off. Fig.11 showed the cracks of Re-2 specimen and CFS-2 specimen.

**c) Strain of reinforcement material**

Fig.12 showed the torque-strain relationship for Re-2 specimen and CFS-2 specimen. Discussed point was difference between reinforcement materials. For Re-2 specimen, the strain of the middle lateral tie and bottom lateral tie were almost same in the first loop. From the second loop, the strain behaviors of lateral tie were similar to Re-1 specimen, the strain of middle lateral tie dramatically increased, and the strain of bottom lateral tie increased moderately. For CFS-2 specimen, in first and second loop, strain of the middle CFS and bottom CFS were almost same. From the latter half of third loop, the strain behaviors of CFS were as same as CFS-1 specimen, strain of bottom CFS increased dramatically faster than middle CFS, especially in the positive direction. With strains of CFS in CFS-2 specimen and in CFS-1 specimen, strain of CFS in CFS-2 specimen was larger than that in CFS-1 specimen. In the later chapter of comparison of the test results, more details will be discussed on the difference between these two specimens.

### 4. COMPARISON AND DISCUSSION OF THE TEST RESULTS

#### (1) Discussions on Reinforcement Effects

**a) Monotonic loading**

By the time finish loading, the maximum torsional

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Torsional moment (kN-m)</th>
<th>angle of twist (rad/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>CFS-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>12.89</td>
<td>11.72</td>
</tr>
<tr>
<td>2</td>
<td>13.88</td>
<td>12.32</td>
</tr>
<tr>
<td>3</td>
<td>14.55</td>
<td>13.17</td>
</tr>
<tr>
<td>4</td>
<td>17.46</td>
<td>15.20</td>
</tr>
</tbody>
</table>

![Fig.10](image)

**Fig.10** Torque-twist relationship for group 2 specimen

![Fig.11](image)

**Fig.11** Cracks on group 2 specimen

![Fig.12](image)

**Fig.12** Torque-strain curves of CFS-2 and Re-2 specimen
moment for Co-1 specimen and Re-1 specimen was 11.80kN-m and it was 23.61kN-m for CFS-1 specimen. As explained earlier, even though the maximum torsional capacity of Re-1 specimen was as same as Co-1 specimen, mechanical properties such as ductility, residual torsional capacity were increased substantially. On the other hand, the torsional moment of CFS-1 specimen did not decrease and maximum torsional moment reached twice higher. The reinforcement effect by CFS was large. Resistance mechanism to torsional moment for specimen covered by CFS had a big difference to specimen reinforced by reinforcing bars. The function of the covering concrete was different accordingly. Here, the difference of strain behaviors for reinforcement material were discussed. When a torsional moment acted on specimen which was reinforced by reinforcing bars, cracks occurred on the surface of concrete when principal tensile stress reached the tensile strength of concrete and the position was in the middle of the specimen. For Re-1 specimen, crack was observed when the load was near the torsional moment at P point, the position was a little bit lower than middle of the specimen. Torsional rigidity became decreased because of the occurrence of the crack and cracks kept on extending. Then cracks went through the covering concrete and finally reached lateral tie. After that, the extension of cracks was restrained by middle lateral tie and lateral tie shared the principal tensile stress which was generated by torsional moment. As described in section 3 (1) c), after passing P point (11.80kN-m, 0.00574 rad/m), strain of the middle lateral tie increased faster than the strain of bottom lateral tie. On the other side, for specimen covered by CFS, it was restrained by CFS instantly after crack occurred and CFS shared the principal tensile stress from torsional moment. As a result that observed in section 3 (1) c), between crossing P point (14.13 kN-m, 0.00675 rad/m) and reaching P’ point (17.04 kN-m, 0.03089 rad/m), strain of middle and bottom CFS still increased in same pace despite of some slips at CFS to concrete. After crossing P’ point (17.04 kN-m, 0.03089 rad/m), the strain of bottom CFS increased faster than strain of middle CFS because CFS reinforcement volume in the place of 200mm-300mm from the bottom of CFS-1 specimen was only half of others. There were cracks occurred in this part and the strain there increased much faster than middle CFS.

b) Cyclic loading
For the reinforcement effects of cyclic loading specimen, Fig.10 showed the torque-angle of twist curve for each specimen. From the beginning to the end, specimen which reinforced by CFS showed higher torsional capacity than specimen which reinforced by reinforcing bar. From third loop and onward, the torsional capacity of specimen which was reinforced by reinforcing bar decreased. The other hand, the torsional capacity of specimen which reinforced by CFS did not decrease, it had higher torsional capacity than specimen which reinforced by reinforcing bar, and it had strong ductility and resilience.

The mechanical interpretation for strain behaviors of reinforcement material which described in section 3 (2) c) was explained in section 4 (1) a).

(2) Discussions on test result based on the loading method difference
a) Torsional moment
Fig.13, Fig.14 showed the relationship between torque and angle of twist for specimen (Re-1, Re-2) and for specimen (CFS-1, CFS-2) at monotonic loading and cyclic loading, respectively. Torsional moment in cyclic loading test was generally lower than in monotonic loading test at same angle of twist. For torsional moment decrease rate at unloading point for each loop, Re-2 specimen was 0.893, 0.852 and 0.903, respectively and CFS-2 was 0.931, 0.944, 0.912, 0.934, 0.996, respectively. It was apparent that the torsional moment of the specimen reinforced by reinforcing bar was lower than the specimen reinforced by CFS.

b) Strain of reinforcement material
Fig.15 showed the angle of twist-strain relationship of Re-1 specimen and Re-2 specimen. Fig.16 showed the angle of twist-strain relationship of CFS-1 and CFS-2 specimen. With the strain of reinforcement material in monotonic loading and in cyclic direction loading test, strains of middle lateral tie for lateral tie reinforced specimen were almost same at the same angle of twist. On the other side, for CFS strengthened specimen, strains of middle CFS at the same angle of twist were almost the same until the latter half of third loop, after that, strains of middle CFS in cyclic loading test was larger than in monotonic loading test. Because the damage of specimen such as cracks in cyclic loading test was larger than in monotonic loading test, especially at the covering concrete. As the cracks on covering concrete was restrained by CFS, the strains of CFS were greatly influenced by cracks on covering concrete, therefore the strains of the CFS in cyclic loading test were larger than in monotonic loading test with increasing of the loading. It was indicated that CFS shared torsional moment more effectively.

(3) Discussions on torsional rigidity
Fig.17 showed the definition of torsional rigidity of cyclic loading specimen in each loop. To obtain tor-
sional rigidity of each loop, the point where the torsional moment was 0 and the point where it was 1/3 of the maximum torsional moment (if it was negative direction, absolute value was used) were connected to calculate the slope at each loading step based on the relationship between torsional moment and the angle of twist. Torsional rigidity ratio was defined by the ratio of torsional rigidity for each loop \( G_k \) to initial torsional rigidity \( G_{k0} \) (torsional rigidity of first loop). Fig.18 showed the relationship between torsional rigidity ratio and the numbers of loops. Decrease rate of the torsional rigidity ratio for specimen reinforced by CFS was much lower than that of the specimen reinforced by reinforcing bar.

5. FEM ANALYSIS

FEM analysis for the member subjected to torsion was more difficult and complicated rather than that of a PC member subjected to bending and shear. It was must to perform three-dimension analysis for a PC member subjected to torsion. The effect based on two-axle stress of compression-tension and material properties of CFS such as anisotropy, different behavior in tension and compression, and bond effect to concrete must be considered. For the FEM analysis of a PC member subjected to cyclic torsion, in addition to the above, the hysteresis property of stress-strain relationship at the time of loading, unloading and reloading must to be considered.

(1) Model of concrete

For concrete stress-strain relationship, the constitutive model based on total strain was used. Cracks of concrete was modeled as smeared cracking model and the fixed crack model based on total strain was adopted for concrete.

For concrete, many models are existed or can be implemented to simulate the nonlinear behavior of concrete. In this research, both monotonic loading and cyclic loading test should be simulated. But existed models in many FEM programs do not consider the hysteresis properties of concrete or secant unloading-reloading is adopted, for instance, origin...
orientation hysteresis model. And therefore residual strain of concrete is neglected. These models are not suitable for simulate the cyclic loading test.

In this research, model of concrete proposed by Maekawa, et al.\(^6\) (so-called Maekawa model) was adopted to simulate the monotonic loading and cyclic loading test. The attractive points of the Maekawa concrete model are that it is defined by engineering parameters such as the tensile and compressive strength and the fracture energy, and that it covers all loading situations. In Maekawa model, not only the tensile and the compressive stress-strain relationship of concrete is defined, but also hysteresis in tensile and compressive unloading-reloading loops is considered. The outline of Maekawa model was shown in Fig.19.

a) Tension model

The modeling of tensile behavior of concrete after cracking is expressed as:

\[
\sigma = f_t \times (\varepsilon_{\text{tu}}/\varepsilon)^\nu
\]

where \(\sigma\) the average tensile stress, \(\varepsilon\) the average tensile strain, \(f_t\) the tensile strength, \(\varepsilon_{\text{tu}}\) the cracking strain and \(c\) the stiffening parameter. \(c=0.2\) was applied for CFS strengthened specimen (confined effect of CFS is considered) and \(c=0.4\) was applied for lateral ties strengthened specimen.

b) Compression model

Compression side of Maekawa model is based on elastic-plastic theory. The fracture parameter of this model in following equation (2) is derived from two dimensional and three dimensional cyclic loading data. Four material parameters (\(K, F, H,\) and \(D\)) are used for concrete with normal aggregate and strength ranging from 15 MPa to 50 MPa\(^6\).

\[
K = K(F) = \exp \left[ - \frac{F}{3.25} \left( 1 - \exp \left( - \frac{F}{0.8} \right) \right) \right] \tag{2}
\]

\[
F = F(I_{1c}, J_{2c}, J_{3c}) = \frac{\sqrt{3} J_{2c}}{0.23 \varepsilon_0 - \sqrt{3} I_{1c}} \times \frac{1}{5} \left( \frac{3 \sqrt{3}}{2} \left( \frac{J_{2c}}{J_{3c}} \right) + 6 \right) \tag{3}
\]

\[
H = H(J_{2c}) = \frac{9}{10} \varepsilon_0 \left( \frac{J_{2c}}{\varepsilon_0} \right)^3 \tag{4}
\]

\[
D = D(I_{1c}, K) = \left\{ \frac{-1 + 2\nu}{2} \sqrt{3} K^2 + \frac{\sqrt{2} I_{1c} + 0.38 \varepsilon_0}{0.28 \varepsilon_0} (1 - 4 K^2) \right\} \tag{5}
\]

Scalars \(I_{1c}, J_{2c}, J_{3c}\) respectively are the first, second and third elastic strain invariants. The material constant \(\varepsilon_0\) adopted as a function of the compressive strength \(f_c\), Young’s modulus \(E\) and Poisson’s ratio \(\nu\).

\[
\varepsilon_0 = 1.6(1 + \nu) \frac{f_c}{E} \tag{6}
\]

Detail information is described in reference \(^6\).

c) Hysteresis model

The Maekawa concrete model checks in the total strain directions whether the total strain is in unloading-reloading or in loading toward new extreme tensile or compressive values. In case of unloading and reloading conditions, the hysteresis behavior is defined as follows:

- **Compressive unloading** (\(\varepsilon > \varepsilon_c, \varepsilon > \varepsilon_0, \varepsilon < 0\))

\[
\sigma = K(E - \varepsilon_0) \alpha \tag{7}
\]

\[
\alpha = K^2 + \frac{\sigma_e}{K(E \varepsilon_0 - \varepsilon_p)} (E - \varepsilon_p)^2 \tag{8}
\]

- **Compressive reloading** (\(\varepsilon > \varepsilon_c, \varepsilon \leq \varepsilon_0, \varepsilon < 0\))

\[
\sigma = \sigma_c - (\sigma_c - \sigma_b) \frac{\varepsilon_c - \varepsilon}{\varepsilon_c - \varepsilon_0} \tag{9}
\]

- **Tensile unloading** (\(\varepsilon < \varepsilon_c, \varepsilon < \varepsilon_0, \varepsilon > 0\))

\[
\sigma = E(\varepsilon - \varepsilon_0) \alpha + \sigma_b \tag{10}
\]

\[
\sigma_b = -f_t \left\{ 0.05 + 0.15 \frac{E}{5} \right\} \tag{11}
\]

\[
\alpha = \left\{ \frac{\sigma_0}{E(\varepsilon - \varepsilon_0)} \right\} \frac{\varepsilon - \varepsilon_0}{\varepsilon_c - \varepsilon_0} \tag{12}
\]

- **Tensile reloading** (\(\varepsilon < \varepsilon_c, \varepsilon \geq \varepsilon_0, \varepsilon > 0\))

\[
\sigma = \sigma_c - (\sigma_c - \sigma_b) \left( \frac{\varepsilon_c - \varepsilon}{\varepsilon_c - \varepsilon_0} + \sigma_b \right) \tag{13}
\]

Where \(\varepsilon\) is the actual total strain and \(\sigma\) is the corresponding stress, \(\varepsilon_p\) is the plastic strain, \(\varepsilon_0\) is the total strain at begin of increment, \(\sigma_0\) is the corresponding stress, \(\varepsilon_c\) is the maximum tensile strain ever experienced, \(\sigma_c\) is the corresponding stress, \(f_t\) is the tensile strength, \(E\) is the Young’s modulus and \(K\) is the damage parameter.

d) Shear stress transfer

Shear stiffness is usually reduced after concrete cracking and shear stress is physically transferred across crack faces due to aggregate interlock and dowel action.

In smeared cracking model, following equations are used and expressed as:

\[
\sigma
\]

\[
\varepsilon
\]

Fig.19 Outline of the Maekawa model for concrete
$$[D_{cr}] = \begin{bmatrix} \beta_n E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & \beta_t G_0 \end{bmatrix}$$  (14)

$$\sigma_{cr} = \alpha' \times f_i$$  (15)

Where, $\beta_n$ and $\beta_t$ are the reduction factor at normal direction and shear direction of crack, respectively, $\alpha'$ is the soften reduced factor as expressed in formula (1), $E$ and $G_0$ are the Young’s modulus and shear modulus before cracking, respectively.

$\beta_n$ is usually set as 0, therefore the shear transfer across crack faces depends on $\beta_t$. $\beta_t$ is so-called shear retention factor varied from 0 to 1. $\beta_t$ is usually adopted as a constant value from cracking to ultimate stage. For CFS strengthened specimens, cracks were restrained by CFS instantly, and the widths and depths of cracks were small because the extending of cracks were restrained by CFS. To improve the precision of analysis results through the whole loading process, variable shear retention factor depended on shear strain was implemented. Fig.20 showed the relationship between shear retention factor and shear strain. From $\gamma_0$ (shear strain at cracking) to $\gamma$ (500$\mu$), $\beta_t$ was decreased linearly and $\beta_t$ was adopted as a constant value from $\gamma$ and onward. Influence of varied shear retention factor on structural response was compared and discussed.

(2) Material model of CFS

Tensile and compressive behavior of CFS like a fabric. Namely it can bear high tensile stress but almost can not bear compressive stress. Therefore Young’s modulus $E$ is $2.30 \times 10^5$ N/mm² and tensile strength is 3400N/mm² in tensile field and a very small Young’s modulus was adopted in compressive field (showed in left of Fig.21). CFS is an anisotropic material, carbon fiber was arranged in one direction (transverse direction of specimen), therefore a very small Young’s modulus (showed in right of Fig.21) was modeled at other directions.

(3) Interface model between CFS and concrete

Between the concrete and CFS, quadrilateral interface element was modeled. Fig.22 showed the stress-relative displacement relations of interface elements. Left of figure was a model of normal direction and right of figure was a model of shear direction. In normal direction, stress-relative displacement relationship was adopted as bilinear properties. $\sigma_{max}$ was adopted as 43.2MPa as the tensile strength of adhesive material, $-\sigma_{cmax}$ was -93.1MPa as the compressive strength of adhesive material. These values are larger than those of concrete. In shear direction, the rigidity was almost zero after reaching maximum values. The values of interfacial shear strength, the followings have been reported: 3.8MPa,$^9$ 5.0MPa,$^9$ 5.4MPa$^{10}$, and 8.0MPa$^{11}$. In this study, the bond stress between the polymer and concrete $\tau_{max}$ and $-\tau_{max}$ were varied as 3, 4, 5, 6, 7, and 8MPa, and their influence was compared. $k_n$ and $k_t$ were adopted as 100 MPa/mm$^{12}$. In cyclic loading condition, linear unloading and reloading paths which intersect the origin were adopted.
(4) Material model of reinforce bars and PC bars
The relationship between stress and strain of reinforcing bars and PC bars were shown in the left side of Fig. 23 as bilinear model. Elastic Young’s modulus $E$ was adopted until reaching yield strength $\sigma_y$, after that, increasing stress by rigidity $E_T$ was $1/100$ of Young’s modulus. The hysteresis relationship between stress and strain of reinforcing bar during unloading and reloading was shown in the right side of Fig. 23.

(5) Analysis model
Fig. 24 showed the mesh of the analysis model for CFS strengthened specimen. Commercial finite element program DIANA was used for analysis. The elements used in the model were 400 solid elements (5mm×5mm×5mm) for concrete, 16 truss elements for PC steel bar, 280 shell elements for CFS and anchor plate which was connected with reinforcement steel plate and PC steel bar, 52 embedded reinforcement elements for reinforcing steel bar. As same as tests, the bottom (200mm) of specimen in the analysis model was fixed at X, Y, Z directions. The upper (300mm) of analysis model was applied for monotonic loading analysis. These values were determined based on internal energy and tolerance for convergence was 0.0001.

6. COMPARISONS OF ANALYSIS RESULT AND TEST RESULT

(1) Monotonic loading test
Fig. 25 showed the final analysis results. The agreement between analysis and test results were very well. The shear retention factor $\beta$ and the bond stress $\tau_{\text{max}}$ (for CFS-1 specimen, $\beta=0.5$, $\tau_{\text{max}}=4$MPa; for Re-1 specimen, $\beta=0.03$) was implemented for monotonic loading analysis. These values were determined based on the following discussion: For CFS-1 specimen, varied shear retention factor $\beta$ and varied bond stress between the polymer and concrete was applied. For Re-1 specimen, only varied shear retention factor $\beta$ was applied. These values were also implemented for cyclic loading analysis.

Fig. 26 showed the torque-angle of twist curve of test result and analytic result for CFS-1 specimen with various value of bond stress $\tau_{\text{max}}$ when the shear retention factor $\beta$ was varied from 0.1 to 0.9. When $\beta=0.03$ was applied, the analysis results corresponded the test results well. Analysis results proved that the shear retention of CFS strengthened specimen was much higher than the lateral ties strengthened specimen and showed the importance of the choice of shear retention factor for specimen subjected to torsion. For CFS strengthened specimen, higher shear retention factor (in this case, $\beta=0.5$) could be applied for FEM analysis because the extension of cracks was restrained by CFS and the exfoliation of covering concrete was prevented due to hoop effect. And for lateral ties strengthened specimen, lower shear retention factor (in this case, $\beta=0.03$) could be applied.

Fig. 28 showed the torque-angle of twist curve of test result and analytic result for CFS-1 specimen with various value of bond stress $\tau_{\text{max}}$ when the shear retention factor $\beta$ was set as 0.5. Analysis results showed that the bond stress $\tau_{\text{max}}$ influenced the torque-angle of twist curve of specimen greatly. For CFS-1 specimen, $\beta$ varied from 0.1 to 0.9, analysis results were in good agreement with test results when $\beta=0.5$ or $\beta=0.7$ was applied and the other values underestimated or overestimated the test results. For Re-1 specimen, $\beta$ was varied from 0.01 to 0.1. When $\beta=0.03$ was applied, the analysis results corresponded the test results well. Analysis results showed that the bond stress $\tau_{\text{max}}$ influenced the torque-angle of twist curve of specimen greatly.
angle of twist curve of specimen a little when bond stress reached the maximum bond stress. The bond stress did not influence the torque-angle of twist curve of specimen at ultimate stage. After the shear stress of interface element reached its maximum shear stress, the shear stress of interface element was released and debonding was occurred between the CFS and concrete (defined as Fig. 21). Then torque-twist curve showed a little drop behavior at that time (zoomed part was shown in Fig. 28). Fig. 29 showed test result and analytic results for strain of middle CFS in CFS-1 specimen. Also slip behavior was calculated and the range of slip was wider with the higher $\tau_{\text{max}}$. From Fig. 28 and Fig. 29, when $\tau_{\text{max}}=4\text{MPa}$ or $\tau_{\text{max}}=5\text{MPa}$ was applied, the analysis results showed better agreement with test results.

Fig. 30 showed the pattern of crack for CFS-1 specimen. Upper figure was the pattern of crack when crack started occurring and the torsional moment was 10.56kN-m, lower figure was the pattern of crack when the torsional moment was 17.40kN-m. Just like in loading test, the cracks were occurred at the center of specimen’s four surfaces, then extended to the other sides with the increase of torsional moment. The development of cracks during the tests was also simulated well.

(2) Cyclic loading test

Fig. 31, Fig. 32 showed the torque-angle of twist curve of test result and analysis result for CFS-2 specimen and Re-2 specimen, respectively. As showed in the figure, the analysis result accurately matched the test result including the residual displacement.

From Fig. 31 and Fig. 32, as same as observed from test, residual angle of twist of CFS-2 specimen was smaller than that of Re-2 specimen. Fig. 33 showed the test and FEM analysis results of relationship between torsional rigidity ratio and loops. From Fig. 33, the analysis results simulated the test results well. Decrease rate of the torsional rigidity ratio for specimen reinforced by CFS was much lower than the specimen reinforced by reinforcing bar in both test and calculation.

7. DISCUSSIONS ON TORSIONAL CAPACITY CALCULATION

There is no reliable design method for torsional reinforcement of PC member covered by CFS. Referring to the test results and analysis results, modified calculation formula of torsional capacity for PC member covered by CFS was proposed. According to the shearing flow theory, the solid cross section member subjected to torsion can be replaced by a hypothetical thin-wall box section with thickness $t$, assumed that torsion is resisted by shearing flow which flowed into the hollow section and calculation can be done on the torsion of thin-wall box section. The area enclosed by the centerline of the thin-wall

![Fig. 27 Torque-twist relationship for Re-1 specimen with various value of shear retention factor $\beta$](image)

![Fig. 28 Torque-twist relationship for CFS-1 specimen with various value of bond stress $\tau_{\text{max}}$](image)

![Fig. 29 Twist-strain curves of middle CFS for CFS-1 specimen with various value of bond stress $\tau_{\text{max}}$](image)

![Fig. 30 Pattern of cracks for CFS-1 specimen (FEM result)](image)
section (the centerline of the shear flow) is the torsional effective cross-sectional area. For PC member (shows in Fig. 34), this area is assumed to be the area enclosed by the centerline of the transverse reinforcement\(^9\). In this case, the torsional effective cross-sectional area is \(A_0=b_0 \times h_0\). For CFS strengthened PC member (shows in Fig. 35), external circumference of shearing flow is the surface of specimen because CFS is covered on the surface of specimen. In this case, the torsional effective cross-sectional area \(A_0=(b-t)(h-t)\). However thickness of CFS is ignored because of very thin thickness.

For CFS strengthened PC member, calculation formula of torsional capacity was shown as below.

\[ T = 2qA_0 \]  
\[ A_0 = (b-t)(h-t) \]  
\[ q = \tau t = \sqrt{q_f q_w} \]  
\[ q_w = q_{tw} = \frac{f_{yd}}{s} \]  
\[ q_l = \sum A_d \cdot f_{yd} \cdot (2(b+h-2t)) \]

Where, \(b\) was the width of section of specimen, \(h\) was the height of section of specimen, \(t\) was the thickness of shearing flow, \(\tau\) was the shearing stress, \(A_{tw}\) was the area of single CFS, \(\sum A_d\) was the area of the longitudinal reinforcing bars, \(f_{sd}, f_{yd}\) were design yield stresses of CFS and longitudinal reinforcing bars, respectively, \(s\) was the longitudinal spacing of CFS.

The following evaluation formula\(^13\) was used for calculating valid thickness \(t\) of shearing flow.

\[ t = 2.79 \frac{\sqrt{q_f q_w}}{f'_c} \]  
\[ q' = \sum A_d \cdot f_{yd} / (2(b+h)) \] and \(f'_c\) was the compression strength of concrete.

This formula considers the peeling off of covering concrete and ignored the torsional moment shared by covering concrete at the ultimate state for the RC member subjected to torsion. At that time, the centerline of the transverse reinforcement was considered as external circumference of shearing flow. This formula was suggested from the relationship with experimental data. \(t\) was calculated from formula (19), based on formula (16), (17) and (18), torsional capacity \(T_{\text{cal}}\) was calculated. In the calculation of torsional capacity, However, when one of \(q_l\) and \(q_w\) was dominant, the reinforcement up to 1.25 times to the lesser was considered as torsion reinforcement. For all the calculations, \(q_w=1.25 \times q_f\). Yield strength was used for strength of reinforcing bar.
Table 5 showed the calculated results of torsional capacity and test results (Details of No. 4 and No. 5 specimen were reported in reference 3). Calculated results generally matched the test results. The calculated result of CFS-1 specimen was lower than other two specimen by compared the respective test result. The reasons could be considered as following two: One was the influence of reinforcement volume of CFS. In calculation of torsional capacity for specimen, the larger one of \( q_l \) and \( q_w \) was limited till 1.25 times of the smaller one. For all calculations, \( q_w = 1.25 \times q_l \) was adopted, but for CFS-1 specimen, \( q_w \) was predicted to be more larger than 1.25\( q_l \) (\( q_w = 10.75 \times q_l \) for CFS-1 specimen, \( q_w = 3.68 \times q_l \) for No. 4 specimen and \( q_w = 1.84 \times q_l \) for No. 5 specimen). The other was the influence of the introduced prestress. In calculation of torsional capacity for specimen, influence of prestress was ignored. But some researches showed that the torsional capacity increases with the higher prestress\(^{14}\). Because introduced prestress for CFS-1 specimen was higher than other two (5N/mm\(^2\) to concrete for CFS-1 specimen and 1N/mm\(^2\) to concrete for No. 4 and No. 5 specimen), therefore B/A was smaller than expected.

### 8. SUMMARY AND CONCLUSIONS

Following conclusions were obtained through loading tests, finite element method analysis and calculation of theoretical torsional capacity.

1. From the results of loading tests, torsional strengthening by using CFS increased the ultimate torsional capacity and the cracking torsional capacity. The cracks of concrete were restrained by CFS, the widths of cracks were smaller, distributing of cracks was dispersed and the exfoliation of covering concrete was prevented. It was confirmed that mechanical properties such as torsional rigidity retention, resilience and ductility were excellent, and residual angle of twist for CFS strengthened specimen subject cyclic torsion was smaller than those of RC member.
2. The resistance mechanism of CFS strengthened specimen subjected to torsion was different from the specimen which was reinforced by lateral tie at the inner side of the covering concrete. The part of the covering concrete was able to resist effectively because the cracks of the covering concrete were restrained by CFS and shear stiffness retention of concrete was high after concrete cracked. Also, since CFS was covered at the surface of specimen, the arm length of torsional moment to specimen central axis became longer, therefore the resistance to the torsion shared by CFS became more effective.

3. In the analysis of finite element method, the nonlinear properties of materials, anisotropic properties of CFS, bond mechanism between concrete and CFS and hysteresis properties of materials were taken into consideration. The mechanical behaviors of PC member strengthened by CFS subjected to torsion, such as skeleton curve of torque-twist, cracks pattern, strains of CFS and bond-slip behaviors between CFS and concrete were nearly quantitatively evaluated both at monotonic loading and cyclic loading tests.
4. Shear retention of concrete after concrete cracked influenced the analysis result greatly. For CFS strengthened specimen, higher shear retention factor and for lateral ties strengthened specimen, lower shear retention factor was appropriate for FEM analysis, respectively.
5. In FEM analysis, bond stress between concrete and CFS influenced the strain behaviors of CFS, but that hardly influenced to skeleton torque-angle of twist curve of specimen.
6. In calculation of torsional capacity based on shearing flow theory, external circumference of shearing flow was adopted to be the surface of the specimen. This concept can explain the resistance mechanism of CFS strengthened specimen.

### ACKNOWLEDGMENT

In finite element method analysis part, the writers would like to gratefully acknowledge Akihoro Nakai, postgraduate student of Waseda University (JIP Techno Science Corporation), for providing the full supports of user-supplied subroutines of Maekawa model, etc in DIANA program.

### REFERENCES

4. He, H.M. and Kiyomiya, O. : Study on mechanical properties of PC members subjected to axial force and cyclically torsion,

(Received June 12, 2007)