The Finite Order Multivariate Normal Universal Portfolio

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Abstract

The finite order multivariate normal universal portfolio is studied in this paper with the objective reducing the implementation time and computer-memory requirements substantially. The finite-order portfolios are run on some selected stock-price data sets from the local stock exchange. The wealths achieved over a modest length of time are recorded. Empirically, the performance of the finite-order multivariate normal universal portfolio is comparable to that of the Dirichlet universal portfolio and yet requiring substantially shorter implementation time.

1. Introduction

The uniform universal portfolio[1] and its generalization, the Dirichlet universal portfolio [2] have been studied by Cover and Ordentlich. In this paper, the multivariate normal universal portfolio is introduced and studied. For simplicity we study the finite order version of the multivariate normal universal portfolio. The implementation of the moving order Dirichlet universal portfolio requires a substantial amount of computer memory growing exponentially fast as a function of number of trading days. To overcome this problem, the memory – saving multivariate normal universal portfolio of finite order is introduced.

2. Universal Portfolios of order ν Generated by the Multivariate Normal Distribution

Let ν be a positive integer. Let $Y = (Y_1, Y_2, ..., Y_m)$ be a random vector having a joint multivariate normal probability density function $f(y_1, ..., y_m)$ defined over $B$, where $B = \{(y_1, ..., y_m): -\infty < y_i < \infty, i = 1, ..., m, f(y_1, ..., y_m) > 0\}$, where
$f_\gamma(y) = \frac{1}{(\sqrt{2\pi})^m |K|^{1/2}} e^{-\frac{1}{2}(y-\mu)'K^{-1}(y-\mu)}$, \\

$K$ is the covariance matrix of $Y$, $\mu = (E(Y_1), E(Y_2), \ldots, E(Y_m))$ is the mean vector. We say $Y$ has the multivariate normal distribution $N(\mu, K)$, $\cdot | \cdot$ means determinate.

The universal portfolio $\hat{b}_{n+1,k}$ of order $\nu$ generated by the joint p.d.f $f_\gamma(y)$ is defined as:

$$\hat{b}_{n+1,k} = \frac{\int y_k \left( y' x \right) \left( y' x \right) \ldots \left( y' x \right) f_\gamma(y) \text{d}(y) \right)}{\int \left( y_1 + \ldots + y_m \right) \left( y' x \right) \ldots \left( y' x \right) f_\gamma(y) \text{d}(y)}$$

(1)

for $k = 1, 2, \ldots, m$; $y = (y_1, y_2, \ldots, y_m)$, $\nu = 0, 1, 2, \ldots$. The theory of universal portfolios of order $\nu$ generated by probability distributions is due to Tan[3].

If $Y \sim N(\mu, K)$, we say (1) is the multivariate normal universal portfolio of order $\nu$.

Similarly, if $Y$ has the Dirichlet joint p.d.f., we say that (1) is the Dirichlet universal portfolio of order $\nu$. In this paper, the Dirichlet universal portfolio is referred to order $\nu = n$ or the moving order universal portfolio.

3. Empirical Results

We use the modified algorithm of Chan[4] for computing the three-stock universal portfolio generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution where $\alpha_i > 0$ for $i = 1, 2, 3$.

We have run the parametric family of Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ universal portfolio on 3 stock data sets chosen from the Kuala Lumpur Stock Exchange. The period of trading of the stocks selected is from January 1, 2003 until November 30, 2004, consisting of 500 trading days. Each data set consists of 3 company stocks. Set A consists of the stocks of Malayan Banking, Genting and Amway(M) Holdings. Set B consists of the stocks of the stocks of Public Bank, Sunrise and YTL Corporation. Finally, set C consists of the stocks of Hong Leong Bank, RHB Capital, and YTL Corporation.

We now consider the finite-order multivariate normal universal portfolios with the objective of reducing the implementation time and computer-memory requirements substantially. The finite-order portfolios are run on same selected stock-price data sets. The wealths achieved over a modest length of time are recorded. Empirically, from the tables 1, 2, 3and 4 we observe the performance of the finite-order portfolios is comparable to that of the Dirichlet universal
portfolios and yet requiring substantially less implementation time. The multivariate normal distribution \(N(\mu, K)\) studied is of special form \(\mu = (\mu_1, \mu_2, \mu_3)\) and \(K = \text{diag}(\sigma_1, \sigma_2, \sigma_3)\), where \(Y_1, Y_2\) and \(Y_3\) are independent.

**Table 1**: The wealths achieved by the Dirichlet universal portfolio and the time needed for implementation after 500 trading days.

<table>
<thead>
<tr>
<th>Set</th>
<th>(\alpha_1, \alpha_2, \alpha_3)</th>
<th>(b_1, b_2, b_3)</th>
<th>(S_{500})</th>
<th>Time needed for implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(0.01, 0.01, 8000)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>1.8750</td>
<td>394.766 second</td>
</tr>
<tr>
<td>B</td>
<td>(0.01, 0.01, 8000)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0761</td>
<td>394.985 second</td>
</tr>
<tr>
<td>C</td>
<td>(0.01, 0.01, 8000)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0761</td>
<td>394.376 second</td>
</tr>
</tbody>
</table>

**Table 2**: The wealths achieved by the multivariate normal universal portfolio where \(\sigma_1 = \sigma_2 = \sigma_3 = \sigma\) and the time needed for implementation for Set A data after 500 trading days.

<table>
<thead>
<tr>
<th>Order</th>
<th>((\mu_1, \mu_2, \mu_3, \sigma))</th>
<th>((b_1, b_2, b_3))</th>
<th>(S_{500})</th>
<th>Time needed for implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(7.1, 0.8, 0.3, 1.0)</td>
<td>(0.8438, 0.1072, 0.0489)</td>
<td>1.9393</td>
<td>0.0144928 second</td>
</tr>
<tr>
<td>2</td>
<td>(7.1, 0.8, 0.3, 1.0)</td>
<td>(0.8246, 0.1155, 0.0594)</td>
<td>1.9442</td>
<td>0.0235865 second</td>
</tr>
<tr>
<td>3</td>
<td>(7.1, 0.8, 0.3, 1.0)</td>
<td>(0.8077, 0.1230, 0.0693)</td>
<td>1.9405</td>
<td>0.0633493 second</td>
</tr>
</tbody>
</table>

**Table 3**: The wealths achieved by the multivariate normal universal portfolio where \(\sigma_1 = \sigma_2 = \sigma_3 = \sigma\) and the time needed for implementation for Set B data after 500 trading days.

<table>
<thead>
<tr>
<th>Order</th>
<th>((\mu_1, \mu_2, \mu_3, \sigma))</th>
<th>((b_1, b_2, b_3))</th>
<th>(S_{500})</th>
<th>Time needed for implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.01, 0.01, 1000, 1.2)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0761</td>
<td>0.0146996 second</td>
</tr>
<tr>
<td>2</td>
<td>(0.01, 0.01, 1000, 1.2)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0353</td>
<td>0.0146996 second</td>
</tr>
<tr>
<td>3</td>
<td>(0.01, 0.01, 1000, 1.2)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0218</td>
<td>0.0146996 second</td>
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</tbody>
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**Table 4**: The wealths achieved by the multivariate normal universal portfolio where \(\sigma_1 = \sigma_2 = \sigma_3 = \sigma\) and the time needed for implementation for Set C data after 500 trading days.

<table>
<thead>
<tr>
<th>Order</th>
<th>((\mu_1, \mu_2, \mu_3, \sigma))</th>
<th>((b_1, b_2, b_3))</th>
<th>(S_{500})</th>
<th>Time needed for implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.01, 0.01, 1000, 1.2)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0761</td>
<td>0.0147734 second</td>
</tr>
<tr>
<td>2</td>
<td>(0.01, 0.01, 1000, 1.2)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0489</td>
<td>0.0147734 second</td>
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<tr>
<td>3</td>
<td>(0.01, 0.01, 1000, 1.2)</td>
<td>(0.00, 0.00, 1.00)</td>
<td>4.0219</td>
<td>0.0624402 second</td>
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References


