Bilinear Robust Control for Vertical Vibration in Railway Vehicle with Semi-Active Suspensions*

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Abstract

It is well known that the vibration control problem for automobiles and railway vehicles with semi-active suspensions is classified as a control problem in a bilinear system. Bullet trains and railway vehicles have lighter body in order to improve acceleration; these vibrations in the body are easily induced by various disturbances due to rigid and elastic dynamics. Currently, passive dampers such as air suspensions and axle springs are installed on railway vehicle trucks as countermeasures for such vibrations. This study presents an effective controller, based on the H_∞ theory, for vibration suppression in railway vehicles and describes a method of synthesizing this robust controller by considering unstructured and structured uncertainties that are applicable to a bilinear system. The performance of the proposed controller and its robustness toward uncertainties are examined by numerical calculations that simulate a railway vehicle subjected to disturbances due to vertical uneven railway tracks, the variations in its mass due to boarding passengers, and the modeling errors caused by non-controlled modes. This enables a comparison of the proposed control method with the conventional one in terms of the robustness toward parameter variation. Thus, this result shows the high robustness and usefulness of the proposed controller.

Key words : Vibration Control, Robust Control, Bilinear System, Flexible Structure, Railway Vehicle, Parameter Variation, Semi-Active Suspension

1. Introduction

Semi-active suspensions are installed for vibration suppression in ground vehicles such as bullet trains, railcars, and automobiles. In recent years, lighter vehicle bodies have been fabricated in order to improve acceleration and for energy conservation. However, as a result of this modification, vibrations in the body are easily induced by various disturbances caused due to rigid and elastic dynamics resulting from track irregularity or unevenness and rough roads. It is also a well-known fact that the semi-active suspension, in comparison to an active damper or a passive damper such as an air suspension and an axle spring, is more effective in vehicle vibration suppression from the viewpoints of performance and energy conservation⁽¹⁾⁻⁽⁷⁾. Vibration control by using a semi-active suspension for a railway vehicle has been described as a control problem in a bilinear system⁽⁸⁾⁻⁽¹¹⁾; therefore, the vibration controller is required to react appropriately to the characteristic variations in a control device. The authors developed an optimal control theory for bilinear systems, which was applied to the semi-

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active isolation of a building^{(12),(13)} and semi-active suspension for vehicles^{(14),(15)}. Moreover, some researchers proposed methods for synthesizing a robust state feedback controller for continuous-time bilinear systems using an approximate solution of the state-dependent algebraic Riccati equation⁽¹⁶⁾ and a nonlinear H_∞ control for automobiles in which the nonlinear function was employed as a weighting for the synthesis^{(17),(18)}. However, each method employed controllers that were synthesized using the approximate solutions.

Therefore, this paper presents a method for synthesizing a controller for continuous-time bilinear systems based on a game-type criterion function. This controller is then designed by an analytical procedure that employs the Riccati algebraic equation and is derived using dynamic programming. Furthermore, in order to enhance the robustness of the proposed controller, unstructured and structured uncertainties^{(19)–(21)} are considered for a generalized plant employed for its synthesis.

The performance and robustness of the proposed controller are examined by numerical calculations that simulate the practical issue of vertical vibration suppression for a railway vehicle. Due to the flexibility of railway vehicles, vertical vibrations are induced by the vertical, pitching, and rolling modes in the vicinity of 1 Hz and the first bending mode between 6 to 10 Hz. These vibrations result in a deterioration in the ride quality of a railway vehicle. Since the first bending mode particularly affects the ride comfort, its suppression is a major issue.

In previous studies, several techniques were employed for the suppression of the first bending vibration mode^{(22)–(24)}. However, few solutions obtained by employing the semi-active suspension have been validated. It was generally considered that it is difficult to reduce the first bending vibration using the semi-active suspensions because the nodes of its vibration mode occur at the positions where the vehicle body is fixed to the trucks. Hence, the authors developed a solution based on the bilinear optimal control theory, which implements the simultaneous suppression of the vertical and first bending vibration modes⁽¹⁴⁾. However, in practice, the controller is required to exhibit robustness toward parameter variation, which depends on the variation in mass due to boarding passengers.

2. Bilinear Robust Control Method

2.1. Construction of generalized plant with unstructured and structured uncertainties

In this paper, we will describe the vibration control for the following bilinear system:

$$\dot{x}(t) = Ax(t) + BX^*(t)u(t) + Gw_d(t), \quad X^*(t) = \text{diag} [\dot{x}_{i1}(t) \cdots \dot{x}_{im}(t)] \quad (1)$$

where $x(t)$ is the state value; $X^*(t)$, the system matrix that affects the control inputs; $\dot{x}_{in}$, the relative velocity in n -th variable damper; m , the number of a variable damper; A , B , and G , the time-invariant matrices with appropriate dimensions; $u(t)$, the control input; $w_d(t)$, the system disturbance. The damping force $X^*(t)u(t)$ is operated by the vibration controller; $u(t)$ is identical to the damping coefficient of a variable damper.

This paper presents a method for synthesizing a bilinear controller that exhibits robustness toward uncertainties. If the controlled object is a system with a flexible structure, it is essential that the exact model be constructed on the basis of a complex formulation that can express the accurate dynamics of the controlled object. It is reasonable to use a numerical model with a lower dimension for the design of a controller. However, the model errors that arise due to the neglected high-order modes cause control and observation spillovers. Moreover, the precise realization of nonparametric or parametric identification in the controlled object, which is a type of bilinear system, is complicated. Thus, it is required to establish a method for synthesizing the bilinear controller using a lower order model that includes descriptions of the parameter variations and model errors.

Based on these arguments, this paper presents a synthesis method for the controller by considering the parameter variations and modeling errors. Initially, we frame the model errors that arise due to the neglected high-order modes as an unstructured uncertainty in the frequency domain and the parameter variations as a structured uncertainty in the time domain.

Fig. 1 A schematic representation of the generalized plant structure with the unstructured and structured uncertainties

Then, the unstructured uncertainty is described by applying shaped filters such as a high-pass filter; moreover, the expression of the structured uncertainty is expressed using weighting matrices. A major issue with regard to controller synthesis by considering only the unstructured uncertainties is that the dimension of the bilinear robust controller increases with the number of shaped filters. In addition, if all the formulated modes are strongly suppressed by the controller by considering only the structured uncertainty, the controller exhibits a high gain for the state in the high-frequency domain; hence, the issue of stability arises. Therefore, in this paper, the proposed method for synthesizing the bilinear robust controller considers these two types of uncertainties.

An augmented system including the two uncertainties in the nominal model (1) is constructed as shown in the following equation:

$$\dot{\mathbf{x}}_z(t) = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{x}}_r(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} + \Delta\mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_r + \Delta\mathbf{A}_r \end{bmatrix} \mathbf{x}_z(t) + \begin{bmatrix} \mathbf{B} + \Delta\mathbf{B}(t) \\ \mathbf{B}_r + \Delta\mathbf{B}_r \end{bmatrix} \mathbf{X}^*(t)\mathbf{u}(t) + \begin{bmatrix} \mathbf{G} \\ \mathbf{0} \end{bmatrix} \mathbf{w}_d(t) \quad (2)$$

$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{C} + \Delta\mathbf{C} & \mathbf{C}_r + \Delta\mathbf{C}_r \end{bmatrix} \mathbf{x}_z(t) + \begin{bmatrix} \mathbf{D}_r + \Delta\mathbf{D}_r \end{bmatrix} \mathbf{X}^*(t)\mathbf{u}(t) \quad (3)$$

where $\mathbf{x}_r(t)$, $\mathbf{A}_r$, $\mathbf{B}_r$, $\mathbf{C}_r$, and $\mathbf{D}_r$ are the state and system matrices of the shaped filter that express the unstructured uncertainty. This shaped filter is designed so as to overcome the differences in the frequency domain between the model that considers the high-order modes and the nominal model as the additional errors from the control inputs to the observed outputs. Furthermore, (2) and (3) include the scaled structured uncertainty Δ that is derived from the differences between the nominal model and the model with considerable parameter variations in the time domain. In other words, this augmented system indicates the model set of the controlled object in which the fluctuation band of the uncertainties is in the time and frequency domains.

2.2. Generalized plant considering uncertainties

For designing the bilinear robust controller, the generalized plant with the two uncertainties is constructed in terms of the above-mentioned augmented system and weighting functions. For the scaled structured uncertainty, the closed-loop expression of the system with the controller substitutes for the open loop one by using the virtual disturbance inputs $\mathbf{w}_a(t)$, $\mathbf{w}_b(t)$, $\mathbf{w}_c(t)$, and $\mathbf{w}_d(t)$ and the weighted performance outputs $\mathbf{z}_a(t)$, $\mathbf{z}_b(t)$, $\mathbf{z}_c(t)$, and $\mathbf{z}_d(t)$. Further, the closed-loop expression of the nominal model with the unstructured uncertainty, expressed by using the shaped filter $\mathbf{W}_n(s)$, is transformed into the open loop one by using $\mathbf{w}(t)$ and $\mathbf{z}_n(t)$. Moreover, in order to adjust the control inputs, the state values are weighted in the frequency domain with the frequency weighting $\mathbf{W}_g(s)$ and performance output $\mathbf{z}_g(t)$. As an example, the open-loop expression of the generalized plant is shown in Fig. 1. In this control problem, the

state-space description of the generalized plant is obtained by the following equations:

$$\dot{\mathbf{x}}_z(t) = \mathbf{A}_z \mathbf{x}_z(t) + \mathbf{B}_1 \mathbf{w}^\#(t) + \mathbf{B}_2 \mathbf{X}^*(t) \mathbf{u}(t) \quad (4)$$

$$\mathbf{z}_h(t) = \mathbf{C}_1 \mathbf{x}_z(t) + \mathbf{D}_{12} \mathbf{X}^*(t) \mathbf{u}(t) \quad (5)$$

$$\mathbf{y}(t) = \mathbf{C}_2 \mathbf{x}_z(t) + \mathbf{D}_{21} \mathbf{w}^\#(t) + \mathbf{D}_{22} \mathbf{X}^*(t) \mathbf{u}(t) \quad (6)$$

$$\mathbf{w}^\#(t) = [\mathbf{w}(t) \ \mathbf{w}_a(t) \ \mathbf{w}_b(t) \ \mathbf{w}_c(t) \ \mathbf{w}_d(t)]^T, \ \mathbf{z}_h(t) = [\mathbf{z}_g(t) \ \mathbf{z}_a(t) \ \mathbf{z}_b(t) \ \mathbf{z}_c(t) \ \mathbf{z}_d(t)]^T$$

We will design the controller to robustly stabilize the above-mentioned augmented system by considering the unstructured and structured uncertainties.

2.3. Synthesis of bilinear robust controller

Firstly, it should be noted that the control input for the bilinear system is derived by switching the positive damping coefficient; this guarantees the stability of the closed-loop system with the bilinear robust controller using the semi-active suspension. Therefore, this section mentions the method of synthesizing the bilinear robust controller by considering the uncertainties in order to satisfy the robust stabilization condition based on the H_∞ norm. In the previously derived generalized plant (4)–(6), the H_∞ norm of the closed-loop transfer function G_{zw} from the virtual worst disturbance $\mathbf{w}^\#(t)$ to the performance output $\mathbf{z}_h(t)$ is described by the L_2 -induced norm in the time domain, as shown by the following equation:

$$\min_{\mathbf{u}} \|G_{zw}\|_\infty = \min_{\mathbf{u}} \sup_{\mathbf{w}^\#} \frac{\|\mathbf{z}_h(t)\|_2}{\|\mathbf{w}^\#(t)\|_2} =: \gamma^* \quad (7)$$

where $\|\cdot\|_\infty$ represents the H_∞ norm; $\|\cdot\|_2$, the L_2 -induced norm; γ^* , the minimum value of the H_∞ norm realized by using the optimal controller $K(y, t)^*$. For $\gamma > \gamma^*$, the control satisfying the following condition is referred to as the quasi optimal control:

$$J(\mathbf{u}^o, \mathbf{w}^{\#o}) = \min_{\mathbf{u}} \max_{\mathbf{w}^\#} \int_0^\infty [\mathbf{z}_h^T(t) \mathbf{z}_h(t) - \gamma^2 \mathbf{w}^{\#T}(t) \mathbf{w}^\#(t)] dt \leq 0 \quad (8)$$

where $\mathbf{w}^{\#o}$ denotes the worst disturbance, which is the solution of the problem

$$J(\mathbf{u}, \mathbf{w}^{\#o}) = \max_{\mathbf{w}^\#} J(\mathbf{u}, \mathbf{w}) \quad (9)$$

and it is expressed by the following equation:

$$\mathbf{w}^{\#o}(t) = \gamma^{-2} \mathbf{B}_1^T \mathbf{X}_w \mathbf{x}_z(t) \quad (10)$$

where $\mathbf{X}_w$ is the unique positive definite symmetric matrix that is the solution of the Riccati algebraic equation derived from (9). Therefore, $\mathbf{u}^o(t)$ is the optimal control input, which is the solution of the minimization problem

$$J(\mathbf{u}^o, \mathbf{w}^\#) = \min_{\mathbf{u}} J(\mathbf{u}, \mathbf{w}^\#) \quad (11)$$

and it is expressed by the following equation:

$$\mathbf{u}^o(t) = -(\tilde{\mathbf{R}} \mathbf{X}^*(t))^{-1} (\mathbf{B}_2^T \mathbf{X}_u + \tilde{\mathbf{S}}) \mathbf{x}_z(t) \quad (12)$$

where $\mathbf{X}_u$ is the unique positive definite symmetric matrix that is the solution of the Riccati algebraic equation derived from (11), $\tilde{\mathbf{S}} = \mathbf{C}_1^T \mathbf{D}_{12}$, and $\tilde{\mathbf{R}} = \mathbf{D}_{12}^T \mathbf{D}_{12}$. The main objective of this control problem is to ensure the convergence of this control to the H_∞ optimal control by iteratively solving the controller $K(y, t)$ by decreasing γ . In other words, the objective of this control problem is to achieve $\gamma \rightarrow \gamma^*$ or $J = 0$. Consequently, the criterion function (8) is transformed into the following equation:

$$J(\mathbf{u}^o, \mathbf{w}^{\#o}) = \min_{\mathbf{u}} \max_{\mathbf{w}^\#} \int_0^\infty [\mathbf{x}_z^T(t) \tilde{\mathbf{Q}} \mathbf{x}_z(t) + 2 \mathbf{x}_z^T(t) \tilde{\mathbf{S}} \mathbf{X}^*(t) \mathbf{u}(t)]$$

$$+ \mathbf{u}^T(t) \mathbf{X}^{*T}(t) \tilde{\mathbf{R}} \mathbf{X}^*(t) \mathbf{u}(t) - \gamma^2 \mathbf{w}^{*T}(t) \mathbf{w}^*(t)] dt \quad (13)$$

where $\tilde{\mathbf{Q}} = \mathbf{C}_1^T \mathbf{C}_1$. In this paper, the problem of obtaining $K(y, t)$, which minimizes the criterion function of (13), is solved by using the optimal control input $\mathbf{u}^o$ in the presence of the worst disturbance $\mathbf{w}^{*o}$.

Furthermore, in the above criterion function, it is considered that the varying damping coefficient $\mathbf{u}(t)$ is not constrained; however, the damping force $\mathbf{X}^*(t) \mathbf{u}(t)$ is constrained in the arbitrary range. Therefore, the weightings on the damping force appear in the second and third terms mentioned above. For the design of the semi-active suspension equipment, the maximum damping force is considered to be an important design parameter. Thus, it is logical to employ this parameter for synthesizing the bilinear optimal and robust controllers based on the criterion equation.

2.4. Synthesis of controller

We begin by considering the controller that achieves robust stabilization for a bilinear system described by employing the generalized plant (4)-(6). A saddle-point solution that satisfies the following inequality condition is obtained for (13):

$$J(\mathbf{u}^o, \mathbf{w}^*) \leq J(\mathbf{u}^o, \mathbf{w}^{*o}) \leq J(\mathbf{u}, \mathbf{w}^{*o}) \quad (14)$$

It is implemented with the following controller in the state-space description⁽²⁵⁾⁻⁽²⁷⁾.

$$\dot{\hat{\mathbf{x}}}(t) = \hat{\mathbf{A}} \hat{\mathbf{x}}(t) + \hat{\mathbf{B}} \mathbf{y}(t) \quad (15)$$

$$\mathbf{u}(t) = \mathbf{F}(t) \hat{\mathbf{x}}(t) \quad (16)$$

$$\hat{\mathbf{A}} = \mathbf{A}_z + (\mathbf{B}_2 + \hat{\mathbf{B}} \mathbf{D}_{22}) \mathbf{X}^*(t) \mathbf{F}(t) + \gamma^{-2} \mathbf{B}_1 \mathbf{B}_1^T \mathbf{X} - \hat{\mathbf{B}} (\mathbf{C}_2 + \gamma^{-2} \mathbf{D}_{21} \mathbf{B}_1^T \mathbf{X})$$

$$\hat{\mathbf{B}} = (\mathbf{I} - \gamma^{-2} \mathbf{Y} \mathbf{X})^{-1} (\mathbf{Y} \mathbf{C}_2^T + \mathbf{B}_1 \mathbf{D}_{21}^T) (\mathbf{D}_{21} \mathbf{D}_{21}^T)^{-1}, \mathbf{F}(t) = -(\tilde{\mathbf{R}} \mathbf{X}^*(t))^{-1} (\mathbf{B}_2^T \mathbf{X} + \tilde{\mathbf{S}}^T)$$

where $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ are obtained by resolving the following Riccati algebraic equations:

$$\mathbf{X} \mathbf{A}_z + \mathbf{A}_z^T \mathbf{X} + \mathbf{X} \mathbf{B}_1 \mathbf{B}_1^T \mathbf{X} / \gamma^2 - (\mathbf{X} \mathbf{B}_2 + \tilde{\mathbf{S}}) \tilde{\mathbf{R}}^{-1} (\mathbf{B}_2^T \mathbf{X} + \tilde{\mathbf{S}}^T) + \tilde{\mathbf{Q}} = \mathbf{0} \quad (17)$$

$$\mathbf{Y} \mathbf{A}_z^T + \mathbf{A}_z \mathbf{Y} + \mathbf{Y} \mathbf{C}_1^T \mathbf{C}_1 \mathbf{Y} / \gamma^2 - (\mathbf{Y} \mathbf{C}_2^T + \mathbf{B}_1 \mathbf{D}_{21}^T) (\mathbf{D}_{21} \mathbf{D}_{21}^T)^{-1} (\mathbf{C}_2 \mathbf{Y} + \mathbf{D}_{21} \mathbf{B}_1) + \mathbf{B}_1 \mathbf{B}_1^T = \mathbf{0} \quad (18)$$

Finally, the damping force must be constrained between the maximum and minimum values. This is because it is possible that a considerably large damping force may be calculated by using the previous output feedback controller for cases in which the input device states of the controlled object shift only slightly.

3. Application to Vibration Control in Railway Vehicles

In this section, the proposed bilinear robust control method is applied to vibration suppression in a railway vehicle due to its flexibility. The vertical vibration of the bullet train caused by uneven rail tracks is considered as the problem to be resolved.

3.1. Finite Element Method (FEM) model

The model used for analyzing the dynamics of the railway vehicle is shown in Fig. 2; further the parameter symbols in Fig. 3 are shown in Table. 1. This model comprises an elastic body, two trucks, single links as coupling elements between the body and the trucks, anti-yaw dampers in the traveling direction, and so on. The natural frequencies and eigenvectors up to 100 Hz are derived using a real eigenvalue analysis for the FEM model. The natural frequencies of the typical vibration modes appear at 1.18 Hz for the vertical vibration mode of the vehicle body, 1.44 Hz for the pitching mode and 8.53 Hz for the first bending mode. In addition, the modal shapes are obtained as shown in Fig. 4.

Fig. 2 The railway vehicle model

Fig. 3 The truck model of the bullet train and the symbols of parameter symbol

Fig. 4 The vibration modes of the railway vehicle:
(a) Vertical (1.18 Hz), (b) Pitching (1.44 Hz),
(c) First bending (8.53 Hz)

Fig. 5 The PSD of the acceleration
(a) at the center of the vehicle body
and (b) above the rear truck

Table 1 Railway vehicle parameters

Device	Symbol	Unit	Value	Symbol	Unit	Value
Axle spring	k_1	N/m	1.12×10^6	c_1	Ns/m	1.96×10^4
Air spring	k_2	N/m	6.00×10^5	c_2	Ns/m	2.00×10^4
Anti yaw damper	k_y	N/m	3.06×10^6	c_y	Ns/m	9.80×10^5
				β_y	—	0.100
Traction device	k_b	N/m	4.90×10^6	β_b	—	0.100
Axle box	k_A	N/m	6.15×10^6	β_d	—	0.100
Distance	h_1	m	0.0500	L_1	m	0.340
	h_2	m	0.0500	L_2	m	0.750
	h_3	m	0.0650	L_3	m	0
	h_4	m	0.940	a	m	2.50

3.2. Construction of state equation

The large localized damping in this system, such as air and axle springs, is typically considered to be a general viscous damping. Although such systems are generally analyzed by a complex eigenvalue analysis, we employ a real eigenvalue analysis without damping in order to construct the state equation of the controlled object. On the basis of the natural angular frequencies, $\Omega_1 - \Omega_N$, derived up to 100 Hz and the eigenvectors, $\phi_1 - \phi_N$, which are obtained from the real eigenvalue analysis, the state equation is obtained using the following equations:

$$\dot{\xi}_s(t) = A_s \xi_s(t) + G \Phi^T w_d(t) \quad (19)$$

$$y(t) = C_s \Phi \xi_s(t) + D_{s2} \Phi G \Phi^T w_d(t) \quad (20)$$

$$A_s = \begin{bmatrix} 0_{N \times N} & I_{N \times N} \\ -K_{N \times N} & -C_{N \times N} \end{bmatrix}, \quad K_{N \times N} = \text{diag}[\Omega_1^2 \quad \dots \quad \Omega_N^2], \quad C_{N \times N} = \Phi^T C^* \Phi$$

where $\xi_s = [\xi_{N \times 1}(t) \quad \dot{\xi}_{N \times 1}(t)]^T$, the damping matrix $C_{N \times N}$ is derived from C^* in the physical coordinate system and the eigenvectors at either end of the body, where the dampers and semi-active suspensions are located. The stiffness matrix $K_{N \times N}$ is a type of a diagonal matrix. However, $C_{N \times N}$ is an off-diagonal matrix. Because all the modes are concerned, the model errors are generated by the discretization of the vibration modes. $\xi(t)$ is the state in the modal coordinate; $w_d(t)$, the actual disturbance; $y(t)$, the system output; $G \Phi^T$, the transformation matrix from the disturbance $w_d(t)$ in the physical coordinate to that in the modal coordinate; $C_s \Phi$,

the transformation matrix from the output in the physical coordinate to that in the modal coordinate, and $D_{s2}\Phi G\Phi^T$, the transformation matrix from the acceleration output in the physical coordinate to that in the modal coordinate. Furthermore, the uneven rail track is considered to be the actual disturbance $w_d(t)$, and it is the signal obtained by repeatedly passing white noise through a shaped filter.

3.3. Verification of model through numerical simulation

For the numerical calculation, the fourth-order Runge-Kutta method is employed for integrating the differential equation; the time interval is 5 ms. In addition, the railway vehicle runs a distance of approximately 3000 m at a constant velocity of 300 km/h; further, there exists a time-delay in the disturbance based on the actual running conditions. Using the previously mentioned parameters, a passive control result is derived for the case in which the varying damping coefficient value is fixed at 2.0×10^4 Ns/m. Figs. 5 (a) and (b) show the power spectrum densities (PSDs) of the accelerations at the center of the body and above the rear truck, respectively. These PSDs are calculated by smoothing the Fourier-transformed data of the time responses in more than 50 simulations. The PSD results are compared qualitatively with those obtained for the actual measurements of the bullet train. It is confirmed from this comparison that the peak amplitude and the frequency at which the vibration decreases is similar in both cases up to a frequency of 10 Hz. Therefore, the previous model can be employed as a valid model for verifying the comfort level of the railway vehicle and the vertical, pitching, and first bending vibrations.

3.4. Model with parameter variation

In this paper, the parameter variation is obtained by considering that the passengers gather in the front portion of the vehicle. By this assumption, the vertical vehicle vibration increases more considerably in comparison with the case in which the entire mass of the vehicle body increases, which is equivalent to considering the worst case scenario. Fig. 6 shows the PSDs of the accelerations at the center of the vehicle body for the case in which the passengers gather in the front portion of the vehicle and the passenger ratio varies from 200% to 400%. The robustness of the proposed controller is verified in these cases.

4. Numerical Calculations

4.1. Conditions for numerical calculation

In this paper, the semi-active suspensions with the variable damping are adopted as air suspensions for the railway vehicle. The bilinear robust control system that uses these suspensions implements the vibration suppression of the railway vehicle, and it is compared with the conventional bilinear optimal control system from the viewpoints of performance and robustness. Furthermore, the actual observable states are limited to the relative displacements of all the suspensions and the absolute accelerations at the top of the front and rear trucks of the railway vehicle.

Fig. 6 The PSDs of the acceleration at the center of the vehicle body as a ratio of the number of passengers in the front of the body

Fig. 7 The shaped filter and model errors

4.2. Design of control system

The model for the synthesis of the bilinear robust controller includes only the following modes: the vertical vibration mode of the vehicle body, and the pitching, rolling, and first bending vibration modes. In addition, it is assumed that the nominal case occurs when all the passengers (100%) board the railway vehicle, and the worst case occurs when 400% of the passengers gather in the front portion of the railway vehicle. Thus, the structured uncertainty is derived from the difference between the state equations of the nominal and worst cases. The unstructured uncertainty is derived by using the high-pass filter to overcome the model errors, as shown in Fig. 7. Moreover, the frequency weighting for the observable states is performed using the bandpass filter with a peak at the same frequency as that of the first bending vibration mode.

For the purpose of comparison, the design of the bilinear optimal controller as the comparative one is based on the previous nominal model, and its state equation is then described by the following equation:

$$\dot{\xi}_o(t) = A_o \xi_o(t) + B_o \Phi^T X^*(t) u(t) + G \Phi^T w_d(t) \quad (21)$$

$$y(t) = C_o \Phi \xi_o(t) + D_{o1} \Phi B_o \Phi^T X^*(t) u(t) \quad (22)$$

The bilinear optimal controller and observer are based on the previous bilinear system. Furthermore, the states and varying damping forces are adopted as the objective functions in the following criterion function:

$$J = E \left[\int_0^\infty \xi_o^T(t) Q \xi_o(t) + u^T(t) X^{*T}(t) R X^*(t) u(t) dt \right] \quad (23)$$

where Q and R are the weightings on the states and control input, respectively. The optimal control input for minimizing the criterion function (23) is given by the following equation:

$$u^o(t) = -[R X^*(t)]^{-1} (B_o \Phi^T)^T P \xi_o(t) \quad (24)$$

where P is the unique positive definite symmetric matrix derived by solving the following Riccati algebraic equation:

$$P A_o + A_o^T P - P B_o \Phi^T R^{-1} (B_o \Phi^T)^T P + Q = 0 \quad (25)$$

Finally, the Kalman filter is adopted as the observer for the estimation of the states of the controlled object.

4.3. Design of semi-active suspension

The varying damping forces derived from the bilinear robust and optimal controllers are calculated by using the damping coefficients limited between the maximum and minimum values as follows.

$$c_{semi}(t) = \begin{cases} c_{max} & \text{if } u^o(t) \geq c_{max} \\ u^o(t) & \text{if } c_{min} < u^o(t) < c_{max} \\ c_{min} & \text{if } u^o(t) \leq c_{min} \end{cases} \quad (26)$$

where the maximum and minimum values of damping are 7.0×10^4 Ns/m and 2.0×10^4 Ns/m, respectively.

Further, it is also assumed that the switching of the varying damping coefficients in the semi-active suspension includes the first-order time delay 10ms. Finally, the configuration of the control systems that include the bilinear robust and optimal controllers is shown in Fig. 8.

5. Control Results and Discussion

5.1. Verification of control methods

This section describes the verification of the bilinear optimal and robust controls and the passive control. The result for the passive control is derived from the numerical calculation for

Fig. 8 A schematic representation of the bilinear optimal and robust controls

Fig. 9 The natural frequency errors for the case in which the passengers gather in the front portion of the vehicle body

Fig. 10 The PSDs of acceleration at the center of the vehicle body in the passenger ratio 100 %: (a) Range from 0.5 to 50Hz, (b) Range from 7 to 10Hz

the case in which the damping coefficient of the semi-active suspension is fixed at a minimum value of 2.0×10^4 Ns/m, in the same manner as that used for deriving the procedure for the result in Fig. 5. Next, the passive control result is assumed as the nominal result, and it is then compared with the semi-active controls from the viewpoint of the robustness toward the parameter variation that arises due to the variation in the passenger ratio.

For the verification of robustness, the nominal performances of the semi-active controllers in vibration suppression are correlated to the case in which all the passengers are aboard the railway vehicle. The robustness of the derived controllers in the nominal case is verified for the case in which 200%-400% of the passengers gather in the front portion of the railway vehicle. The variations in the natural frequencies in each control mode for the verification case are depicted in Fig. 9, and the indicators show the differences between the natural frequencies for the parameter-varying and nominal cases. In Fig. 9, it is confirmed that the variation in the natural frequency in the first bending vibration mode is greater than that in the other modes; in other words, this variation has the most significant effect on controller robustness.

5.2. Comparison of the control results

Firstly, Fig. 10 shows the PSDs of the absolute acceleration at the center of the vehicle body for the nominal case. In addition, the time histories of the varying damping coefficient in the rear-right semi-active suspension with the bilinear robust and optimal controls are shown in Fig. 11. It is then confirmed that the responses obtained using the two controllers exhibit approximately identical behavior. Secondly, Fig. 12 shows the distribution of the ride comfort level in the range of 0.5 to 80 Hz, and in Fig. 12(a), it is verified that the control performances of the two controllers are identical in the nominal case. Furthermore, Figs. 12 (b), (c), and (d) show the results of the robustness of the two controllers for the case in which the passenger ratio varies from 200% to 400%. It is evident that the robust controller exhibits a higher robustness than the optimal one, that is, it performs the effective robust stabilization under the parameter variations. Thirdly, Fig. 13 shows the PSDs of the absolute acceleration at the center of the vehicle body for the case in which the passenger ratio is 400%, and Fig. 13 (b)

Fig. 11 Time histories of varying damping coefficient in the rear right semi-active suspensions in the passenger ratio 100 %: (a) Bilinear optimal control, (b) Bilinear robust control

Fig. 12 The distributions of the ride comfort level in the railway vehicle with respect to the corresponding front passenger ratios

Fig. 13 The PSDs of the acceleration at the center of the vehicle body in the passenger ratio 400 %: (a) Range from 0.5 to 50 Hz, (b) Range from 7 to 10 Hz

Fig. 14 Time histories of the varying damping coefficient in the rear right semi-active suspensions in the passenger ratio 400%: (a) Bilinear optimal control, (b) Bilinear robust control

shows that the bilinear robust controller derives better suppression performance for the first bending vibration in the vicinity of 7.5 Hz. Particularly, the acceleration in the vicinity of the frequency at which the first bending vibration mode occurs strongly affects on the comfort level of the railway vehicle; this is evident in Fig. 12 (d). Finally, Fig. 14 shows the time histories of the varying damping coefficient in the rear-right semi-active suspension with the bilinear robust and optimal controls. Thus, by using the robust controller, fewer variations in the damping force are observed in comparison to the optimal one, which slightly suppresses the vibration in the first bending mode.

From the previous results, it is clear that the bilinear robust control performs a more effective suppression of the first bending vibration for the case in which the passenger ratio is large. This is because the bilinear robust controller is designed by considering the structured

uncertainty, which is expressed as the parameter variation. Therefore, for the semi-active control of the railway vehicle, the bilinear robust control, in comparison with the bilinear optimal control, improves the feasibility of implementing robust stabilization in a railway vehicle.

6. Conclusions

This paper described the method for synthesizing a bilinear robust controller, which is an improvement over the bilinear optimal controller with regard to the robustness toward uncertainties. The proposed controller was then designed by using a generalized plant with the unstructured and structured uncertainties in the time and frequency domains, respectively. Furthermore, the proposed method was verified by a numerical calculation that simulated vertical vibration control in the railway vehicle. The following conclusions were obtained; (1) the damping forces generated by the semi-active suspensions should be evaluated for vibration control in the bilinear system; (2) the semi-active suspensions, which are located on the trucks of the railway vehicle, can effectively reduce the first bending vibration mode; (3) the robust controller that considers the uncertainties, in comparison to the conventional optimal controller, shows the higher robustness toward considerable parameter variation due to passengers boarding the railway vehicle, although its dimension increases with the number of shaping filters.

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