Prediction of Vibration Energy Levels on Structures Using Wave Intensity Analysis Based on Experimental Data*

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Abstract
This paper describes a Wave Intensity Analysis (WIA) method based on experimental data, which is named Experimental WIA (EWIA). EWIA uses coupling and internal loss factors obtained experimentally through the power injection method, commonly applied in Experimental Statistical Energy Analysis (ESEA). The directional dependency of the energy is introduced by deriving a transmission coefficient for each boundary at each frequency band. Then, EWIA is applied to predict the vibration energy of simple plate systems and the results are compared to directly measured energies, WIA and ESEA energy predictions. The results show that EWIA can improve the WIA and ESEA predictions without any additional effort. Moreover, EWIA provides more information than ESEA about the characteristics of the energy field, which allows a more detailed analysis of the structure.

Key words: Statistical Energy Analysis, Wave Intensity Analysis, Method of Vibration Analysis, Vibration of Continuous System, Modeling

1. Introduction

Vibration is a field of great interest in engineering since it can produce fatigue failure, noise emissions and inaccuracy in high precision machinery. There are different methods to estimate vibration energy levels in structures. These methods cover either low or high frequency ranges but there is still no single method that can cover all the frequency range. High or low frequency range refers to those frequency bands where the modal density (number of modes per frequency band) is, respectively, higher or lower than unity.

In the low frequency range, the most popular method is the Finite Element Method (FEM)\(^1\), but FEM becomes impracticable and expensive in terms of computer power and simulation cost as we enter the mid frequency range. The reason is that, as frequency increases, the wavelengths are shorter and more elements are needed to create the model.

In the high frequency range, the most popular method is the Statistical Energy Analysis (SEA)\(^2\). In SEA, the required parameters, Internal Loss Factors (ILFs), Coupling Loss Factors (CLFs) and modal densities, can be determined theoretically. For complex structures, these parameters are estimated experimentally, for example, by applying the power injection method (PIM)\(^2\). However, the SEA method is limited by a number of assumptions like high modal overlap and diffuse field. When those assumptions are not satisfied, the SEA energy predictions are unreliable. Langley\(^3,4\) formulated the Wave Intensity Analysis (WIA) method to improve the SEA predictions in the high frequency range. WIA relaxes the SEA assumption of diffuse field using intensity vectors to describe...
the characteristic power flow equation. An energy field is highly diffuse when the same amount of wave energy propagates in every direction. When the energy field is not highly diffuse, SEA produces prediction errors due to the so-called ‘energy filtering effect’ that occurs at every connected boundary. On the other hand, WIA can describe non-highly diffuse fields and, as a result, the energy filtering effect is taken into account. Thus, energy predictions are improved compared to SEA.

However, WIA calculates the angular distribution of the energy using theoretical models, which are accurate in the high frequency ranges, but they become inaccurate at lower frequency ranges. Thus, WIA is limited to high frequencies and it would be advantageous to extend its application to lower frequency ranges. This may be achieved using experimental models because experimental data is more accurate than theoretical data for a wider frequency range. However, WIA methods using experimental data had not been formulated because the process to acquire WIA data such as intensity vectors is very cumbersome, and the measurement of wave intensities is too complex in comparison to the procedures of SEA, ESEA and WIA.

This paper proposes a novel WIA method using experimental data, which is named ‘EWIA’. The main advantage of the EWIA method is avoiding the difficulties of creating WIA experimental models. This is achieved by employing Experimental SEA (ESEA) models, which are easy to obtain. The results show that EWIA improves WIA predictions at mid and low frequency ranges. EWIA also improves the ESEA predictions for structures with non-highly diffuse energy fields because EWIA predictions take into account the energy filtering effect. For structures with highly diffuse fields, the EWIA and ESEA results are very similar because the energy filtering effect is negligible. However, the results show that EWIA provides more information about the energy field such as estimation of energy distribution in the subsystems, which allows a more exhaustive analysis of the structure.

2. Outline and comparison of SEA and WIA

2.1 SEA

In SEA, a system is divided into smaller subsystems, and the energy levels of the subsystems are calculated using the following power balance equation.

$$P_j = \omega \eta_j E_j + \omega \eta_i E_i - \omega \eta_0 E_i$$

(1)

where $P_j$ is the external power input into subsystem $j$, $\omega$ is the central frequency of the considered frequency band, $\eta_i$ is the internal loss factor (ILF), $E_j$ is the stored energy in subsystem $j$, and $\eta_0$ is the coupling loss factor (CLF) between subsystems $j$ and $i$.

In SEA, the subsystems need to satisfy a series of assumptions in order to guarantee the accuracy of the result. These assumptions have been the focus of many researches because they limit the applicability of SEA. One of those assumptions is that the energy field of the subsystems must be highly diffused. A highly diffuse field is equivalent to assuming the same amount of wave energy in every direction of propagation. Then, $P_j$, $E_j$, and $\eta_i$ in Eq. (1) are spatially averaged and do not depend on the direction of propagation of the energy.

In some cases, complex systems cannot satisfy all of the SEA assumptions. For example, structures that include non-diffuse subsystems or configurations such as a series of rectangular panels connected in a single row. In those cases, SEA cannot predict the energy levels accurately because the energy transmitted through a boundary does not propagate equally in every direction and the waves at some angles carry less energy than others. That is called ‘energy filtering effect’ and it is incorporated into the WIA formulation. As a result, non-diffuse energy fields are described and SEA results can be improved.

2.2 WIA

In SEA, the power terms of Eq. (1) are described in terms of energy carried by the resonant modes of the structure. WIA uses the same power balance equation as SEA, but the
power terms are described as energy carried by waves that travel in different directions. Then, the angle of propagation of the wave energy becomes a variable in the WIA formulation. Therefore, WIA can allow for different amount of energy in each direction and, as a result, it can describe non-diffuse fields. For a highly diffused system, the energy predictions of WIA and SEA are very similar but for systems with non-diffuse fields WIA can predict energy levels more accurately than SEA, as Refs. (3) and (4) show. The WIA power balance equation for a wave type \( j \) is

\[
P^\text{in}_j (\theta, \omega) = P^\text{diss}_j (\theta, \omega) + P^\text{co}_j (\theta, \omega) + P^\text{ci}_j (\theta, \omega)
\]

where \( P^\text{in}_j \) is the power input into a wave \( j \), \( P^\text{diss}_j \) is the power dissipated, \( P^\text{co}_j \) and \( P^\text{ci}_j \) represent output and input power at the boundaries, \( \omega \) is the center frequency, and \( \theta \) is the direction of propagation of wave \( j \). Figure 1 illustrates the power exchanged in a WIA subsystem.

Now, only flexural waves are considered for simplicity. Following Ref. (3), the power terms of Eq. (2) are expanded in the space domain as

\[
P^\text{in}_j (\theta, \omega) = \omega \eta_j E_j (\theta, \omega)
\]

where \( \eta_j \) is the ILF, \( E_j (\theta, \omega) \) is the mean stored energy in direction \( \theta \) and \( j \) represents a wave type. WIA uses transmission coefficients, \( \tau(\theta) \), to express the coupling terms in the power balance equation (from here, \( \tau(\theta) \) is written as \( \tau \) for simplicity). \( \tau \) depends on the angle of transmission, in contrast with the CLF of SEA, which is constant for all the angular range. The coupling terms of Eq. (2) are expressed in terms of wave intensities \( I_j (\theta, \omega) \) as

\[
P^\text{co}_j (\theta, \omega) = \int_{0}^{2\pi} \int_{0}^{\pi} I_j (\phi, \omega) \cdot nL \tau(\phi, \omega) d\phi d\theta
\]

and the intensity of a wave at a given direction and frequency is

\[
I_j (\theta, \omega) = e_j (\theta, \omega) = |c_j (\phi, \omega)| c_{j2} (\phi, \omega) (\phi)
\]

where \( e_j (\theta, \omega) \) is the energy density of wave \( j \), \( c_j (\phi, \omega) \) is its group velocity and \( (\phi) \) is a unit vector in \( \theta \) direction. Then, Substituting Eqs. (3) ~ (6) into Eq. (2) results in

\[
P^\text{co}_j (\theta, \omega) = \omega \eta_j E_j (\theta, \omega) + \left( \omega L / 2\pi c_j \right) \left[ E_j (\theta, \omega) / \nu_j \right] \sum_k L_k \cos(\theta + \pi / 2 - \psi_k) \]

\[
- \left( \omega L / 2\pi c_j \right) \sum_m \int_{0}^{2\pi} E_j (\phi_m, \omega) / \nu_j \sum_{n} [N_j (\phi_m, \omega) / \nu_j] L_n \cos(\theta + \pi / 2 - \psi_m) \tau^n (\phi_m, \pi / 2 - \psi_m)
\]

Equation (7) can be simplified by applying a Fourier series expansion of the energy terms, then using Galerkin procedure the equation can be written in a matrix form

\[
CE = P
\]

where \( \vec{E} \) represents a vector with the components \( E_j / \nu_j \). The components of \( P \) and \( C \) are given by Eqs. (9) and (10)
where \( N_j \) and \( N_i \) represents the corresponding sine and cosine terms of the Fourier series. In Eq. (10), \( \Theta_i \) and \( \Theta_o \) are the angular ranges of the input and output boundaries.

### 2.3 Comparison between SEA and WIA

#### 2.3.1 ILF

The ILF determines the amount of energy dissipated in each subsystem. Both SEA and WIA consider the energy dissipated in the subsystems to be proportional to the mean stored energy. Thus, the dissipated energy is averaged for all the angular range and does not depend on the angle of transmission. Consequently, the ILF of any subsystem is same for the SEA and WIA formulations, as shown in Eqs. (1) and (3).

\[
P_{j}^{diss}(\theta; \omega) = \omega \eta_j E_j(\theta; \omega) = \omega \eta_j E_j(\omega)
\]  

#### 2.3.2 CLF and transmission coefficient

The CLF and transmission coefficient, \( \tau \), determine the amount of energy transmitted between subsystems. SEA uses constant CLF to describe the coupling between subsystems. Then, the transmitted energy is equally distributed for every angle of transmission. On the other hand, WIA describes the couplings using \( \tau \), and it can consider the different amount of energy transmitted at different angles. Relationships between CLF and \( \tau \) are available in the literature\(^7\) for different type of systems and boundaries. For the case considered in this paper (line connections between plates), the relationship is given by Eqs. (12) and (13).

\[
\eta_j^m = \left( L_m c_g / \omega A, \pi \right) \left\{ c^m \right\}
\]

\[
\left\langle \tau^m \right\rangle = \frac{1}{2} \frac{1}{\Theta_o} \int_{\Theta_o} \cos(\Theta) \tau(\Theta) d\Theta
\]

where \( L_m \) is the length of the connecting boundary, \( c_g \) is the group velocity, \( A \) is the area of subsystem \( j \), \( \left\langle \tau^m \right\rangle \) is the angular average of \( \tau \) from subsystem \( j \) to \( i \) at boundary \( m \), and \( \Theta_o \) is the angular range of boundary \( m \).

### 3. Experimental WIA

#### 3.1 Outline

This paper proposes the formulation of a novel WIA method, EWIA. The important characteristic of the EWIA formulation is that it avoids the calculation of transmission coefficients from measurements of wave intensities. This is achieved by combining ESEA models with WIA formulation. As a result, EWIA preserves the simplicity characteristic of SEA, ESEA and WIA, and, at the same time, it improves the energy predictions. In EWIA, the boundaries between subsystems are simplified, and the actual angular distribution of \( \tau \) at each boundary is considered proportional to the theoretical distribution of the simplified model. Under that assumption, the \( \tau \) of each boundary is estimated from the relationship between the theoretical \( \tau \) and CLF measured experimentally at that boundary by ESEA procedures.

#### 3.1.1 Advantages of EWIA

There are three main advantages of EWIA which are: improvement of results respect to WIA, improvement of results respect to ESEA, and more detailed information about the characteristics of the energy field of the structure than in ESEA models. EWIA improves WIA results at lower frequency ranges because experimental models (EWIA or ESEA models) are more accurate than theoretical models for a wider frequency range.

Furthermore, EWIA also improves ESEA predictions when the energy field is
non-highly diffuse because EWIA takes into account the energy filtering effect in the predictions. The filtering effect occurs at every connected boundary and contributes to generate non-highly diffuse fields because it creates a predominant direction of transmission. Figure 2 illustrates the energy filtering effect for a three-plate system. Structural details and analysis of that system are presented in § 5. Therefore, the filtering effect becomes more important as the number of boundaries crossed by the wave energy increase and the energy field, gradually, becomes less diffuse. Since ESEA models neglect the energy filtering effect, the predictions become less accurate as the energy field becomes less diffuse. On the other hand, the filtering effect does not decrease the accuracy of EWIA, and the results are improved respect to ESEA.

In the case that all of the subsystems in the structure have highly diffuse field, the EWIA and ESEA results should be very similar. However, in both cases, highly and non-highly diffuse field, the EWIA method gives more detailed information about the structure than ESEA. For example, EWIA estimates the energy distribution in each subsystem. This information could be very useful to analyze and understand the characteristics of the structure, as shown in § 5.

3.2 Estimation of EWIA parameters - ILF and transmission coefficients

3.2.1 ILF

As shown in § 2.3.1, the ILF of any subsystem is same in both, SEA and WIA methods. Therefore, the ILFs of EWIA can be estimated using ESEA modeling by applying the PIM.

3.2.2 Transmission coefficients - $\tau (\theta)$

EWIA transmission coefficients are estimated from the relationship between measured CLFs and the theoretical $\tau$. $\tau$ must be estimated at each frequency, for each boundary and for every coupling direction. Measured CLFs are obtained by employing, for example, the PIM(2).

For any system, the actual amount of transmitted energy between subsystems and its angular distribution often differs from the theoretical model. The reason is that the direction of the intensity vectors can vary due to irregularities of the boundary and non-uniformity of the material. Thus, predominant directions of transmitted energy may not be perpendicular to the boundary. The EWIA method assumes that the irregularities at or near the boundary do not greatly affect the angular distribution of the energy. Hence, a theoretical model of the energy distribution could be used. Under that assumption, the actual $\tau$ at each boundary should be directly proportional to the theoretical $\tau$ for each angle of transmission. Then, a simple relationship between $\tau$ and experimental CLFs can be derived to calculate that proportionality factor. Therefore, $\tau$ can be estimated without measuring wave intensities. The actual $\tau$ of a boundary is estimated by scaling the theoretical $\tau$ with a scaling factor, $Cnst$, at each frequency for every boundary and coupling direction. Scaling the theoretical $\tau$ ensures that the amount of energy transmitted matches the energy measured in the experiments. Figure 3 shows the comparison of $\tau$ for an L-shape plate system. The estimated $\tau$, given by $Cnst*TAU$, is directly proportional to the theoretical $\tau$, given by $TAU$. 

![Fig. 2 Wave transmission at connecting boundaries](a) Without filtering effect (ESEA)  
(b) With filtering effect (EWIA)
at every angle of transmission. Structural details and analysis of this system are given in § 4.

![Figure 3](image-url)  
*Fig. 3* Example of theoretical and estimated transmission coefficient, $\tau(\theta)$, at 1600Hz in 1/3 octave band, for the L-shape plate system simply supported at the corners with four strings

### 3.3 Calculation of scaling factors – $Cnst$

The scaling factors of EWIA are determined by combining ESEA and WIA. A criterion for that combination is obtained by comparing the energy terms in the power balance equations of the two methods.

Regarding the power dissipated in the subsystems, both, WIA and ESEA, consider this power to be diffused. Then, the first term on the right hand side of Eq. (1) and the right hand side of Eq. (3) should be same. On the other hand, the exchanged power of ESEA (second and third terms on the right hand side of Eq. (1)) should be same as the angular average of the exchanged power of WIA (angular average of second and third terms on the right hand side of Eq. (10)). The reason is that the angular average of the exchanged power in WIA represents the diffuse energy field, and the diffuse part of the transmitted energy should be same for both methods. Therefore, the ESEA model and the diffuse part of the WIA model can be compared. In the WIA method, the power balance equation of the diffuse part energy field is obtained by including only a single term in the Fourier series (3,4). Thus, the ratio between the transmitted power in SEA and ESEA and the angular average of transmitted power in WIA should be same, as shown in Eq. (14)

$$\frac{P_j}{P_i} = \frac{\omega \eta^{m,p}_j \left\langle E_j \right\rangle_e}{\omega \eta^{m,p}_i \left\langle E_i \right\rangle_e} = \frac{\omega \eta^{m,p}_j \left\langle E^{\omega \pi}_j \right\rangle_e}{\omega \eta^{m,p}_i \left\langle E^{\omega \pi}_i \right\rangle_e}$$

where $P_j$ and $P_i$ are the energies exchanged between subsystems $j$ and $i$, $\left\langle E_j \right\rangle_e$ is the energy of subsystem $j$ obtained via experiments, and $\left\langle E^{\omega \pi}_j \right\rangle$ is the energy of subsystem $j$ predicted by WIA using a single term in its Fourier series. The terms $\eta^{m,p}_j$ and $\eta^{m,p}_i$ are respectively the CLF of boundary $m$ estimated via experiments and theoretically from material properties using Eqs. (12) and (13). In real systems, the ratios in the mid and right hand sides of Eq. (14) are usually different to each other. Thus, an scaling factor, $Cnst$, is required to ensure that Eq.(14) is true. Since the energy ratios in the mid and right hand side of Eq. (14) are same under diffuse field assumption, these and the frequency can be cancelled out. Then, multiplying one side by the scaling factor, $Cnst$, results in

$$\frac{\eta^{m,p}_j}{\eta^{m,p}_i} = Cnst \ \frac{\eta^{m,p}_j}{\eta^{m,p}_i}$$

### 3.4 Estimation of transmission coefficient using $Cnst$

An experimental angular average of $\tau$ is obtained from measured CLFs using Eq. (12)

$$\frac{\eta^{m,p}_e \omega A, \pi}{L_m c_{\omega}} = \left\langle \tau^{m,p}(\theta) \right\rangle_e$$

where the subscript $e$ indicates that the value is calculated from experimental data. Since measured CLFs depend on the frequency, $\left\langle \tau^{m,p}(\theta) \right\rangle_e$ also depends on the frequency, while its theoretical value depends only on the material properties and the angle of incidence $\theta$. The theoretical average of the transmission coefficient is calculated using material data as
\[ \frac{1}{2} \int_{\sigma_a} \cos(\theta) \tau_j(\theta) d\theta = \langle \tau_n^m(\theta) \rangle, \tag{17} \]

where the subscript \( \tau \) indicates that the data is obtained using the corresponding theoretical equation. The experimental and the theoretical averaged \( \tau \) of a given boundary are different especially at lower frequency ranges. At higher frequency ranges, the experimental value becomes closer to the theoretical value. Combining Eqs. (15) and (16), \( Cnst \) is written as

\[ Cnst = \frac{L_w c_p}{\omega A \pi} \langle \tau_n^m \rangle, \frac{L_w c_p}{\omega A \pi} \langle \tau_n^m \rangle \]

\[ = \langle \tau_n^m \rangle, \langle \tau_n^m \rangle, \tag{18} \]

If \( Cnst \) is defined for each coupling direction, as shown in Eq. (19), it can be rewritten as

\[ Cnst_{ij} = \frac{\tau_{ij}}{\tau_{ji}}, \tag{19} \]

\[ Cnst = \frac{Cnst_{ij}}{Cnst_{ji}}. \tag{20} \]

Hence, assuming a theoretical distribution of the transmitted energy and combining Eqs. (13) and (19), a relationship between the theoretical and experimental transmission coefficient for a given coupling direction is

\[ \int_{\sigma_a} \cos(\theta) \tau_j(\theta) d\theta = \int_{\sigma_a} \cos(\theta) \tau_{ij}(\theta) d\theta \tag{21} \]

Equation (21) indicates that the distribution of \( \tau \) has same angular distribution as the theoretical \( \tau \), but its value is scaled at each angle of transmission. This ensures that the average of the exchanged power in EWIA matches the averaged of the exchanged power in ESEA. Eq. (18) is also rewritten in the form of Eq. (22) using Eq. (12)

\[ \int_{\sigma_a} \cos(\theta) \tau_j(\theta) d\theta = \frac{\eta_{\phi}^m}{\eta_{\phi}^m} A c_p Cnst \int_{\sigma_a} \cos(\theta) \tau_{ij}(\theta) d\theta \tag{22} \]

Equation (22) indicates that the average energy in a given coupling direction for a given frequency is proportional to the average energy in the opposite direction. The factor of proportionality is determined by \( Cnst \), the corresponding measured CLFs, and material properties of the subsystems.

4. Results of a highly diffuse system

In this chapter, the EWIA method is applied to a simply supported L-shaped plate system, shown in Fig. 3, which has highly diffuse condition. The results are compared to directly measured subsystem energies, the ESEA and WIA predictions to validate the EWIA method. The system consists of a steel plate bended to a 90 degrees angle into an L-shape with Young’s modulus \( E=1.9995 \times 10^{11} \) N/m\(^2\), Poisson’s ratio=0.35, and density \( \rho=7834.6 \) kg/m\(^3\). It is divided into two rectangular subsystems. The lengths of the subsystems are: Plate_1 = 0.5 m and Plate_2 = 0.4 m. The width of the subsystems is 0.35 m.

Experimental data (ILFs and CLFs) obtained using the PIM is shown in Figs. 4 and 5.
The measurements of each subsystem were taken by six accelerometers positioned at random locations to measure the out-of-plane response. Power was input by hammer impacts at three random locations in each subsystem.

### 4.1 Transmission coefficient, $\tau(\theta)$, and scaling factor, $Cnst$

The theoretical value of the integral of Eq. (17) depends only on the angle of incidence and material properties of the connected plates. Therefore, the value is constant for all frequency range and the values in both coupling directions are same. On the other hand, the experimental estimation of $\tau$ using $Cnst$ becomes frequency dependent, and depends also on the coupling direction because the value of $Cnst_{ij}$ at each frequency may be different from $Cnst_{ji}$, as shown in Fig. 6. Figure 7 demonstrates that the theoretical and the experimental values of the integral are different especially at lower frequency ranges. As frequency increases the experimental values of the integral tend to be close to the theoretical values. The reason is that, at high frequency ranges, the system meets the SEA or WIA assumptions such as high modal density and highly diffuse field better than in the low frequency ranges.

Thus, the theoretical models of SEA and WIA become more accurate at high frequency ranges and both give similar results. Consequently, at high frequency ranges, the theoretical and experimental models are more similar to each other and the SEA, ESEA, WIA and EWIA predictions are also similar, as shown in Figs. 8 and 9 (these figures are discussed in detail in the following chapter).

The theoretical CLF and ILF curves decay exponentially\(^2\). In contrast, experimental CLF and ILF curves are not smooth exponential curves and present an irregular decay, as shown in Figs. 4 and 5. The peaks that appear in the measured CLF curves for example, at 250 and 400 Hz in Fig. 5, also appear in the $Cnst$ curve of Fig. 6 because $Cnst$ is obtained from the CLF data, as shown in Eq. (15). These peaks are caused by irregularities in material properties and boundaries or measuring errors during the experiments. They can also be produced when the system does not meet the SEA or WIA assumptions like highly diffuse field, weak coupling or low modal density, especially in the lower frequency ranges.
4.2 Energy predictions

4.2.1 Comparison of WIA and ESEA

Figure 8 shows a comparison between measured energy ratio, ESEA prediction and analytical WIA prediction in two different cases: one applying constant ILFs, which is estimated by averaging the ILF in the higher frequency range, and the other applying experimental ILF data. ESEA gives a very good prediction at all frequencies because of the simplicity of the system. WIA using constant ILF shows good agreement at mid frequency ranges and very good agreement at the higher frequency ranges but tends to overestimate the energy at lower ranges. The predictions of the WIA using experimental ILFs, in general, follow the measured data. At the lower frequency ranges, there is in general a good improvement compared to the WIA predictions using constant ILFs. On the other hand, the results are less accurate than the ESEA prediction. At higher frequency ranges, the WIA prediction with experimental ILFs tends to follow the measured data. The results become poorer than the ESEA estimations at 500 and 630 Hz. At these frequencies, the WIA model fails to follow the measured energy. In the higher frequency ranges, the prediction becomes very close to the measured data at each frequency and improves the results of WIA using constant ILFs and the ESEA estimation.

4.2.2 Comparison of WIA and EWIA

Figure 9 compares the measured data with both WIA and EWIA predictions. At the lower frequency ranges, the EWIA greatly improves the results of both WIA predictions. At some frequencies, the EWIA predictions are very close to the measured data. In the mid frequency ranges, the EWIA predictions also follow the measured data. Good improvements are achieved especially at 500 and 630 Hz comparing to the WIA using experimental ILF. In the higher frequency ranges, the results are in good agreement with measured data and similar to both WIA cases.

The reason for the general good estimations of EWIA especially at 500 and 630 Hz is the use of the scaling factor, \( C_{nst} \), to adjust the transmission coefficients as explained in §§ 3.3 and 3.4. This highlights that the difference between the theoretical (WIA) and experimental (EWIA) transmission coefficients becomes more important at lower frequency ranges, where experimental estimations are more accurate than their theoretical estimations. Therefore, the energy predictions can be improved at these frequency ranges. For this reason, EWIA can be expected to improve WIA results in more complex systems and lower frequency ranges, since experimental models are more accurate for complex systems and for a wider frequency range than theoretical models.

4.2.3 Comparison of ESEA and EWIA

Figure 10 shows a comparison between measured data, the ESEA and EWIA predictions. The EWIA and ESEA estimations give a similar accurate prediction at all frequencies because the system analyzed in this paper can be considered highly diffuse. The first subsystem is highly diffuse because power is equally input in every direction. Since the material properties of the first and second subsystem are same, except for the length, the wave energy filtered at the boundary between the first and second subsystems is very small. Then, similar amount of energy would be transmitted in every direction and the energy field of the second subsystem could be diffuse. Structures with a very regular single boundary create predominant energy directions perpendicular to the boundary. However, the filtering effect is too small and do not have effect, so that the non-diffuse model cannot be much more accurate than the diffuse model. Differences between the ESEA and EWIA predictions are not important as expected from highly diffuse fields and, as a result, the results validate the EWIA method proposed in this paper.
4.3 Maximum and minimum values of scaling factor, $Cnst$

WIA introduces the Fourier series expansion to describe the energy terms in the power balance equation. Thus, boundary conditions can be defined in some extent what gives the possibility of adjusting the WIA model. The EWIA method introduces also the scaling factor $Cnst$, which can be calculated in several ways. Depending on the way this factor is calculated, the models can be farther adjusted, especially in the lower frequencies.

Equation (22) shows that the theoretical $\tau$ in one coupling direction (coupling from subsystems $j$ to $i$) is proportional to the theoretical $\tau$ in the opposite direction (coupling from subsystem $i$ to $j$). Moreover, $Cnst$ was defined in Eq. (20) as the ratio between the scaling factor of each coupling direction, $Cnst_{ji}$ and $Cnst_{ij}$. Then, Eq. (22) scales the theoretical $\tau$ from subsystem $i$ to $j$. However, Eq. (22) could also have been presented scaling $\tau$ in the opposite coupling direction, from subsystem $j$ to $i$. The only difference is that the ratio between the scaling factors of Eq. (20) in one coupling direction is the inverse of the ratio in the opposite coupling direction. Thus, the values of each term in Eq. (22) do not change. But it is often found that depending on what coupling direction is scaled there are some difference in the predictions, especially at lower frequency ranges as shown in Fig. 11.

Two approaches are considered in this paper. The first one scales the direction with higher $Cnst_{ji}$ value for each frequency, Maximum curve in Fig. 11. The second one scales the direction with the lower $Cnst_{ji}$ value for each frequency, Minimum curve in Fig. 11. Figure 11 shows the discrepancy between the two approaches. A better prediction is obtained when using the higher $Cnst$ values. However, the difference is negligible for simple systems and the selection of $Cnst$ does not greatly affect the results.

5. Results of a non-highly diffuse system

In this chapter, EWIA is applied to a row of three steel plates. The configuration of this structure promotes energy transmission in the longitudinal direction of the structure as shown in Fig. 2(b) and the energy field becomes gradually less diffuse as the wave energy crosses each boundary. EWIA and ESEA results are compared to measured energies for this system. The structure consists of three steel plates welded at the connecting boundaries. All of the plates have Young’s modulus $E=2.11\times10^{11}$ N/m$^2$, Poisson’s ratio=0.3 and density $\rho=7800$ kg/m$^3$. The dimensions of each plate are shown in Table 1. The experimental data (ILFs and CLFs) was obtained using the PIM. The structure was supported at the corners and both extremes of the junctions between subsystems by small rubber blocks. Then, out-of-plane response was measured at six random locations in each plate and power was
input by hammer impacts at two random locations for each subsystem. The energy predictions were obtained with input energy in Subsystem 1.

Table 1  Material properties of plates [Units: m]

<table>
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<tr>
<th>Plate</th>
<th>Thickness</th>
<th>Length</th>
<th>Width</th>
</tr>
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<tr>
<td>#1</td>
<td>0.004</td>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td>#2</td>
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<td>0.7</td>
<td>1</td>
</tr>
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</tbody>
</table>

Figure 12 shows a comparison between measured energies, and the EWIA and ESEA predictions for each subsystem. WIA and SEA predictions are not included since the analysis would be similar to the one presented in § 4. The results show that EWIA improve ESEA results at all frequency ranges especially for Subsystem 3. The reason is that the energy field becomes less diffuse as wave energy is transmitted through the boundaries.

The deviation for the EWIA and ESEA predictions respect to the measured energies averaged for all the frequencies is shown in Table 2. This table also includes the average error at each subsystem. The results shown in Table 2 indicate that EWIA slightly improves the ESEA predictions for Subsystem 1. The EWIA and ESEA predictions are very similar for Subsystem 2. The reason for this similitude is that the energy filtering effect between Subsystem 1 and 2 is not important because both subsystems are separated by one single boundary. On the other hand, for Subsystem 3, the wave energy is transmitted across two boundaries from the source (Subsystem 1) and the energy filtering effect becomes more relevant. Thus, greater differences appear between the EWIA and ESEA energy predictions for this subsystem. As a result, the error of the ESEA prediction for Subsystem 3 is clearly reduced by EWIA, which is about a 38% lower (Table 2).

Although EWIA improved the ESEA results, there is still a difference between the predicted and measured energies at every subsystem. The ESEA method does not provide any data to explain the errors in the ESEA results but the EWIA method provides additional information about the energy field that can be used to explain those errors through a more detailed analysis. Under ESEA assumptions, the energy field of Subsystem 1 is expected to be highly diffused because the input power is same in every direction. On the other hand, Fig. 13 shows that the distribution of the energy field in Subsystem 1 is not highly diffused.
These results suggest that the subsystems are strongly coupled and an important amount of energy is transmitted from Subsystem 1 to the other subsystems. Since ESEA assumes a weak coupling between subsystems, the error can be attributed to the experimental data and therefore affects both, ESEA and EWIA predictions. Subsystems 2 and 3 are also non highly diffuse since the filtering effect contributes to energy transmission in direction perpendicular to the connecting boundaries (x-direction), as shown in Fig. 3.

\[
\begin{align*}
[E(\theta)/E_{\text{max}}] \sin(\theta), \\
[E(\theta)/E_{\text{max}}] \cos(\theta)
\end{align*}
\]

Fig. 13 Wave energy distribution in 3-plate system.

6. Conclusions

This paper describes a formulation of a novel experimental WIA method, which is named EWIA. The EWIA method requires two parameters, ILFs and \(\tau\). ILFs are measured by the PIM and \(\tau\) is estimated by scaling theoretical \(\tau\) at each frequency and coupling direction. The factors to scale the theoretical \(\tau\) are named \(\text{Cnst}\) and these are calculated from measured CLFs using simple mathematical relationships between CLF and \(\tau\). Therefore, calculating \(\tau\) from measured intensity vectors is not necessary in EWIA. Instead of the intensity measurement, EWIA uses ESEA models (CLF and ILF), thus the experiments are simplified. The advantages of EWIA are the possibility of using simple ESEA models, improvement of the WIA results at mid frequency ranges, improvement of ESEA results for non-highly diffuse fields and providing additional information about the energy field.

The EWIA method was applied to a simple L-shaped and a three-plates system. Then, measured energies, and the ESEA and WIA predictions were compared. The results show that EWIA improves the WIA predictions at lower frequency ranges. The reason is that the experimental models used by EWIA are more accurate for a wider frequency range than theoretical models used by WIA. EWIA also improves ESEA results when the energy field is non-highly diffuse because it takes into account the filtering effect of the transmitted energy, which is neglected by ESEA. The results also show that the EWIA provides very useful additional information about the energy field, which allows a more detailed analysis of the structure. Next paper concerning EWIA will be the application to complex systems.

References