Active Vibration Control for Suspension by Considering Its Stroke Limitation*

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Abstract

When large external forces come from the road, a suspension stroke reaches its limitation and riding comfort may decrease. To overcome this problem, we propose a new control method for an active suspension that can avoid reaching the stroke limitation. A sliding mode controller is designed by considering the rigidity variation of a spring. Also, in order to estimate the internal state of the suspension, a variable structural system (VSS) observer is designed without the information of nonlinear force occurring in the rigidity variation when the suspension reaches the stroke limitation. By carrying out simulation and experiment of a quarter-car model, it is verified that the performance of the controller is superior to that of the method, which switches to a passive damper near the stroke limitation from a linear quadratic regulator (LQR) in a small stroke range.

Key words : Active Suspension, Robust Control, Stroke Limitation, Sliding Mode Control, VSS Observer

1. Introduction

When large external forces come from the road, a suspension reaches the stroke limitation and exhibits a nonlinear property due to an increase in stiffness. This may make riding comfort worse. A patent of a semiactive suspension system has been filed, where sky-hook control is applied in a small stroke and a high damping coefficient in a large stroke in order to avoid reaching the limitation⁽¹⁾. However, there is a possibility of reaching the stroke limitation by applying only the high damping coefficient such that riding comfort may become worse. In the case of an active suspension, it might be possible to control the actuator used to avoid reaching the stroke limitation.

There are some studies on the active suspension that consider stiffness variation using μ synthesis⁽²⁾, sliding mode control⁽³⁾, and fuzzy control⁽⁴⁾. Although in Refs. (3), (4) control experiments have been performed by considering spring nonlinearity in which stiffness increases when the stroke exceeds a certain range⁽⁵⁾, the control strategy used has not assumed that the spring reaches the stroke limitation, but has only a narrow stiffness variation range.

In Ref. (6), a method of avoiding riding comfort reduction by decreasing the gain of the controller in a large road disturbance, which is estimated from the amplitude of the suspension stroke, has been proposed. Although the controller has been designed so as to maintain the stroke in a certain range based on a regulated road disturbance⁽⁷⁾, the suspension stroke might reach the limitation due to various road shapes.

To overcome these problems, we propose a new control method for an active suspension that can avoid reaching the stroke limitation. A sliding mode controller is designed by considering the rigidity force variation smoothly given by the actuator before the stroke reaches the limitation. Also, in order to estimate the internal state of the suspension, a variable structural

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system (VSS) observer is designed without the information of nonlinear force occurring in the rigidity force variation. It is verified from the results of the simulation and experiment of a quarter-car model that the performance of the controller designed is superior to that of the method, which switches to a passive damper near the stroke limitation from a linear quadratic regulator (LQR) in a small stroke range.

2. Modeling

An experimental setup of a quarter-car model is shown in Fig. 1 (a), and its specifications are shown in Table 1. The resonant frequencies of the sprung and unsprung masses are 1.19 and 11.7 Hz, respectively. Sprung mass acceleration is detected using an acceleration sensor, and the relative displacement between the sprung and unsprung masses is detected using a laser sensor. A linear motor is installed between the sprung and unsprung masses as the actuator, whose normal thrust is 73 N. The lowest plate is excited using an AC servo motor connected through a ball screw to generate velocity disturbance from the road.

Figure 1 (b) shows the dynamical model. By using the relative displacement $x_s = x_b - x_w$ between the sprung and unsprung masses, the relative displacement $x_t = x_w - x_r$ between the unsprung mass and the road, the sprung mass velocity $\dot{x}_b$, the unsprung mass velocity $\dot{x}_w$, and the velocity disturbance $d = \dot{x}_r$, the equations of motion of the dynamical model are described as

$$M_b \ddot{x}_b = -K_s x_s - C \dot{x}_b + C \dot{x}_w + u + F_{nl}, \quad (1)$$

$$M_w \ddot{x}_w = K_s x_s - K_t x_t + C \dot{x}_b - (C + C_t) \dot{x}_w - u + C_t d - F_{nl}, \quad (2)$$

where u is the control input from the linear actuator. The acceleration of the sprung mass $\ddot{x}_b$ is used as a feedback signal. F_{nl} is the following nonlinear input introduced to change the suspension stiffness in the stroke range from 5 to -5 mm.

$$F_{nl} = \hat{F}_k + K_s x_s + f_c(x_s, \dot{x}_s), \quad (3)$$

where $\hat{F}_k$ is given by

$$\hat{F}_k = -\hat{K}_s x_s \quad (4)$$

Fig. 1 Quarter-car suspension system

Table 1 Parameters of experimental setup

Parameter	Value	Unit
M_b	94.0	kg
M_w	13.1	kg
K_s	5720	N/m
K_t	64450	N/m
C	8	Ns/m
C_t	10.5	Ns/m

Fig. 2 Characteristics of spring force $\hat{F}_k$

and the variable spring constant $\hat{K}_s$ is defined by a hyperbolic function of x_s ⁽⁸⁾ as

$$\hat{K}_s = \begin{cases} K_s \left(\frac{\tanh(x_s/\beta)^5}{(x_s/\beta)^5} \right)^{-1} & (|x_s| \leq \beta) \\ K_s \frac{1}{\tanh(1)} \left(\frac{x_s}{\beta} \right)^8 & (|x_s| > \beta) \end{cases} \quad (5)$$

$\beta = 0.00374$.

Figure 2 (a) shows the relation between x_s and $\hat{K}_s$. Using this continuous $\hat{K}_s$ variation, we obtain the relation between x_s and $\hat{F}_k$, as shown in Fig. 2 (b). The damping force f_c is given by Eq. (6) according to the stiffness variation.

$$f_c(x_s, \dot{x}_s) = \begin{cases} -800\dot{x}_s & (|x_s| > 0.004) \\ 0 & (|x_s| \leq 0.004) \end{cases} \quad (6)$$

The second term on the right-hand side of Eq. (3) draws the spring force from $\hat{F}_k$ in the linear range.

To eliminate the offset and noise from the detected acceleration signal of the sprung mass, the band-pass filter of 0.4 to 30 Hz as shown in Fig. (3) is used and the system is augmented including the state x_f of the filter. The state equation (7) and output equation (8), where y_f is the output from the filter, are

$$\dot{x} = Ax + B_u u + B_w d + B_{nl} F_{nl}, \quad (7)$$

$$y_f = Cx + D_u u + D_w d + D_{nl} F_{nl}, \quad (8)$$

where

$$x = \begin{bmatrix} x_s & x_t & \dot{x}_b & \dot{x}_w & x_f^T \end{bmatrix}^T.$$

3. Controller design

3.1. Design of sliding mode control

Applying sliding model control to the systems (7) and (8), the control input u and switching function σ are given as Eqs. (9) and (10).

$$u = u_l + u_n, \quad (9)$$

$$\sigma = Sx, \quad (10)$$

where

$$u_l = -F_s x, \quad u_n = -k \frac{\sigma}{|\sigma| + \eta},$$

$$k = k_c (F_s B_u)^{-1}, \quad S = F_s.$$

Fig. 3 Transfer function of band-pass filter

Here, u_l is the linear term of the control input and u_n is the nonlinear term due to a switching function. k is the switching width of u_n , and η and k_c are positive constants. The feedback gain F_s based on the optimal control theory is applied as a switching hyperplane S . F_s is obtained as

$$F_s = \begin{bmatrix} 1.75 & -390.8 & 299.3 & -122.4 & O_{1 \times 6} \end{bmatrix},$$

minimizing the following criterion function

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \quad (11)$$

$$Q = \begin{bmatrix} 8000 & 0 & 0 & 0 \\ 0 & 910 & 0 & 0 \\ 0 & 0 & 35000 & 0 \\ 0 & 0 & 0 & 8200 \\ O_{6 \times 4} & O_{6 \times 6} \end{bmatrix}, R = 0.4$$

subjected to the system (7). In the criterion function (11), not only a large weight on the sprung mass velocity $\dot{x}_b$ is used on the basis of the sky-hook control theory, but also similar large weights on the stroke x_s and unsprung mass velocity $\dot{x}_w$ are used to avoid the resonant vibration of the unsprung mass.

To transit the system to a sliding mode, σ is required to satisfy the following inequality.

$$\sigma \dot{\sigma} < 0 \quad (12)$$

From Eqs. (10) and (12), when $\eta = 0$, the sliding mode condition is

$$\begin{aligned} \sigma \dot{\sigma} &= x^T F_s^T F_s \{Ax + B_u(u_l + u_n) + B_w d + B_{nl} F_{nl}\} \\ &= x^T F_s^T F_s (A - B_u F_s) x - k_c |\sigma| + \sigma F_s (B_w d + B_{nl} F_{nl}) < 0 \end{aligned} \quad (13)$$

The first term on the right-hand side of Eq. (13) is negative definite because the closed loop is stabilized by the feedback gain F_s . Since $F_s^T F_s$ is positive semidefinite and $x^T F_s^T F_s (A - B_u F_s) x \leq 0$, Eq. (13) can be described as

$$-k_c |\sigma| + \sigma F_s (B_w d + B_{nl} F_{nl}) < 0. \quad (14)$$

From Eq. (14), if a positive constant k_c satisfies

$$k_c > |F_s (B_w d_{max} + B_{nl} F_{nlmax})|, \quad (15)$$

$\sigma \dot{\sigma} < 0$, where d_{max} is the maximum amplitude of d , and F_{nlmax} is that of F_{nl} . In the nominal model, where $F_{nlmax} = 0$, Eq. (15) is describes as

$$k_c > |F_s B_w d_{max}|. \quad (16)$$

From the constraints of the nominal thrust (73 N) of the linear actuator and the mechanism of the experimental setup, we use $d_{max} = 0.03$ m/s and $F_{nlmax} = 70$ N. In this case, we set k_c to be 900, since the value of term on the right side of Eq. (15) is approximately 886. To avoid input chattering, $\eta = 50$ is applied.

3.2. Design of VSS observer

To estimate the state for sliding mode control, a VSS observer (10)–(12) is designed for the system involving the nonlinear terms of Eqs. (7) and (8). Equations (17) and (18) are the state and output equations, respectively.

$$\dot{\hat{x}} = (A_0 - GC_0)\hat{x} + (B_u - GD_u)u_v + Gy_f + P_0^{-1}C_0^T F_o^T H, \quad (17)$$

$$\hat{y}_f = C_0\hat{x} + D_u u_v, \quad (18)$$

$$u_v = u + 800\dot{x}_s, \quad (19)$$

where $\hat{x}$ is the estimated augmented system state and $\hat{y}_f$ is the estimated output y_f . G is the linear observer gain when $H = 0$. When an observer is designed for the system Eqs. (7) and (8), it is difficult to assign the poles far from the imaginary axis. A damping coefficient of 800 Ns/m is added to the observed systems (7) and (8) in order to improve estimation accuracy. After the observer design, the damper force is removed from the system as an external force in the estimation process⁽⁹⁾. To distinguish damper inclusion, A and C in Eqs. (7) and (8) are changed to A_0 and C_0 , respectively.

H is a structural variation term in the VSS observer as

$$H = -\rho_o \frac{F_o(\hat{y}_f - y_f)}{\|F_o(\hat{y}_f - y_f)\| + \eta_o}, \quad (20)$$

where F_o is a constant satisfying

$$C_0^T F_o^T = P_0 B_u. \quad (21)$$

P_0 is a unique positive symmetric solution of the Lyapunov equation

$$A_0^T P_0 + P_0 A_0 = -Q_0. \quad (22)$$

After the trial and error to improve the accuracy, we selected $F_o = -0.189$ and $Q_0 = 0.07A_0A_0^T$. Note that since the observed output is scalar, F_o is also scalar. ρ_o , the switching width of H , is set to be 10 for accurate estimation even if F_{nl} exists. η_o , a positive constant for avoiding H chattering is set to be 1.

3.3. Verification by simulation

The results of control performance and estimation using the sliding mode control and VSS observer are verified from simulation results. Figure 4 (a) and (b) show the frequency

Fig. 4 Gain of frequency responses of nominal model

response functions of the sprung mass acceleration and unsprung mass velocity to the road velocity in the case of the nominal model without stiffness variation, respectively. The solid line indicates the gain of a passive damper whose damping coefficient is 200 Ns/m, and square plots show that of the sliding mode control. Since the sliding mode control is nonlinear, the road velocity amplitude is fixed at 0.02 m/s and the stationary responses are used to obtain Fig. 4. From Fig. 4, it is seen that the sliding mode controller can obtain a gain reduction of 10 dB compared with the passive damper in the frequency range of the sprung mass resonant frequency. In the range from 2 to 10 Hz, the gain is almost the same.

Figure 5 shows the time-history response of the system with stiffness variation to the road velocity whose amplitude is 0.03 m/s and frequency is 1.2 Hz. Figure 5 (a) shows the sprung mass acceleration, (b) unsprung mass velocity, (c) stroke between the sprung and unsprung masses, and (d) control input. In (a) to (c), the broken line indicates the response of the passive control whose damping coefficient is 200 Ns/m and the solid line indicates that

Fig. 5 Simulation results with stroke limitation

Fig. 6 Comparison of real and estimated states using VSS observer

of the sliding mode control. From Fig. 5, it is seen that the sliding mode control can suppress the high-frequency vibrations of the sprung and unsprung masses even if x_s reaches 4 mm compared with the passive damper. From Fig. 5 (d), it is verified that the sliding mode control exerts the nonlinear control input u_n to compensate the excessive acceleration which the passive damper causes.

Figure 6 shows a comparison between the real and estimated states obtained using the VSS observer. The estimated errors of the sprung and unsprung masses are less than 20 %, and the estimation delay is less than 0.1 s. Figure 6 shows the usefulness of the VSS observer designed.

4. Control experiment

A control experiment was performed, assuming the road disturbance of trapezoidal shape shown in Fig. 7. The time-history response of the nominal system without stiffness variation

Fig. 7 Trapezoidal road displacement

Fig. 8 Experimental results without stroke limitation

Fig. 9 Experimental results with stroke limitation

is shown in Fig. 8 where the solid line indicates the response of the sliding mode control and the broken line indicates that of the LQR. The stroke velocity shown in (c) is calculated by the differentiation of the signal detected by the laser displacement sensor. Since the LQR is designed so as to have the same control performance as that of the sliding mode control, the responses coincide with each other. The states are estimated using the VSS observer.

The solid line in Fig. 9 shows the time-history response of the nonlinear model with stiffness variation by the sliding mode control. The broken line indicates the results of switching control in which the LQR is used in the amplitude range of ± 4 mm and the passive damper whose damping coefficient is 350 Ns/m is used in the upper range. Because even in the simulation the LQR made the system unstable at the stroke limitation, we intended to stabilize the system using a passive damper. From Fig. 9, although the switching control can stabilize the system at about 0.72 s even if the stroke x_s reaches -4 mm and the sprung mass acceleration reaches 0.75 m/s^2 , it becomes unstable at about 1.14 s when x_s reaches the limitation of 5 mm. The passive damper switched could not stabilize the system at that moment.

The sliding mode control keeps the system stable, avoiding reaching the stroke limitation, and also suppresses the sprung mass acceleration to 0.6 m/s^2 at about 7.2 s by 20 % improvement compared with the switching control. The sliding mode control does not generate such a large input as the switching control in Fig. 9 (d) at about 0.72 s and can suppress the sprung mass vibration. The nonlinear input F_{nl} in the case of the sliding mode control ranges from about 70 to 90 N. These results show that the stiffness variation is generated as we assumed by the road disturbance shown in Fig. 7 and that the sliding mode control designed can not only avoid reaching the stroke limitation, but also generate an appropriate control input to suppress the sprung mass acceleration and unsprung mass velocity.

5. Conclusions

A sliding mode control system involving state estimation using a VSS observer was de-

signed for a quarter-car active suspension model with its stiffness variation to avoid reaching the stroke limitation. We also performed a simulation and an experiment to verify the performance of control. The results are summarized as follows:

- We introduced the stiffness variation according to the suspension stroke and added the variation force by the actuator thrust to avoid reaching the stroke limitation.
- The sliding mode control system based on the VSS observer was designed, and its effectiveness in suppressing the sprung and unsprung mass vibrations was verified from the simulation results.
- A control experiment was performed with the road disturbance of trapezoidal shape, and we verified that the sliding mode control system can suppress the sprung mass acceleration and unsprung mass velocity, although the switching control of the LQR for a small stroke and the passive damper near the stroke limitation becomes unstable.

Note that the performance of control cannot be guaranteed owing to the thrust limitation of the actuator. This study shows that the improvement of riding comfort in the stroke limitation range is possible in the specification of an actuator.

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