Contacts within Valve Train Simulations: a Comparison of Models*

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Abstract

In this work different contact models within valve train simulations are investigated. Two basic approaches are opposed, namely rigid and flexible models. The rigid model is described by a unilateral constraint, for the group of flexible models a linear spring-damper-element and different variants of HERTZIAN contacts are used. Frictional effects are considered by COULOMB's law in the original and a regularized formulation.

The models are discussed in terms of their influence on the dynamics of the valve train system, especially when they are used in diverse configurations.

For the numerical analysis the system size has been varied and extended to the entire valve train with all cylinders and valve mechanisms.

Key words : Valve Train Simulation, Multi-Body Dynamics, Contact, Friction

1. INTRODUCTION

During the last years much effort has been devoted to develop sophisticated models for the multi-body simulation of valve trains. In order to account for all important dynamical effects it is not sufficient to describe the motion of the valve train via a kinematical constraint between the cam and the opponent. A detailed contact model becomes necessary.

To describe the contact, two basic approaches can be distinguished within multi-body simulations: the first approach assumes the contact zone to be rigid and therefore no penetration of the bodies is allowed. As a consequence, the contact forces can not be calculated explicitly with respect to the state coordinates. For their computation, additional mathematical constraints are needed.

The second approach comprises several models which are all based on an analytical relation between penetration depth and contact force. The elasticity as well as the damping within the contact zone is considered using different empirical or theoretical models.

Although the general assets and drawbacks of both approaches are well known and investigated, especially for isolated contact problems, the choice is not always obvious for problems where the main focus lies on the global system dynamics.

A suitable example to compare both approaches is an automotive valve train, as it is a complex industrial problem having several different types of contacts. To begin with, the mechanical system – a valve train with a rocker arm setup – is described. Next, different contact models are outlined briefly, beginning with an exemplary formulation of the contact kinematics for the contact between cam and roller body. For the rigid approach, a formulation in terms of an unilateral constraint is presented. Two widespread representatives of the second approach, namely a linear spring-damper-element and HERTZIAN contact models, have been chosen to be analyzed. Independent of the underlying contact model, friction is also taken into account. COULOMB-friction as well as a regularized form of COULOMB-friction is utilized in this study. In Sec. 4 the model of the valve train is described. Thereby, numerous variations are considered, using different models for the contacts as well as for the bearings. Afterwards, the simulation results are presented, comparing the multiple valve train models. In this context,

Fig. 1 Valve Train in rocker arm setup⁽²⁾.

Fig. 2 Kinematics of the contact between cam and roller bodies.

attention is drawn onto the influence of a certain contact model on the dynamics of the valve train, especially with respect to the contact forces in the cam-roller contact. Moreover, the models are evaluated with respect to computing times.

2. MECHANICAL SYSTEM

In this work, a valve train of a six-cylinder diesel engine is investigated (see Fig. 1). It has six cylinders in a row and one single cam-shaft to actuate the inlet and exhaust valves as well as the unit injectors. The valve train design is a classical overhead camshaft layout with three rocker arms per cylinder. Each rocker arm setup consists of the cam with its profile, the roller follower, the rocker arm, the valve side masses and the valve springs. The rocker arms are shaft-mounted. Due to the dynamics of the system in combination with a finite valve lash, impacts can occur between the cam and the roller or between the rocker arm and the caliper. The valves are kinetically excited by the cylinder pressure. Similarly, the injector rocker arms are excited by the force from the injection pressure.

The valve spring pushes the rocker against the cam contour to ensure a persisting contact and the desired valve lift. In case the eigenfrequencies of the valve springs are excited, it is possible that the contact force decreases and the contact opens. This may also happen due to inertia forces at high engine speeds.

3. CONTACT MODELS

3.1. Contact Kinematics

The evaluation of the contact kinematics needs to be carried out for all contacts in the system, no matter which specific contact model is used to describe the corresponding contact. As an example for the approach, the necessary steps for the contact between cam and roller (compare Fig. 2) are pointed out briefly, a detailed description can be found in Ref. (4).

At a particular time, the positions $\mathbf{q}$ and velocities $\mathbf{u}$ of the system are known. For the description of the cam contour, a cubic spline interpolation with 50-100 segments, depending on the actual cam contour, is used. The contour is described by its coordinates $K2y = K2y(\alpha)$

and ${}_{K2}z = {}_{K2}z(\alpha)$ being 2π -periodic functions of the angle. Therefore, a match of the first and second derivative of the first and the last point has to be ensured. The vector $\mathbf{r}_{PS2}$ from the rotational axis of the cam body to a contour point follows

$${}_{K2}\mathbf{r}_{PS2} = {}_{K2}\mathbf{r}_{PS2}(\alpha) = \begin{pmatrix} 0 \\ {}_{K2}y(\alpha) \\ {}_{K2}z(\alpha) \end{pmatrix}. \quad (1)$$

The normal and the tangential vectors $\mathbf{n}$ and $\mathbf{t}$ as well as the curvature κ of the cam contour are obtained using standard plane geometry formulations⁽¹⁾.

Solving the necessary condition

$$f(\alpha, \mathbf{q}, t) = \begin{pmatrix} \mathbf{r}_D^T \mathbf{t}_1 \\ \mathbf{n}_1^T \mathbf{t}_2 \end{pmatrix} = 0, \quad (2)$$

the current contour parameter α is obtained. The distance g_N between the two bodies results for example from $g_N(\mathbf{q}, t) = -\mathbf{r}_D^T \mathbf{n}_1$. In order to calculate the velocities

$$\dot{g}_N = \mathbf{W}_N^T \dot{\mathbf{q}} \quad (3a)$$

$$\dot{g}_T = \mathbf{W}_T^T \dot{\mathbf{q}} \quad (3b)$$

in the normal and the tangential direction, the matrices $\mathbf{W}_N$ and $\mathbf{W}_T$ are needed:

$$\mathbf{W}_N^T = \mathbf{n}_1^T \mathbf{J}_{C1} + \mathbf{n}_2^T \mathbf{J}_{C2} \quad (4a)$$

$$\mathbf{W}_T^T = \mathbf{t}_1^T \mathbf{J}_{C1} + \mathbf{t}_2^T \mathbf{J}_{C2} \quad (4b)$$

$\mathbf{J}_{C1}$ and $\mathbf{J}_{C2}$ are the Jacobians of the two bodies in the contact point.

3.2. Force Laws In Normal Direction

In order to establish different force laws for the normal direction, a decomposition of the contact force $\vec{F}$ in the form

$$\vec{F} = \vec{n} \lambda_N + \vec{t} \lambda_T \quad (5)$$

is used. The scalar λ_N is the absolute value of the contact force in the normal direction and λ_T is the scalar friction force, being projected into the tangential direction by $\vec{t}$. Furthermore, for all force laws except for the rigid contact, λ_N is split into a stiffness induced part λ_{Nc} and the damping influence λ_{Nd}

$$\lambda_N = \lambda_{Nc} + \lambda_{Nd}. \quad (6)$$

3.2.1. Rigid Contact For the rigid approach the contact is described by a discrete point model where the stiffness in the contact zone is assumed to be infinite, resulting in a constraint and *not* an explicit functional relation with respect to the state variables. This assumption can be described by the *Signorini-Fichera*-condition

$$g_N \geq 0, \quad \lambda_N \geq 0, \quad g_N \lambda_N = 0. \quad (7)$$

It ensures that the normal contact force is zero if the contact is open. In case the contact is closed, λ_N is calculated such that the kinematical constraint ($g_N \geq 0$) is satisfied.

The force law of a unilateral constraint on a position level according to (7) holds for continuous contact and detachment. A closing contact must be treated separately as it implies a discontinuity in the relative velocities of the colliding bodies in order to avoid penetration. Moreover, the effect of an impact of a specific contact may concern all other unilateral constraints in the system. Considering a collision at the discrete point in time t_i , i.e. $g_N(t_i) = 0$, this requirement can be expressed with a relation between the post-impact velocity

$$\dot{g}_N^+ = \lim_{\varepsilon \rightarrow 0^+} \dot{g}_N(t_i + \varepsilon) \quad (8)$$

and the impulsive force

$$\Lambda_N = \int_{t_i^-}^{t_i^+} \lambda_N dt \quad (9)$$

in terms of

$$\dot{g}_N^+ \geq 0, \quad \Lambda_N \geq 0, \quad \dot{g}_N^+ \Lambda_N = 0. \quad (10)$$

From the physical point of view, the impact law (10) corresponds to a completely inelastic shock. It should be mentioned that this impact behavior can not be modeled with any non-rigid approach, e.g. spring-damper-elements or HERTZIAN contacts.

One way to achieve a non-plastic impact behavior is to substitute the post-impact velocities in (10) by an interpolation of the form

$$\dot{g}_N = \varepsilon_N \dot{g}_N^- + \dot{g}_N^+ \quad (11)$$

leading to a unilateral formulation of Newton's impact law. The restitution coefficient $0 \leq \varepsilon_N \leq 1$ allows to choose an impact behavior between fully plastic and fully elastic.

3.2.2. Linear Spring-Damper-Element The simplest way to model a contact considering elasticity is to add a spring-damper-element with a very high stiffness between the bodies that pushes them apart as soon as the normal distance becomes negative. The scalar force value λ_N is zero when the two bodies are separated, i.e. $g_N > 0$. In case g_N is negative, the bodies are penetrated and the scalar value λ_{Nc} is given by

$$\lambda_{Nc} = -c g_N. \quad (12)$$

The scalar value λ_{Nd} can easily be obtained using

$$\lambda_{Nd} = d \dot{g}_N \quad (13)$$

and, depending on the application, it can be set to zero when the two bodies are moving apart as sometimes tractive forces are not present and therefore not desired to be considered.

3.2.3. Hertzian Contacts In case two elastic bodies with curved surfaces (cylinders or spheres) are pressed together, the elasticities of the bodies lead to a flattening at the contact point and to a certain contact area. HERTZ has analyzed the characteristic pressure distribution along this contact area and stated a theory how to calculate the size and the shape of the pressure area as well as the distribution of the stresses below the contact areas. The theory can also be used to calculate the contact forces as stated below. For a detailed description of the theory see Ref. (5).

The equivalent YOUNG's modulus $\hat{E}$ in order to account for different materials of the two contact bodies follows

$$\hat{E} = \frac{E_1 E_2}{E_1(1 - \nu_2^2) + E_2(1 - \nu_1^2)} \quad (14)$$

where E_1 and E_2 denote the individual YOUNG's moduli and ν_1 and ν_2 the POISSON's ratios of both materials.

One idea of the HERTZIAN theory is to describe the actual contact between two bodies with an equivalent contact of a sphere or a cylinder body with a plane. The equivalent radius $\hat{r}$ in our case is a function of the contour parameter α and follows

$$\hat{r}(\alpha) = \frac{r_{Cam}(\alpha) r_{Roller}}{r_{Cam}(\alpha) + r_{Roller}}. \quad (15)$$

The equivalent radius $\hat{r}(\alpha)$ needs to be evaluated in every time step after the contact kinematics are solved and the contour parameter α is known

$$r_{Cam}(\alpha) = \frac{\|\mathbf{r}'_{PS2}\|^3}{\|\mathbf{r}'_{PS2} \times \mathbf{r}''_{PS2}\|}. \quad (16)$$

Fig. 3 Different force laws for contacts.

The contact area itself can be considered to be a point or a line leading to different derivations for the pressure distribution and the resulting force laws. In the first case the normal force is computed by⁽⁵⁾

$$\lambda_{Nc} = \frac{4}{3} \hat{E} \sqrt{\hat{r}} |g_N|^{\frac{3}{2}}, \quad (17)$$

whereas one possible empirical solution for the line contact with the length l yields⁽⁶⁾

$$\lambda_{Nc} = 0.495 \hat{E} l^{0.919} |g_N|^{\frac{1}{0.925}}. \quad (18)$$

In analogy to the spring-damper-model, a linear damping force

$$\lambda_{Nd} = d \dot{g}_N \quad (19)$$

is considered, in order to account for the dissipation in the contact.

All contact models described above are opposed in Fig. 3. One can see the explicit functional relations of the flexible contact models (dashed and dotted lines) in contrast to the set-valued force law of the rigid approach (solid line).

3.3. Friction

In order to take also friction forces into account when a contact is closed, COULOMB's law as well as a regularized version of COULOMB's law is used.

3.3.1. Coulomb Friction With the scalar normal force λ_N and the relative tangential velocity $\dot{g}_T$ according to equation (3), COULOMB's law for planar friction is given by

$$\dot{g}_T = 0 \Rightarrow |\lambda_T| \leq \mu_0 \lambda_N \quad (20a)$$

$$\dot{g}_T \neq 0 \Rightarrow \lambda_T = -\frac{\dot{g}_T}{|\dot{g}_T|} \mu \lambda_N, \quad (20b)$$

where μ_0 and μ denote the corresponding friction coefficients. Relation (20a) describes the sticking phases whereas Eq. (20b) is associated with sliding.

In the presence of unilateral constraints, an impact event also affects the friction force. In analogy to the normal direction (10), COULOMB's friction law is transferred to the impulsive level

$$\dot{g}_T^+ = 0 \Rightarrow |\Lambda_T| \leq \mu_0 \Lambda_N \quad (21a)$$

$$\dot{g}_T^+ \neq 0 \Rightarrow \Lambda_T = -\frac{\dot{g}_T^+}{|\dot{g}_T^+|} \mu \Lambda_N \quad (21b)$$

and holds for all active constraints with $g_N = 0$.

Fig. 4 Exact and regularized COULOMB's law for planar friction.

Fig. 5 Setup of a single valve train module.

3.3.2. Regularized Coulomb Friction The time integration of the friction model described in the previous section can be done by a time-stepping or an event-driven integration scheme, see Ref. (3). In case an ODE-integrator is to be used, a regularized form of COULOMB's law is needed. One possibility is to smooth the set-valued force law of COULOMB as follows:

$$|\dot{g}_T| < \varepsilon \Rightarrow \lambda_T = -\frac{\dot{g}_T}{|\dot{g}_T|} \mu \left(1 - e^{-\frac{|\dot{g}_T|}{v_{\text{ref}}}}\right) \lambda_N \quad (22a)$$

$$|\dot{g}_T| > \varepsilon \Rightarrow \lambda_T = -\frac{\dot{g}_T}{|\dot{g}_T|} \mu \lambda_N \quad (22b)$$

In comparison to (20a), Eq. (22a) does not permit real sticking, but approximates it by a steep e -function (steepness $\frac{1}{v_{\text{ref}}}$), if the relative tangential velocity $\dot{g}_T$ falls below a predefined bound ε . This difference can also be seen in Fig. 4, where both friction curves are depicted.

4. CASE STUDIES

In Fig. 5 the setup of a single valve train module is shown. The planar model consists of the cam body, the roller, the rocker arm and a lumped mass representing the valve and the valve caliper.

The bodies have two translatory and one rotatory degree of freedom except for the lumped mass which moves along the valve axis only. As this work focuses on the contact models the valve spring exceptionally may be represented by a simple force element having a constant stiffness.

The roller follows the cam contour due to a frictional contact between these two bodies. The rotatory motion of the cam actuates the valve via the rocker arm. The valve spring is to ensure the realization of the desired lift curve by pressing the roller against the cam contour. To close the valve, it is pushed into the valve seat by the valve spring. The valve lash allows

Table 1 Combinations of the different models investigated in this work.

Instance No.	Bearings	Contacts	
		Cam-Side	Valve-Side
1	Rigid	Rigid	Rigid
2	Rigid	Spring-Damper	Rigid
3	Rigid	HERTZIAN Point	Rigid
4	Rigid	HERTZIAN Line	Rigid
5	Rigid	Rigid	Spring-Damper
6	Rigid	Spring-Damper	Spring-Damper
7	Rigid	HERTZIAN Point	Spring-Damper
8	Rigid	HERTZIAN Line	Spring-Damper
9	Spring-Damper	Rigid	Rigid
10	Spring-Damper	Spring-Damper	Rigid
11	Spring-Damper	HERTZIAN Point	Rigid
12	Spring-Damper	HERTZIAN Line	Rigid
13	Spring-Damper	Rigid	Spring-Damper
14	Spring-Damper	Spring-Damper	Spring-Damper
15	Spring-Damper	HERTZIAN Point	Spring-Damper
16	Spring-Damper	HERTZIAN Line	Spring-Damper

the contacts between cam and roller and rocker arm and lumped mass to open, which happens when the valve is closed and the roller is in the base circle of the cam contour.

The camshaft is modeled as three masses per cylinder. The masses are connected to each other by torsional springs and to the cylinder head via translational springs or rigid bearings.

The valves are excited kinetically by the cylinder pressure. Similarly, the injector rocker arms are excited by a force due to the injection pressure. The kinematic excitation of the camshaft is given as a function of time and is a result of a previous simulation of the entire engine including the vibrational behavior of the crankshaft and the timing gear drive⁽²⁾.

Due to the dynamics of the system in combination with a valve lash, impacts can occur between the cam and the roller, between the rocker arm and the caliper as well as in the valve seat. For each of the three contacts per valve module, the different contact models of Sec. 3 may be used. In Tab. 1 an overview of the investigated combinations including the bearing models for the camshaft segments and the rocker arms is given.

5. RESULTS

5.1. Model Properties

Firstly, some general aspects of the different contact models can be discussed regarding Fig. 6. In the illustrations of this figure the contact model for this contact is varied only. On top, the normal contact forces λ_N between the cam and the roller at the inlet valve is shown during one actuation cycle. The transient behavior of the rigid model and the spring-damper model is almost equivalent in this case. In the middle, the different penetration depths of the models become apparent. The influence on the contact forces are mainly noticeably in the frequencies and only have a comparably minor impact on the magnitudes. The last illustration in Fig. 6 corresponds to Fig. 3 and shows the different force laws within the simulation. The points in the plot are evaluated during one actuation cycle of the valve. Besides the characteristic corner law of the rigid approach at $g_N = 0$, the linear and the nonlinear behavior of the elastic models can be seen. The cloud of the HERTZIAN Point model is due to the different equivalent radius $\hat{r}(\alpha)$ in Eq. (17).

Fig. 7 compares the contact force for a completely rigid and a completely flexible model. The latter model uses spring-damper-elements for the bearings as well as for the contacts. In contrast to the flexible model, no oscillations occur in the rigid model but small perturbations due to the kinematic excitation of the camshaft are visible. Two other aspects have to be pointed out: firstly, the rigid model hits almost for the entire cycle the average value of the flexible model; secondly, the lag of the flexible model to build up the contact force can be seen at the beginning of the cycle.

Fig. 6 Normal contact forces λ_N (top) and distance g_N (middle) as a function of time t as well as the normal forces depending on the penetration depth (bottom) for the different contact models (Instances Nos. 13-16).

Fig. 7 Comparison of the contact force for a completely rigid and a completely flexible model (Instances Nos. 1 and 14).

Fig. 8 Tangential contact forces λ_T (top) and relative tangential velocities $\dot{g}_T$ (bottom) as a function of time t for the COULOMB and the regularized COULOMB law (Instances Nos. 13 and 14).

Fig. 9 Qualitative comparison of the normal contact force λ_N for $0.1 \text{ s} \leq t \leq 0.115 \text{ s}$ with rigid bearings (top) and flexible bearings (bottom).

As illustrated in Fig. 8, the regularization of COULOMB's friction law leads only to marginal differences regarding the tangential contact forces. Therefore, the influence of this approximation on the global dynamics of the system can be neglected, compared to the original friction law. However, no statement about possible sticking phases can be given using the regularized version (compare Fig. 8 bottom).

In complex technical systems, it is often difficult to trace back the characteristics of a simulation result (e.g. the transient behavior, the frequency, the amplitudes, etc.) to a component or even to a certain model type of this component. On the other hand, measurements give an idea or even the exact knowledge of the system behavior but it is not obvious which model is needed in order to build up a simulation which is as exact as possible to describe this specific system. In addition to that, the task gets more difficult as the combination of several model types may interfere with, erase or amplify each other.

Different models for certain components may lead to significant differences, even in a relative small system, when analyzing the transient behavior or the frequencies. Figure 9 gives qualitative results for an inlet valve module. The illustrations show variations of the bearing model (top and bottom row) as well as different combinations of the contact models (columns).

5.2. Numerical analysis

The computing time of a multi-body simulation is still a major aspect in the practical application, especially when performing parameter studies or optimizations in complex systems. Therefore, the influence of the system size of the valve train is analyzed, starting with the inlet valve mechanism only, adding the exhaust and inject to one cylinder module and finally ending up with the entire valve train consisting of six cylinder modules. Besides that, Tab. 2 compares also different integration methods for the flexible contact models (LSODE non-stiff / stiff, DOPRI5) and the rigid contact models (time-stepping algorithm⁽³⁾). The CPU-times are relative values compared to the simulation time for the inlet mechanism using the time-stepping integrator.

6. CONCLUSION

A detailed comparison between four different contact models for a valve train has been presented. The analysis was not limited to one contact, instead investigations have been performed how the contact models and their combination within the system may affect the global system dynamics. A general statement about the "correct" contact model can not be given.

Only a systematic variation of different models can provide a deeper insight into the

Table 2 Comparison of the computing times for different integration methods.

Integrator	System	Rel. CPU-time t_{rel}	
		Flexible bearings	Rigid bearings
LSODE (non-stiff) TOL = 10^{-6}	Inlet only	1.5	1.1
	1 module	8.4	6.0
	6 modules	98.0	76.5
LSODE (stiff) TOL = 10^{-6}	Inlet only	2.6	1.4
	1 module	25.6	14.6
	6 modules	> 1000	> 1000
DOPRI5 TOL = 10^{-6}	Inlet only	1.5	1.1
	1 module	10	5.5
	6 modules	term.	48.0
Time-stepping (rigid model) $\Delta t = 10^{-6}$	Inlet only	1.0	1.0
	1 module	2.7	2.8
	6 modules	16.6	16.8

system behavior. Even though rigid contact models might be insufficient at first sight, they are a powerful tool in order to blank certain effects (frequencies) where the source is not known beforehand. Moreover, the amount of unknown parameters can be reduced if a rigid contact model is suitable for a specific component.

Regarding the computing times, it is possible to achieve very fast simulations using time-stepping schemes, especially for large systems.

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