Analytical Study of Active Prosthetic Legs*

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Abstract

Walking with prosthesis has not been well analyzed mathematically and it seems that the design of powered prosthesis has been done empirically so far. This paper presents a dynamic simulation of a normal human walking and walking with an active prosthesis. We also studied the two controlling methods of a powered thigh prosthesis based on multi-body simulation of human walking. First we measured the normal human walking gait, then, we showed that a 3-DOF human walking model can walk on level ground by applying tracking control to the measured walking gait within a certain range of tuned walking period. Next, we applied the tracking control and self-excited control to the powered thigh prosthesis and compared the robustness and efficiency of the two control methods by numerical simulation. As a result, we found that the self-excited control can significantly decrease the hip joint torque and specific cost to 1/3 compared with the tracking control. Moreover, the self-excited control is superior to the tracking control because tuning for the walking period is not needed for the active prosthetic leg.

Key words: Biped Walking, Human Walking, Prosthesis, Powered Thigh Prosthesis, Active Prosthetic Leg, Simulation, Tracking Control, Self-Excited Control, Specific Cost, Robustness

1. Introduction

Prosthetic legs for disabled persons have been developed through the empirical skill and trials put forth by both medical doctors and designers for a long time. Recently there have been several studies conducted to perform mathematical analysis of walking with a prosthetic leg and to utilize the analytical results for making prosthetic legs and enhancing walking rehabilitation1-7). However, there have not been any mathematical studies done on a prosthetic leg with an active hip joint.

Patients which have prosthetic legs with passive hip and knee joints swing the artificial leg by swinging their hips, but it is difficult to generate an appropriate walking gait without risk of a collision or becoming unbalanced and falling. Since motion of the prosthetic leg is mainly determined by the initial conditions, it is not easy to achieve a stable walking gait. Thus, a reasonable degree of power and a controlled technique are required of the existent leg in order to achieve stable walking with a prosthetic leg. For this reason it is very important to research the use of an active prosthetic leg.

In terms of a simple control strategy for natural human walking, Ono et al.(8,9) proposed self-excited walking, by which a biped with passive knee joints can walk stably on level ground by using only one hip actuator. In this study we focused on the potential of an active thigh prosthetic leg based the self-excited walking principle. For comparison we also studied another control method where hip and knee joint are feedback controlled so as to follow the trajectories of the average person’s walking gait.

In Section 2, we first measured the common walking gait of an average person in order to
determine a reference for walking trajectories. In Section 3, we introduce a three-degree-of-freedom (DOF) biped model incorporating sagittal plane, walking algorithm, and basic equations. In Section 4 we show the simulation results of stable walking for the average, unimpeded person based on feedback control to the reference trajectories. In section 5, we describe the walking algorithm of an active prosthetic leg for both the tracking control and self-excited control methodologies. Following this, the simulation results of patients with two types of active prosthetic leg are comparatively described in terms of input torque, efficiency and robustness regarding variations in the walking period. In Section 5, conclusions are drawn from this study.


We first measured the walking motion of an average, unimpeded person by using a motion capture system in order to simulate the walking motion of a common human and to use it for reference trajectories for the healthy leg as it applies to walking with a prosthetic leg. From the measured three dimensional walking data, two-dimensional walking motion in a sagittal plane was obtained.

Figure 1(a) shows a walking model of the average human. If the hip joint angle and knee joint angle from the vertical line are denoted by \( \theta_2 \) and \( \theta_3 \) in a counterclockwise direction, respectively, as shown in Fig. 1(a), the measured values of \( \theta_2 \) and \( \theta_3 \) in the first three steps of natural walking are plotted in Fig. 1(b). The steady swing motion \( \theta_2 \) and \( \theta_3 \) of the swinging leg is referred to as the reference trajectories (\( \theta_{2r} \) and \( \theta_{3r} \)). From the motion capture data of the average human gait, we found that the motions of the ankle, knee and hip joints in normal human walking are very similar to those of a 3-DOF biped model without feet, under the assumption that the exchange of support and swing leg occur instantly. Therefore, numerical simulations for average, unimpeded walking and those for walking with an active thigh prosthetic leg were performed based on a 3-DOF biped model without feet.

3. Analytical Model and Basic Equations

3.1 Walking Algorithm and Three-Link Biped Model

Figure 2(a) shows the walking algorithm for one step of natural human walking, while Fig. 2(b) shows the walking algorithm for the three-link biped model. In natural human walking, \( t_0 \) is the period when both legs are on a ground, \( t_1 \) is the period from the start of the swing motion to knee collision, and \( t_2 \) is the period from knee collision to touch-down of the swinging leg.

Although human walking has three phases, it is assumed that the three-link model without feet has only two phases. The first phase includes period \( t_0 + t_1 \) because the tip of the
shank is regarded as being the ankle. Since the support leg is assumed to have no relative motion at the knee joint, the first phase can be modeled by three-links. The second phase involves period $t_2$, the same as natural human walking. The second phase biped motion is modeled by a two-link system assuming that the knee joint is fixed after knee collision.

### 3.2 Analytical Model and Basic Equations

Figure 3 shows the three-link analytical model used for analysis of the first phase walking motion in a sagittal plane. The support leg is assumed to have no relative motion between the thigh and shank, thus, it is represented by one link. The support leg and the thigh and shank of the swing leg are represented by links 1, 2 and 3, respectively. Actuators are located at both the hip and knee joints in the case of normal walking. Length, mass, moment of inertia about mass center, and position of mass center for link $i$ are denoted by $l_i$, $m_i$, $I_i$, and $a_i$, respectively.

Equations for translation motion of mass and rotational motion of inertia with respect to mass center were derived for the three links. By using the constrained equations, these six equations are reduced to three basic equations for angular displacement vector $[\theta_1, \theta_2, \theta_3]^T$, as follows:

$$
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & \dot{\theta}_1 \\
M_{21} & M_{22} & M_{23} & \dot{\theta}_2 \\
Sym & M_{32} & M_{33} & \dot{\theta}_3
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2 \\
\ddot{\theta}_3
\end{bmatrix}
+ 
\begin{bmatrix}
C_{12} & C_{13} & 0 \\
C_{21} & C_{22} & 0 \\
C_{31} & C_{32} & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
$$

Fig. 3 Three-link biped walking model
Table 1 Parameter values used for calculation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>link 1</th>
<th>link 2</th>
<th>link 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length $l_i$ [m]</td>
<td>0.86</td>
<td>0.40</td>
<td>0.46</td>
</tr>
<tr>
<td>Mass $m_i$ [kg]</td>
<td>28.8</td>
<td>8.82</td>
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<tr>
<td>Center of mass $a_i$ [m]</td>
<td>0.65</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>Moment of inertia $I_i$ [kgm²]</td>
<td>3.1</td>
<td>0.12</td>
<td>0.099</td>
</tr>
</tbody>
</table>
4.2 Simulation Results and Optimal Walking Conditions.

Although the measured time of one step of human walking was 0.65 s, the link parameter values in Table 1 may be different from those of the person measured. In addition, an optimal time for taking one step referring to the biped model may be different from 0.65 s. Therefore, we changed the one step time frame of the reference trajectories ($\theta_2$ and $\theta_3$) from that of the measured trajectories shown in Fig. 1(b). This time frame is termed reference period $T_r$ below. We simulated human walking motion by changing $T_r$ and feedback gains $k_2$ and $k_3$. Then we obtained the ranges of $T_r$, $k_2$, and $k_3$, by which stable walking can be achieved.

Figure 4 plots stable walking ranges for the reference period and one of the feedback gain values, keeping the other feedback gain values constant. Figure 4(a) shows the range of the reference period and $k_2$ in the 1st phase that enables stable walking when $k_3=2$ Nm/rad in the 1st phase and $k_2=30$ Nm/rad in the 2nd phase. We note from this figure that stable walking is possible for a reference period from 0.56 s to 0.78 s. With a large reference period, stable walking is possible for a wide range of $k_2$ from 10 Nm/rad to over 100 Nm/rad. Thus, $k_2=70$ Nm/rad was selected as the optimal gain because a relatively large range in the reference period for stable walking can be obtained without increasing the $k_2$ value. In this case, walking is stable for a reference period of 0.68 s ±0.1 s (±15%).

Under the condition where $k_2=70$ Nm/rad in the
1st phase and $k_3=30$ Nm/rad in the 2nd phase, region $k_2$ in the 1st phase and $T_r$ which enable stable walking, are shown in Fig. 4(b). From this figure we selected $k_3=2$ N/rad as it produced robust stable walking for a wide range of $T_r$.

Under the condition where $k_2=70$ Nm/rad and $k_3=2$ Nm/rad in the 1st phase, region $k_2$ in the 2nd phase and $T_r$ that enable stable walking, are plotted in Fig. 4(c). From this figure, we selected $k_2=30$ Nm/rad in the 2nd phase in terms of robustness and efficiency. Under this condition stable walking is possible for reference period range of $0.7\pm 0.08$ s ($\pm 11\%$).

Figure 5 shows a stick figure illustrating the walking simulation under the initial conditions described above. It is noted from this figure that walking becomes steady from the 4th step on. From these simulation results for natural human walking, it is considered that robust and efficient walking is possible when $k_2=70$ Nm/rad and $k_3=2$ Nm/rad in the 1st phase, and $k_2=30$ Nm/rad in the 2nd phase. Therefore, we used these feedback gain values for the tracking control of the healthy leg for walking with a prosthetic leg.

5. Simulation of Walking With Prosthetic Leg
5.1 Walking Algorithm and Control Method of Walking with Prosthetic Leg

Next let us consider the walking algorithm and control method for walking with an active prosthetic leg. Figure 6 illustrates one cycle of walking with a prosthetic leg. The upper section illustrates the case where the prosthetic leg is in the support phase while the normal leg is in the swing phase. This phase is divided into the 1st phase; from start of the swing motion to knee collision, and the 2nd phase; from knee collision to touch-down of the healthy leg. The lower section illustrates the case where the prosthetic leg is in the swing phase. This phase is divided into the 3rd phase; from the start of swinging to knee collision, and 4th phase; from knee collision to touch-down of the prosthetic leg.

We assume that the active prosthetic leg has a actuator at the hip and an active lock mechanism at the knee. In the 1st and 2nd phases in Fig. 6, we assume that the feedback control torque values of Eqs. (4) and (5) are input to the hip and knee of the normal swing leg. During this period, it is assumed that the knee joint of the supporting prosthetic leg is locked. On the other hand, in the 3rd and 4th phases where the prosthetic leg is in the swing motion, we assume that the knee is a passive joint and that control torque is input only to the hip joint. For control torque, we comparatively considered two control methods based on tracking control to the reference trajectory and self-excited control. In the tracking control method, the hip input torque is given by

$$T_2 = -k_1(\theta_2 - \theta_{2r})$$

In self-excited control method, the hip input torque is given by

$$T_2 = -k_2\theta_3$$

We simulated walking with an active prosthetic leg by using two different control inputs and compared the two methods in terms of power consumption and robustness of

![Fig. 6 Model of walking with prosthetic leg](image-url)
stable walking. When the healthy leg is in the swing phase, tracking control torque values of Eqs. (4) and (5) were applied to the hip and knee joints. The feedback gains used for normal leg control were $k_2=70$ Nm/rad and $k_3=2$ Nm/rad in the 1st phase and $k_2=30$ Nm/rad in the 2nd phase.

5.2 Comparison of Stable Walking Conditions

Fig. 7(a) and (b) plot a stable walking region of $k_t$ in the 3rd phase and $T_r$ based on the tracking control method. In Fig. 7(a), the control input in the 4th phase was zero by letting $k_t=0$ Nm/rad. In contrast, in Fig. 7(b), the feedback control torque with $k_t=20$ Nm/rad is input into the hip joint. From these figures, we note that the $k_t$ value that gives a wide range of $T_r$ is small in Fig.(b) when compared with that in Fig.(a). Thus, from Fig. 7(b) we chose $k_t=40$ Nm/rad in the 3rd phase and $k_t=20$ Nm/rad in the 4th phase for the most suitable control condition in terms of a robust and efficient tracking control method. Under these conditions, stable walking is possible for the reference period of $T_r=0.69\pm0.08$ s ($\pm12\%$).

Figures 8(a) and (b) show the stable walking region $k_s$ in the 3rd phase and $T_r$ based on a self-excited control method. In Fig.(a), no torque was input by putting $k_s=0$ in the 4th phase, whereas in Fig.(b), the self-excited control torque with $k_s=2$ Nm/rad in the 4th phase was applied to the hip joint. From these figures we note that the stable walking regions of $k_s$ in the 3rd phase and $T_r$ are almost the same as each other. Since keeping power consumption low for control of the prosthetic leg is the most important factor, we chose $k_s=17$ Nm/rad that gives the widest range of $T_r$ for the most suitable condition in terms of the robust and efficient tracking control. Under these conditions, stable walking is possible for the reference period of $T_r=0.7\pm0.06$ s.
From these results, it is known that the robustness to a variation of $T_r$ in self-excited control is worse than tracking control by 4%. However, it should be noted that input torque in Eq. (7) to the prosthetic leg does not contain either $\theta_2r$ or $\theta_3r$, so that input torque in self-excited control is not affected by $T_r$. Only the tracking control input to the able-side leg has a constrained condition for $T_r$. Since the walking period of the able-side leg can be automatically controlled to achieve stable walking under practical conditions, it is considered that the self-excited control is inherently robust to physical parameter values of the prosthetic leg.

From the above discussions, we determined the following conditions as standard control conditions: In the self-excited control, a feedback torque with $k_s=17$ Nm/rad was input into the prosthetic hip joint only in the 3rd phase. The reference period for the feedback torque in the swing motion of able-side leg was $T_r=0.70$ s. Conversely, in the tracking control a feedback control torque with $k_t=40$ Nm/rad in the 3rd phase and $k_t=20$ Nm/rad in the 4th phase were input into the prosthetic hip joint. The reference period was $T_r=0.69$ s. The stick figures under these conditions are comparatively illustrated in Fig. 9. As seen in these figures, it is noted that the walking gaits in both cases are very similar to that of natural human walking shown in Fig. 5. The walking motion becomes steady after 3 steps.

5.3 Comparison of Input Torque and Specific Cost

Next, let us compare input torque and efficiency pertaining to active prosthetic walking based on tracking control and self-excited control. Figure 10 shows the hip input torque of able-side leg and unable-side leg, comparing tracking control and self-excited control. Solid and dashed lines represent input torque values of unable-side leg and able-side leg, respectively. From these figures it is noted that the input torque of the unable-side leg under self-excited control is about one third of that under tracking control. Moreover, input torque of
the able-side leg under self-excited control is smaller than that under tracking control. This result is certainly very attractive because it is usual that the input torque to the able-side leg must be increased if the input torque to the unable-side leg is decreased. Although not shown here, it was also confirmed that the input torque to the able-side leg under self-excited prosthetic walking is almost equal to that of natural human walking, whereas the maximum input torque to the able-side leg under tracking control prosthetic walking is a little larger than that of natural human walking.

Generally, for evaluating the efficiency of locomotion, specific cost (SC) has been used as a figure of merit. SC is a non-dimensional value defined as,

\[
SC = \frac{P}{mgV} \quad (8)
\]

where \( m \) is total mass of the walking body, \( g \) is acceleration of gravity and \( V \) is the average walking speed. \( P \) is the average input power given in the form,

\[
P = \frac{1}{T} \int_{0}^{T} \dot{\theta} k \dot{\theta} dT \quad (9)
\]

where \( T \) is the walking period. In the following calculation, \( m=41 \) kg was used.

Figure 11(a) and (b) respectively show SC values of prosthetic walking with tracking control and self-excited control as a function of the feedback gain, taking reference period as a parameter. We note that the SC value in self-excited control takes 0.05 to 0.3, whereas that in tracking control takes 0.2 to 1.0. Therefore, it is clear that the SC in self-excited control is about one third of that in tracking control. From these results we can deduce that in terms of magnitude of input torque and walking efficiency, self-excited control is much better than tracking control.

6. Conclusion

Based on a numerical simulation of three-DOF human walking model, we first demonstrated that normal walking is possible by inputting the feedback torque proportional to the error from the reference trajectory measured natural walking gait if the walking period is properly selected. Stable, normal walking is possible for a ±11% variation of walking period of the reference trajectory. Next, we compared tracking control and self-excited control applied to an active prosthetic leg walking in terms of robustness to walking period variation and walking efficiency. As a result, it was found that tracking control allows a ±12% variation in walking period of reference trajectory compared with a ±8.6% in self-excited control. However, we should note that this tolerance of walking period is only necessary for the able-side leg in self-excited control, whereas the tolerance...
of ±12% variation in walking period is necessary for both able-side and unable-side legs using tracking control. Under practical conditions while walking with an active prosthesis leg, it is considered that the walking period of able-side leg will be autonomously optimized with either control methods. In self-excited walking, the unable-side leg does not need any reference period. Therefore, it can be said that self-excited control is more robust to the variation of walking period than tracking control. In terms of efficiency, input torque required of the prosthesis leg and specific cost, self-excited control is about one third that of tracking control. Accordingly, it is concluded that self-excited control is much more superior to tracking control for walking with an active prosthesis leg in terms of both robust stability and efficiency.

This study was done in Tokyo Institute of Technology from 2002 to 2004.

References