Forced Vibration Isolation System with Stiffness On-Off Control *

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Abstract
Semi-active vibration systems expend a small amount of energy to change the system parameters such as damping and stiffness. The variable damping and stiffness semi-active systems with base excitations have demonstrated excellent performances in the resonant and high frequency regions. For a forced vibration, it plays an important role in describing a building system with wind loading or a vehicle suspension with an engine vibration force; and the system stiffness has a large influence on the frequency response in the low frequency region. However, forced vibration systems with variable stiffness control have not been received significant attention. To address this issue, the performance of a variable stiffness system with a force excitation is studied in theoretical calculations and experiments. The responses to sinusoidal and random inputs showed that the system with stiffness on-off control had advantages of both soft and stiff systems in the whole frequency.

Key words: Variable Stiffness, MR Damper, Semi-Active Control, Forced Vibration

1. Introduction

In recent years, vibration isolation systems have been studied broadly and in great depth. Traditionally, based on mechanism and energy requirements, vibration control can be categorized as: passive, active or semi-active. Semi-active control systems fill the gap between passive and active control systems and represent a compromise between performance improvement and simplicity of implementation. They only expend a small amount of energy to change the system parameters such as damping and stiffness. Although the idea of using variable damping in vibration systems has been shown by many researchers to provide effective vibration control (1)-(4), there is still room for further improvement because the system stiffness is often fixed. Variable stiffness systems have been studied by a few researchers(5),(6).

Kobori proposed a variable stiffness system to suppress the building’s response to earthquakes(6). The aim of Kobori’s work was to achieve a non-stationary and no resonant state during earthquakes. Youn and Hac used an air spring in a suspension system to vary the stiffness among three discrete values(5). A change of stiffness occurred only when the required control force could not be generated by variable damping alone. The vehicle system with variable stiffness demonstrated a good performance compared to a semi-active system with a variable damping and fixed stiffness. The authors of this paper had proposed...
a structure using two Voigt elements (each one is composed of a controllable damper and a constant spring) in series to realize the variable damping and stiffness. The structure was experimentally implemented using two magnetorheological (MR) fluid dampers for the controllable dampers. The sinusoidal and random responses of one degree-of-freedom (DOF) and 2-DOF systems showed that the system with damping and stiffness on-off control had a good vibration isolation performance. Because two dampers were in series in the proposed structure, the damping and stiffness could not be changed independently. An improved variable stiffness system using a Voigt element and a spring in series and a damper in parallel was proposed. The responses to sinusoidal and random base excitations showed that system with damping and stiffness on-off control had demonstrated excellent performances.

However, for a forced vibration, the stiffness dominates the system transmissibility in the low frequency region. If the stiffness is small, the transmissibility becomes large; inversely, if the stiffness is large, the transmissibility is small in the low frequency region. Therefore, it is necessary to study the performance of the variable stiffness system with a force input. In this paper, the performance of the system with a force excitation is studied by theoretical calculations and experiments.

2. Forced vibration system with variable stiffness

2.1 Forced system with variable stiffness

A one-degree-of-freedom (1-DOF) variable stiffness system with an exciting force $F$ is shown in Fig. 1 (a). In this system, there are two dampers, damper 1 and 2 (corresponding damping coefficients of $c_1$ and $c_2$), and two springs, spring 1 and 2 (corresponding stiffness of $k_1$ and $k_2$). The stiffness values of the two springs are not varied; however, the apparent total stiffness of the system can be varied by changing the controllable damping coefficient $c_2$. Damper 2 and spring 2 comprise a Voigt element. The Voigt element and spring 1 are in series. The variable $x_m$ is the displacement of the point between the Voigt element and spring 1; and $\dot{x}$ is the response of mass $m$. The equivalent model of this system is shown in Fig. 1 (b). Here $k'$ and $c'$ are equivalent spring stiffness and damping coefficient, respectively.

If $c_2$ is small, the total system stiffness approaches the series stiffness of $k_1$ and $k_2$ and it is soft. However, if $c_2$ is large, the total stiffness approaches the stiffness $k_1$, and it becomes stiff. Therefore, modulating damper 2 can be used to realize variable stiffness for the system. Damper 1 provides damping for the system.

2.2 Equations of motion

The equations of motion for the system shown in Fig. 1 (a) are

\[ m\ddot{x} = -k_1(x - x_m) - c_2(x - \dot{x}_m) - c_1\dot{x} - F, \]

\[ k_1x_m = k_2(x - x_m) + c_2(\dot{x} - \dot{x}_m), \]

where $\dot{x}_m$ is the velocity of the point between the Voigt element and spring 1; $\dot{x}$ and
\[ \dot{x} \] are the acceleration and velocity of the mass, respectively. The transfer function of the system is

\[ \frac{X}{F_0} = \frac{-1}{-m\omega^2 + [k_1 - \frac{k_1^2 (k_1 + k_2)}{(k_1 + k_2)^2 + \omega^2}] + \frac{k_1^2 c_2}{(k_1 + k_2)^2 + \omega^2} \omega} \]

where \( F = F_0 e^{i \omega t} \), \( x = X e^{i \omega t} \), and \( \omega \) is the excitation frequency. Comparing Eq. (3) with the transfer function of the equivalent model shown in Fig. 1 (b)

\[ \frac{X'}{F_0} = \frac{-1}{-m\omega^2 + k' + ic' \omega} \]

the equivalent spring stiffness and damping coefficient are

\[ k' = k_1 - \frac{k_1^2 (k_1 + k_2)}{(k_1 + k_2)^2 + \omega^2}, \]

\[ c' = c_1 + \frac{k_1^2 c_2}{(k_1 + k_2)^2 + \omega^2}. \]

Based on Eqs. (5) and (6), \( k' \) is independent of \( c_1 \), and \( k' \) and \( c' \) are influenced by \( c_2 \). When \( c_2 = \infty \) (Damper 2 is in the on-state), \( k' = k_1 \) and \( c' = c_1 \). When \( c_2 = 0 \) (Damper 2 is in the off-state), \( k' = k_1/k_2(k_1 + k_2) \) and \( c' = c_1 \). Therefore, when damper 2 is on and off controlled, the system stiffness can be varied from soft to stiff. Figure 2 shows how \( k' \) and \( c' \) can be modified by varying the damping ratio \( \varsigma_2 \) (\( \varsigma_2 = c_2/2\sqrt{mk_2} \)). In this example, \( \varsigma_1 = 0.1, 0.3 \) and \( 0.5 \) (\( \varsigma_1 = c_1/2\sqrt{mk_1} \)); \( k_1/k_2 = 1/3, m = 1 \) kg (These values give the natural frequencies of 0.5 Hz for \( \varsigma_2 = 0 \) and 1 Hz for \( \varsigma_2 = \infty \)). The values of \( k_1, k_1/k_2(k_1 + k_2) \), and corresponding \( c_1 \) are also shown by dotted lines in the figures. Based on Fig. 2 (a), \( k' \) can be changed from \( k_1/k_2(k_1 + k_2) \) to \( k_1 \) by varying \( \varsigma_2 \). Moreover, while varying \( \varsigma_2 \), \( c' \) also changes a little as shown in Fig. 2 (b).

![Fig. 2](image-url)

**3. Control algorithms**

The force produced by damper 2 is controlled according to the following

\[ f_{d2} = \begin{cases} -c_{2on}(\dot{x} - \dot{x}_m) & \text{if } \dot{x} \dot{x}_m > 0 \\ -c_{2off}(\dot{x} - \dot{x}_m) & \text{if } \dot{x} \dot{x}_m \leq 0 \end{cases} \]

where \( f_{d2} \) is the damping force of damper 2, the damping coefficient \( c_2 \) is equal to \( c_{2on} (=2\varsigma_{2on}m/o2) \) in the on-state and \( c_{2off} (=2\varsigma_{2off}m/o2) \) in the off-state. Herein the natural frequency \( o_{n2} = \sqrt{k_2/m} \). Since in this study, a magnetorheological (MR) fluid damper is used for \( c_2 \), \( \varsigma_2 \) can not be zero and \( \infty \). The damping ratios \( \varsigma_{2off} = 0.1 \), and \( \varsigma_{2on} = 10 \) are used in the following analysis. Three types shown in Table 1 are investigated in this study. For Type 1, damper 2 are always in the off-sates and the total stiffness is small (called “Soft system”). For Type 2, damper 2 is always in the on-state and the total stiffness is large (called “Stiff system”). For Type 3, damper 2 is on-off controlled as given by Eq. (7).
Consider a sinusoidal force excitation as

\[ F = F_0 \sin(\omega t), \]  

(8)

where \( t \) is the time, \( F_0 \) is the amplitudes of \( F \). Figure 3 shows the transmissibility, \( |X/F_0| \), of the system with damping ratios of damper 1, \( \zeta_1 = 0.01, 0.1, 0.3 \) and 0.5 (\( c_1 = 2\zeta_1m\omega_n, \omega_n = \sqrt{k_1/m} \)). In this example, \( F_0 = 0.04\pi^2 \) N, \( k_1 = 4\pi^2 \) N/m, \( k_2/k_1 = 1/3 \), and \( m = 1 \) kg. These values give the system natural frequencies of about 0.5 Hz for \( \zeta_2 = 0.1 \) and 1 Hz for \( \zeta_2 = 10 \).

According to Fig. 3, the response of soft system is larger than that of “S on-off” control system in the low frequency region. The response of “S on-off” control system is smaller than that of stiff system in the high frequency region. At two resonances (0.5 Hz and 1 Hz), there are no resonant responses of “S on-off” control system. When damping ratio of damper 1 increases, the response of “S on-off” control system is decreased in the low frequency region. Therefore, the “S on-off” control system has the advantages of both soft and stiff systems in the whole frequency region.
4. Experiments

4.1 Experimental setup

Figure 4 shows the experimental setup with a force excitation. Two magnetorheological (MR) fluid dampers (RD 1097 made by Lord Cooperation) are used to implement the system control. When the applied current $I_2$ of the damper 2 is zeros, the damping ratio is small and the total stiffness becomes small. When $I_2$ is large, the total stiffness approaches spring 1, which is hard. Damper 1 applied current $I_1$ is used to provide damping for the system.

In the experiment, the mass is supported by leaf springs (spring 1 and 2); and it is shaken in horizontal direction using an electromagnetic vibration exciter and a signal generator. The exciting force $F$ is measured by a force transducer which is installed between a connecting rod and a fixed structure on the exciter platform. The displacement $x$ is measured by a laser displacement sensor. The velocity $\dot{x}$ is obtained by differentiating the displacement in the controller. The software used to implement the control algorithm is MATLAB 6.5 with Simulink. The A/D and D/A boards are dSPACE (DS1104).

4.2 Parameter values in experiments

Values for the parameters shown in experimental setup are listed in Table 2. The mass of the experimental structure is included in $m$. In the experiments, the currents applied on damper 2, $I_{2on}=0.48$ A and $I_{2off}=0.0$ A, are used to realize large and small stiffness for the system. The currents applied on damper 1, $I_{1on}=0.19$ A and $I_{1off}=0.0$ A, are used to provide...
high and low damping. In general, MR fluid dampers are friction dampers\(^8\). Therefore, the equivalent damping coefficients are obtained by experiments. The damping coefficients \(c_{\text{off}}\), \(c_{\text{on}}\), \(c_{\text{off}}\), and \(c_{\text{on}}\) are shown in Table 2.

### Table 2 Parameter values in the experiment.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values (Ns/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>10.5 kg</td>
<td>(c_{\text{off}})</td>
<td>3.62 \times 10</td>
</tr>
<tr>
<td>(k_1)</td>
<td>4.68 \times 10^3 N/m</td>
<td>(c_{\text{on}})</td>
<td>1.55 \times 10^2</td>
</tr>
<tr>
<td>(k_2)</td>
<td>2.51 \times 10^3 N/m</td>
<td>(c_{\text{off}})</td>
<td>2.01 \times 10</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>10.8°</td>
<td>(c_{\text{on}})</td>
<td>6.00 \times 10^3</td>
</tr>
<tr>
<td>(\theta_2)</td>
<td>42.9°</td>
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</tbody>
</table>

### Table 3 Values of equivalent stiffness and damping coefficients in the experiment.

<table>
<thead>
<tr>
<th>Damper 1</th>
<th>Damper 2</th>
<th>(k') (N/m)</th>
<th>(\zeta') (Ns/m)</th>
<th>(f_n) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>off</td>
<td>off</td>
<td>1.64 \times 10^3</td>
<td>4.47 \times 10</td>
<td>0.170</td>
</tr>
<tr>
<td>off</td>
<td>on</td>
<td>4.67 \times 10^3</td>
<td>4.44 \times 10</td>
<td>0.100</td>
</tr>
<tr>
<td>on</td>
<td>off</td>
<td>1.64 \times 10^3</td>
<td>1.64 \times 10^2</td>
<td>0.622</td>
</tr>
<tr>
<td>on</td>
<td>on</td>
<td>4.67 \times 10^3</td>
<td>1.63 \times 10^2</td>
<td>0.368</td>
</tr>
</tbody>
</table>

Based on Eqs. (6) and (7), when \(\omega = \omega_n\), the values of \(k'\), \(\zeta'\), damping ratio \(\zeta'\) and resonance frequency \(f_n\) are shown in Table 3. Herein \(\zeta' = c'/\sqrt{mk'}\) and \(f_n = \sqrt{k'/m}/2\pi\). According to Table 3, the maximum value of \(k'\) is more than 2.8 times of the minimum value of \(k'\). Damper 1 in the off and on states provides different damping values for the system.

### 4.3 Experimental results

#### 4.3.1 Frequency responses to a sinusoidal excitation

Figures 5 and 6 show the system steady-state responses of \(X/F_0\) in the theoretical calculation and experiments when damper 1 is in the off and on states. The force amplitude is 30 N and the frequency is from 1 Hz to 10 Hz of a sinusoidal wave.

![Frequency responses to a sinusoidal excitation](image)

**Fig. 5** Frequency responses to a sinusoidal excitation (damper 1: off).
According to Figs. 5 and 6, the calculation results are in agreement with the experimental results. The response of soft system is larger than that of “S on-off” control system in the low frequency region and the response of “S on-off” control system is smaller than that of stiff system in the high frequency region. At two resonances (about 2 Hz and 3.35 Hz), there are no resonant responses of “S on-off” control system. Therefore, the system with “S on-off” control has good performances in the whole frequency region.

**4.3.2 Responses to a random excitation**

Figure 7 show the frequency responses of the system with a random force excitation. The values of $\frac{X}{F_0}$ are derived from power spectral density (PSD) values of $X$ divided by that of $F$. The PSD values of the displacement and force are obtained by averaging the experimental data over 15 minutes. The time history of exciting force and PSD value of the excitation are also shown in the figure.
The frequency responses of the systems with the random input shown in Fig. 7 are similar to those of the system with the sinusoidal input shown in Figs. 5 and 6. At two resonances corresponding to the soft and stiff systems, the responses of “S on-off” control do not exhibit the resonant peaks. The system with “S on-off” control has good frequency responses to a random force excitation. Figure 8 shows the time responses of the displacement to the random force excitation. The root mean square (RMS) values are shown in Table 4. Based on Fig. 8 and Table 4, the system with “S on-off” control has good responses to the random excitation.

![Graphs showing time responses to a random force excitation](image)

Fig. 8  Time responses to a random force excitation.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>RMS values of the displacement (x: mm).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Damper 1: off</td>
</tr>
<tr>
<td>Soft system</td>
<td>2.78</td>
</tr>
<tr>
<td>Stiff system</td>
<td>3.54</td>
</tr>
<tr>
<td>S on-off</td>
<td>2.33</td>
</tr>
</tbody>
</table>

5. Conclusions

A forced vibration system with stiffness on-off control was studied in theoretical calculations and experiments. The calculation and experimental results were in good agreement. The time and frequency responses to sinusoidal and random force excitations were studied for three types of control system: soft system, stiff system and stiffness on-off control system. In the experiments, magnetorheological fluid dampers were used as the controllable dampers.

Based on the calculation and experimental results, the response of “S on-off” was smaller than that of soft system in the low frequency region, moreover, it was smaller than
that of stiff system in the high frequency region. Therefore, the system with stiffness on-off control had the advantages of both soft and stiff systems in the low and high frequency regions. At two resonances corresponding to the soft and stiff systems, the responses of “S on-off” control do not exhibit the resonant peaks.

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